Human Capital Formation and Cross-Country Comparison of Inequality*

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Abstract  
The paper studies the effects of cross-country differences in human capital formation on income distribution and growth. Our overlapping generations economy has the following features: (1) consumers are heterogenous with respect to parental human capital and ability; (2) intergenerational transfers take place via parental education and, public investments in education financed by taxes (possibly, with a level determined by majority voting); (3) due to investment in human capital, which is a factor of production, we have endogenous growth. Besides exploring several cross-country variations in the production of human capital, some attributed to ‘home-education’ and others related to ‘public-education’, we indicate how the level of public education can lead to poverty traps and affect the intragenerational income inequality along the equilibrium path.

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1 Introduction

The well-documented literature on the growth performance of countries has tried to substantiate a negative relationship between inequality and growth. Early tests of this hypothesis by Alesina and Rodrick (1994), Persson and Tabellini (1994) and others report evidence of an inverse relationship, while more recent empirical findings, for instance, by Barro (2000) and Forbes (2000), suggest a positive relationship instead. From an empirical point of view this lack of consensus is not surprising as the variations in inequality and growth are too different to obtain robust relations between the two [see Quah (2002a)]. While the causality between inequality and growth and the functional form relating them are well-studied issues, other important observations from the data suggest a number of open theoretical questions. First, the data show significant shifts of the distribution of income in the more recent past for various countries [Quah (2002b)]. Second, while the Gini coefficients within countries vary little around their mean over time [see, Deininger and Squire (1998)], a high variation in inequality measures is observed across countries. For example, data on US income reveal a Gini coefficient of 0.401 and 0.426 in 1989 and 1994 respectively. For Chile, the corresponding coefficients are 0.573 and 0.565 [see, Tabatabai (1996)]. Therefore, the questions of what determines income inequality in equilibrium and why it differs across countries assume considerable importance. This paper focuses on the process of human capital formation to examine these issues.

Statistical offices of international organizations compile extensive lists of indicators that compare scholastic achievements across countries. A primary common element of these indicators is that education, learning and acquisition of knowledge occur differently in various parts of the world. The involvement of parents, the level and efficiency of public education, the human capital of teaching staff, the use of existing technologies (like internet) vary across countries and differences can be large. If we believe that processes describing the accumulation of human capital affect output and income distribution, then we need to find out how they may matter.
This is the purpose of this paper.

We consider an overlapping generations economy that produces a single good using two types of production factors: physical capital, and human capital represented by a continuum of skills. Each individual lives for three periods, where during the 'youth' period (in which no economic decision is made) education is acquired. Intergenerational transfers in our economy take place via two channels: investments made by parents in educating their own offspring at home and the provision of public education financed by taxing wage incomes. Home education is provided by the close family and carried out mainly through parental tutoring, social interaction, learning devices available at home (such as computer and the internet), etc. In this case the human capital of parents and the time they dedicate to tutoring are important factors. Public education includes formal education in schools, public expenditure related to schooling, the 'outside' social interactions and other activities like the media etc. A government has two tasks in our economy: first, in organizing public education and determining its level and, second, in financing it by taxing wage incomes. In our framework the level of public education represents the effective educational inputs related to teaching and not the public education expenditures.

Our framework allows for the following properties to hold in equilibrium: (a) in some cases utility maximization leads some parents not to participate to the education of their own child, which is a stylized fact of some countries; (b) the mere choice of the public education level allows, under some conditions, for a poverty trap although we do not assume scale economies; (c) due to investments in human capital the economy exhibits endogenous growth; (d) a political equilibrium regarding the level of public education readily follows (using the median-voter theorem).

We show that traditional explanatory factors of income inequality like international trade and technological progress in production play no role in our model in affecting the equilibrium distribution of income. In contrast, initial endowments matter in the sense that a country that starts from a lower level of human capital, not necessarily less equal, has a better chance to experience more inequality over time. Nevertheless, trade and physical capital mobility that are based on these differences in endowments do not affect the income inequality of both countries, although intertemporal effects on output and welfare exist. These results single out processes of human capital formation as one of the main explanations for income inequality differences.

We show that when the government is absent from the education process our economy
generates an endogenously determined intragenerational income distribution. Inequality em-
anates, in this case, from the innate ability, the heterogeneity in parents’ human capital (due
to their role in home education) and it is independent of initial conditions. The contribution of
public education in our framework is to dampen differences arising from families’ human capital
and thereby reduce inequality in the distribution of human capital and income. Put differently,
if one compares two countries that are similar in all respects except for the level of public edu-
cation, the country that invests less in public schooling faces higher income inequality along the
whole equilibrium path. Moreover, we show that when the level of public education is ”low”
the economy may converge to a 'poverty trap'; namely, the stock of human capital declines
over time. On the other hand, higher levels of public education guarantee that the aggregate
human capital increases over time. This claim holds for any given (positive) provision of public
schooling and it is only reinforced if this level is determined under majority voting.

In this work we take no explicit stance regarding the causality between inequality and
growth. Basically, we point out that the way in which countries enhance human capital matters:
If the gap between countries is mainly in the 'home' component of human capital formation it
results in higher growth while income inequality rises. In contrast, when this gap occurs in the
'public' part, then the higher growth is accompanied by less income inequality.

The remainder of the paper is organized as follows. The next section examines the litera-
ture. Section 3 presents an OLG model with heterogenous agents and analyzes the properties of
the model. Section 4 studies cross-country variations in education systems on intragenerational
income inequality. Section 5 quantifies the response of income inequality to various education
systems. To that end, a dynamic computable general equilibrium model is developed on the
basis of our theory and calibrated on statistics from the Netherlands over the period 1975-2000.
Section 6 concludes. To facilitate the reading we relegate all proofs to the Appendix.

2 Related Literature

Our aim is to study the cross-country differences in income distribution, the components of
education and the formation of human capital. The cross-country empirical evidence uses
various measurements of ’quality and education’, ignoring some important features of the human
capital production process. Becker and Chiswick (1966) demonstrate (in the US) that income
inequality is positively correlated with schooling inequality and negatively correlated with the
average level of schooling. Later, based on cross-section data from nine countries, Chiswick (1971) shows that earnings inequality increases with educational inequality. Later studies, based on larger sample of countries, support this result showing as well that higher level of schooling reduce income inequality [see, e.g., Adelman and Morris (1973), Chenery and Syrquin (1975)].

Though human capital formation is a complex process, economic models have assumed some particular mechanisms describing it. Due to tractability reasons, these processes concentrate on very few parameters [see, e.g., Eckstein and Zilcha (1994), Orazem and Tesfatsion (1997), Hanushek (2002)].¹ In our framework the production function for human capital exhibits two important properties. First, individuals from below-average human capital families have a greater return to investment in public schooling than those from above-average families. In addition, the effort, and therefore cost, of acquiring human capital for the younger generation is smaller for societies endowed with relatively higher levels of human capital [see, e.g., Tamura (1991), Fischer and Serra (1996)]. Second, the importance of parental human capital in forming the human capital of a child has been established [see, e.g., Hanushek (1986)]. For example, Glaeser (1994) divides the education’s positive effects on economic growth into parts, and concludes that children in families with educated parents obtain a better education than children without support. Also, Burnhill et al. (1990) find that parental education influences entry to higher education in Scotland over and above the influence of parental social class. More recently, Lee and Barro (2001) find that family characteristics, such as income and education of parents, enhance student’s performance. A reason that is put forward is that parental education elicits more parental involvement (including related private investment) at home.

Income distribution is a key economic issue and a large literature has improved our understanding of its underlying determinants. Besides trade and technical progress, some believe that social norms are crucial determinants of earnings inequality [e.g., Atkinson (1999), Corneo and Jeanne (2001)]. Others have thoroughly studied the role of human capital accumulation on income distribution in various contexts [see, e.g., Loury (1981), Becker and Tomes (1986), Galor and Zeira (1993), Benabou (1996), Chiu (1998), Fernandez and Rogerson (1998)]. However, as information technology advances and computers are being integrated into the learning technology, new issues like the increasing technological contribution to the process of learning arise. These technological changes do not affect the formation of human capital similarly in the

¹A different approach was used by Eicher (1996), who assumes endogenous absorption of new technologies into the production process.
various countries. We distinguish between cross-country technological gaps which affect mostly the 'home-component' vs. technological differences which affect mainly the 'public-component' of the education process.

The literature also contains work on how education systems come about. For example, Glomm and Ravikumar (1992) establish that majority voting results in a public educational system as long as the income distribution is negatively skewed. Cardak (1999) strengthens this result by considering a voting mechanism where the median preference for education expenditure, rather than median income household, is the decisive voter. The equilibrium we consider in Sections 4 and 5 is an application of the median-voter theorem.

Our model assumes that a continuum of skills is used in the production of a single final good. It does not consider an arbitrary segmentation of the labor market between skilled and unskilled worker like in studies which look at the time pattern of the wage of skilled labor relative to that of unskilled labor [see, e.g., Slaughter (1998)]. Nevertheless, this so-called skill premium can be analyzed both theoretically and numerically in our framework by comparing any two agents along the equilibrium path.

As was demonstrated in various ways endogenous growth models provide an extremely efficient analytical tool in studying issues related to growth, convergence and income inequality in equilibrium [see, e.g., Loury (1981), Tamura (1991), Glomm and Ravikumar (1992), Fischer and Serra (1996), Fernandez and Rogerson (1998), van Marrewijk (1999), Galor and Moav (2000), Viaene and Zilcha (2002)]. The main emphasis has been on the role played by human capital as an engine for growth [see, e.g., Razin (1973), Lucas (1988), Azariadis and Drazen (1990)]. Our model in the stationary state is an AK-model where all variables grow at the same rate as effective labor. However, we consider only non-stationary competitive equilibria.

3 The Model

3.1 Human Capital Formation

Consider an overlapping generations economy with a continuum of consumers in each generation, each lives for three periods. During the first period each child is engaged in education/training, but takes no economic decision. Individuals are economically active during the working period which is followed by the retirement period. We assume no population growth,
hence population is normalized to unity. At the beginning of the ‘working period’, each parent gives birth to one offspring. Each household is characterized by a family name \( \omega \in [0, 1] \). Denote by \( \Omega = [0, 1] \) the set of families in each generation and by \( \mu \) the Lebesgue measure on \( \Omega \).

Agents are endowed with *two units* of time in their second period. One unit is inelastically supplied to labor, while the other is allocated between leisure and self-educating the offspring. Though the supply of labor is inelastic, each family’s supply of human capital is the result of utility maximization. Consider generation \( t \), denoted \( G_t \), namely all individuals \( \omega \) born at the outset of date \( t - 1 \), and let \( h_t(\omega) \) be the level of human capital of \( \omega \in G_t \).

We assume that the production function for human capital is composed of two components: informal education initiated and provided by parents at home and public education provided by the government by hiring ‘teachers’, constructing schools etc. The ‘home-education’ depends on the time allocated by the parents to this purpose, denoted by \( e_t(\omega) \), and the ‘quality of tutoring’ represented by the parent’s human capital level \( h_t(\omega) \). The time allocated to public schooling ( i.e., the level of public education) is denoted by \( e_{gt} \). The human capital of the teachers determine the ‘quality’ of ‘public education’ in the formation of the younger generation’s human capital. We also assume that the (random) innate ability of individual \( \omega \in G_{t+1} \), denoted by \( \theta_t(\omega) \), is known when parents make their decision about investment in education. Moreover, all the random variables \( \theta_t(\omega) \) across individuals and across generations are i.i.d., hence, without loss of generality, we take each \( \theta_t(\omega) \) to be distributed as some random variable \( \tilde{\theta} \). Let \( \tilde{\theta} \) assume values in \( [\underline{\theta}, \overline{\theta}] \), where \( 0 < \underline{\theta} < \overline{\theta} < \infty \), and denote its mean by \( \overline{\theta} \) where, without loss of generality, \( \overline{\theta} = 1 \). We assume that for some constants \( \beta_1 > 1, \beta_2 > 1, v > 0 \) and \( \eta > 0 \), the evolution process of a family’s human capital is given as follows. For all \( \omega \in G_{t+1} \):

\[
h_{t+1}(\omega) = \theta_t(\omega)[\beta_1 e_t(\omega)h_t^v(\omega) + \beta_2 e_{gt}\overline{h}_t^{\eta}]
\]

(1)

where the human capital involved in public schooling, denoted \( \overline{h}_t \), is the *average* human capital of generation \( t \). This is justified if we assume that instructors in each generation are chosen randomly from the population of that generation. The parameters \( v \) and \( \eta \) measure the externalities derived from parents’ and society’s human capital respectively. The constants \( \beta_1 \) and \( \beta_2 \) represent how efficiently parental and public education contribute to human capital: \( \beta_1 \) is affected by the home environment while \( \beta_2 \) is affected by facilities, the schooling system, size of classes, neighborhood, social interactions, and so forth.

The production function for human capital given by (1) exhibits the property that
public education dampens the family attributes. As it is common to all, individuals from below-average families have, therefore, a greater return to human capital derived from public schooling than those born to above-average human capital families. In addition, the effort of acquiring human capital is smaller in countries endowed with relatively higher levels of human capital. An important difference between our process of human capital acquisition and most cases discussed in the literature is the representation of the private and the public inputs in the production of human capital via allocation of time. Our approach suggests that the *time spent learning*, coupled with the human capital of the instructors, and not the expenditures on education, should be the relevant variables in such a process. This is in line with Hanushek (2002) who argues in favor of considering the ‘efficiency’ in the public education provision rather than ‘expenditure’ on public education. This distinction is important since in a dynamic framework the cost of financing a particular level of human capital fluctuates with relative factor rewards.

Consider the lifetime income of individual \( \omega \), denoted by \( y_t(\omega) \). Since the human capital of a worker is observable and constitutes the only source of income, it depends on the effective labor supply. Let \( w_t \) be the wage rate in period \( t \) and \( \tau_t \) is the tax rate on labor income, then

\[
y_t(\omega) = w_t(1 - \tau_t)h_t(\omega)
\]

Under the public education regime the taxes on incomes are used to finance education costs of the young generation. Making use of (1) and (2), balanced government budget means:

\[
\int_{\Omega} w_t e_{gt} h_t d\mu(\omega) = \int_{\Omega} \tau_t w_t h_t(\omega) d\mu(\omega)
\]

or equivalently,

\[
e_{gt} = \tau_t
\]

that is, the tax rate on labor is equal to the proportion of the economy’s effective labor used for public education.\(^2\)

\(^2\) Under a decentralized system, namely under a fully private education regime, both \( \tau_t(\omega) \) and \( e_{gt}(\omega) \) are decision variables of each agent, hence the individual’s budget constraint on private education is: \( \tau_t(\omega)w_t h_t(\omega) = w_t e_{gt}(\omega) \bar{h}_t \), where the level of teachers’ instruction \( e_{gt}(\omega) \) is chosen freely while their average human capital is the same as their corresponding generation.
3.2 Equilibrium

Production in this economy is carried out by competitive firms that produce a single commodity, using effective labor and physical capital. This commodity is both consumed and used as production input. There is a full depreciation of physical capital. The per-capita effective human capital in date \( t \), \( h_t \), is an input in the aggregate production process. In particular we take the (per-capita) production function to be:

\[
q_t = F(k_t, (1 - e_{gt})h_t) \quad (4)
\]

where \( k_t \) is the capital stock and \((1 - e_{gt})h_t = (1 - \tau_t)h_t \) is the effective human capital used in the production process. \( F(\cdot, \cdot) \) is assumed to exhibit constant returns to scale, it is strictly increasing, concave, continuously differentiable and satisfies \( F_k(k_t, 0) = \infty \), \( F_h(k_t, 0) = \infty \), \( F(0, (1 - \tau_t)h_t) = F(k_t, 0) = 0 \).

Given the public education provision, agent \( \omega \) at time \( t \) maximizes lifetime utility, which depends on consumption, leisure and income of the offspring. Thus:

\[
\max_{e_t, s_t} u_t(\omega) = c_{1t}(\omega)^{\alpha_1} c_{2t}(\omega)^{\alpha_2} y_{t+1}(\omega)^{\alpha_3} [1 - e_t(\omega)]^{\alpha_4} \quad (5)
\]

subject to

\[
c_{1t}(\omega) = y_t(\omega) - s_t(\omega) \geq 0 \quad (6)
\]

\[
c_{2t}(\omega) = (1 + r_{t+1}) s_t(\omega) \quad (7)
\]

where \( h_{t+1}(\omega) \) and \( y_{t+1}(\omega) \) are given by equations (1) and (2). The \( \alpha_i \)'s are known parameters and \( \alpha_i > 0 \) for \( i = 1, 2, 3, 4 \); \( c_{1t}(\omega) \) and \( c_{2t}(\omega) \) denote, respectively, consumption in first and second period of the individual’s economically active life; \( s_t(\omega) \) represents savings; leisure is given by \( (1 - e_t(\omega)) \); \( (1 + r_{t+1}) \) is the interest factor at date \( t \). The offspring’s income \( y_{t+1}(\omega) \) enters parents’ preferences directly and represents the motivation for parents’ investment in tutoring and formal education expenditure.

Given some tax rates \( (\tau_t) \), initial human capital distribution \( h_0(\omega) \) and \( k_0 \), a competitive equilibrium is \( \{e_t(\omega), s_t(\omega), k_t, w_t, r_t\} \) which satisfies: For all \( t \) and all individuals \( \omega \in G_t \), \( \{e_t(\omega), s_t(\omega)\} \) are the optimum to the above problem given \( \{w_t, r_t\} \). And, the following market clearing conditions hold:
\[ w_t = F_h(k_t, (1 - e_{gt})h_t) \]  
(8)

\[ (1 + r_t) = F_k(k_t, (1 - e_{gt})h_t) \]  
(9)

\[ k_{t+1} = \int_\Omega s_t(\omega) d\mu(\omega) \]  
(10)

Equations (9) and (10) are the clearing conditions on factor markets. Condition (11) is a market clearing condition for physical capital, equating the aggregate capital stock at date \( t + 1 \) to the aggregate savings at date \( t \). After substituting the constraints, the first-order conditions that lead to the necessary and sufficient conditions for an optimum are:

\[ \frac{c_{1t}}{c_{2t}} = \frac{\alpha_1}{\alpha_2(1 + r_{t+1})} \]  
(11)

\[ \frac{\alpha_4}{(1 - e_t(\omega))} = \frac{\beta_1 \alpha_3(1 - \tau_{t+1})w_{t+1}h_t^i(\omega)\theta_{t}(\omega)}{y_{t+1}(\omega)}, \quad \text{if} \quad e_t(\omega) > 0 \]  
(12)

\[ \geq \quad \text{if} \quad e_t(\omega) = 0. \]  
(13)

From (7), (8) and (11) we obtain:

\[ c_{1t}(\omega) = \left(\frac{\alpha_1}{\alpha_1 + \alpha_2}\right) y_t(\omega) \]  
(14)

\[ s_t(\omega) = \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right) y_t(\omega) \]  
(15)

Equation (12) allocates the unit of nonworking time between leisure and the time spent on education by the parents. In fact, we find that whenever \( e_t(\omega) > 0 \):

\[ e_t(\omega) = \left(\frac{\alpha_3}{\alpha_3 + \alpha_4}\right) \left[1 - \frac{\alpha_4 \beta_2 \tau h_t^i(\omega)}{\alpha_3 \beta_1 h_t^i(\omega)}\right] \]

Hence, \( e_t(\omega) \) increases with the parents’ human capital \( h_t(\omega) \) but decreases with the tax rate \( \tau_t \). It is also \textit{independent of the ability} of their offspring. We use the above relations
to obtain a useful expression for income at date $t+1$, $y_{t+1}(ω)$. To that end we apply (12) and (13) and make use of (1), (2) and (3) to obtain:

$$y_{t+1}(ω) = (1 - τ_{t+1})w_{t+1}h_{t+1}(ω)$$  \hspace{1cm} (16)

where,

$$h_{t+1}(ω) = \left( \frac{α_3}{α_3 + α_4} \right) θ_t(ω) \left[ β_1 h^ω_t(ω) + β_2 τ_t \overline{h}^ω_t \right]$$  \hspace{1cm} (17)

whenever $e_t(ω) > 0$, and

$$h_{t+1}(ω) = β_2 θ_t(ω) τ_t \overline{h}^ω_t , \text{ whenever } e_t(ω) = 0$$  \hspace{1cm} (18)

Equations (16)-(18) determine the income at the future date in terms of the net wage at date $t+1$, the parents’ human capital, society’s level of human capital at date $t$, the current education input ($τ_t = e_{gt}$) and the externalities in education.

### 3.3 Non-participation of Parents

The retreat of parents from the education process is an important stylized fact of education systems in some OECD countries that has attracted the attention of policymakers. This situation, where utility maximization is attained at $e_t(ω) = 0$, occurs under certain conditions. To derive these recall that (12) and (13) establish a negative relationship between the two types of education, that is, public education substitutes for parental tutoring. For each individual there exists a particular tax rate such that $e_t(ω) = 0$, namely, when the marginal utility of leisure is larger than the utility gain obtained from a marginal increase in the offspring’s human capital due to parental tutoring. Consider the families which optimally choose $e_t(ω) = 0$ and denote this set of families in generation $t$ by $A_t \subset G_t = [0,1]$. In fact, condition (13) holds if:

$$1 - e_t(ω) < \frac{α_4}{β_1 α_3} \left[ β_1 e_t(ω) + β_2 e_{gt} \overline{h}^ω_t(ω) \right]$$

Hence, for each individual in $G_t$ we obtain $e_t(ω) = 0$ and hence, $ω \in A_t$ if:
\[ h_t^\nu (\omega) \leq \frac{\alpha_4 \beta_2 e_{gt}}{\alpha_3 \beta_1} \frac{h_t}{h_t^\nu} \]  

(19)

Parental and public education being substitutes, inequality (19) shows that the set \( A_t \) increases as the level of public provision of education \( e_{gt} \) increases. It is clear that this set includes individuals with low levels of human capital.

3.4 Under-provision of Public Education and Poverty Trap

As parental education is crowded out by public schooling, let us examine the net contribution of public education to the long-run human capital stock. We show that a higher provision of public education results, along the whole equilibrium path, in a positive growth rate of the stock of human capital, while decreasing the level of public education below a certain threshold may result in a negative growth rate. Our model allows, therefore, for a poverty trap although we do not assume scale economies [Benhabib and Farmer (1994)].

To simplify our analysis we assume, in this section only, a stationary provision of public education, i.e., \( e_{gt} = e_g = \tau \) for all \( t \). Let us impose some restrictions on the parameters in our economy in order to demonstrate the following: The level of public education is critical to the positive or negative accumulation of human capital. We also assume in this section only that the parameters in our economy satisfy the following conditions:

(A1) \( \frac{\alpha_3^3}{\alpha_3 + \alpha_4} < 1 - \xi \), for some \( \xi > 0 \).

(A2) The initial distribution \( h_0(\omega) \) satisfies: \( h_0 \geq 1 \).

(A3) \( \eta = 1 \) and \( v = 1 \).

(A4) \( \alpha_4 > \alpha_3 \).

Proposition 1 Assume that (A1) - (A4) hold. Then:

(a) If \( e_g \) satisfies:

\[ e_g \leq \left[ 1 - \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \right] \beta_2^{-1} \]  

(20)

then, along the equilibrium path, the aggregate human capital decreases, namely, \( \bar{h}_{t+1} < \bar{h}_t \) for \( t = 0, 1, 2, \ldots \).

(b) If \( e_g \) satisfies:
\[ e_g \geq \left[1 - \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \right] \frac{\alpha_3 + \alpha_4}{\alpha_3 \beta_2} \]

then the aggregate human capital increases, i.e., \( \overline{h}_{t+1} > \overline{h}_t \) for all \( t \).

This result underlines the important role played by the level of public education. To emphasize this point, let us compare two countries which differ in the provision of public education and in their initial distributions of human capital, given that each economy satisfies (A1)-(A4). If \( e_g \) is chosen to be low in one country, assuming that condition (20) holds, while in the other country it is higher, say condition (21) holds, then we obtain a poverty trap in the former country while the latter has a positive rate of growth. The human capital indicators of the World Bank site documents examples of countries with declining human capital.

### 3.5 Endogenous Growth

Consider the competitive equilibria for some given initial conditions and compare the long run properties of this economy under the various regimes of education we have considered. Define the growth factor of aggregate labor supply as:

\[ \gamma_t \equiv \frac{\int_\Omega h_{t+1}(\omega)d\mu(\omega)}{\int_\Omega h_t(\omega)d\mu(\omega)} \]

(22)

Since, by our assumptions, ability \( \theta_t(\omega) \) is independent of \( h_t(\omega) \) and \( h^\nu_t(\omega) \), substituting (17) in (22) gives rise to an alternative expression for \( \gamma_t \):

\[ \gamma_t = \left( \frac{\alpha_3}{\alpha_3 + \alpha_4} \right) \left[ \beta_1 \frac{\int_\Omega h^\nu_t(\omega)d\mu(\omega)}{\int_\Omega h_t(\omega)d\mu(\omega)} + \beta_2 \tau_t \overline{h}_t^{\eta-1} \right] \]

(23)

The growth rate is positive as long as (23) is greater than 1. The two terms in the square brackets represent the two channels through which income distribution can matter for the growth factor of aggregate labor: (1) via parental education and (2) via the endogenous determination of \( \tau_t \) like in a median-voter equilibrium. If education has constant returns to scale [hence, \( \nu = \eta = 1 \)], then:

\[ \gamma_t = \frac{\alpha_3}{\alpha_3 + \alpha_4} [\beta_1 + \beta_2 \tau_t] \]

(24)
The growth factor $\gamma_t$ is larger than unity for $\beta_1$ and $\beta_2$ sufficiently large. The following monotonicity results can be verified:

$$\frac{\partial \gamma_t}{\partial \alpha_3} > 0 \quad \text{and} \quad \frac{\partial \gamma_t}{\partial \alpha_4} < 0$$ (25)

Also, the next two derivatives will be useful when we discuss the effects of efficiency in the schooling system on growth:

$$\frac{\partial \gamma_t}{\partial \beta_1} > 0, \quad \frac{\partial \gamma_t}{\partial \beta_2} > 0$$ (26)

It is clear from (24) that the time independence of $\tau_t$ implies time independence of $\gamma$ as well. In addition, when we take the aggregate production function to be of the Cobb-Douglas type, then by direct computation we obtain that: $q_{t+1}/q_t = \gamma$. In the stationary state our model is then an AK-type endogenous growth model where all variables grow at the rate $(\gamma - 1)$.

4 Income Distribution

The objective of this section is to consider changes in the intragenerational income distribution, in equilibrium, due to variations in education systems. Such variations can be attributed to numerous factors but the ones considered here reflect cross-country differences in the process describing the accumulation of human capital. This section deals mainly with differences in levels of public education, differences in education technology and in factor endowments. We use second degree stochastic dominance to rank inequality [see Atkinson (1970)].

4.1 Inequality without Public Education

Let us consider first a situation in which the government plays no role in the process of human capital formation. Thus, we take $\tau_t = 0$ for all $t$. In this case:

$$y_{t+1}(\omega) = w_{t+1}h_{t+1}(\omega)$$ (27)

From (19) we know that the set $A_t$ is empty, and from (12) we obtain that:
\[ e_t(\omega) = e^* = \frac{\alpha_3}{\alpha_3 + \alpha_4} \quad \text{for all } \omega \quad (28) \]

We see that in the absence of public education the only source of income inequality is the initial distribution of human capital. This is clear from:

\[ y_{t+1}(\omega) = \beta_1 w_{t+1} e^* h_t'(\omega) \theta_t(\omega) \quad (29) \]

We conclude from these observations that:

**Proposition 2** In the absence of public education income inequality (i) declines over time under decreasing returns to parental human capital (i.e., if \( \nu < 1 \)), (ii) increases over time under increasing returns (i.e., if \( \nu > 1 \)), and (iii) remains constant over time under constant returns (i.e., if \( \nu = 1 \)).

Our economy generates, in equilibrium, an intragenerational income distribution whose inequality is endogenously determined by the externality in the home-part of education. Inequality may decrease even in the absence of public schooling. When \( \nu > 1 \) a family ‘poverty trap’ arises in that \( h_t(\omega) \) goes to zero for some families whose initial endowment of human capital is too low. More precisely, this occurs for family \( \omega \) such that:

\[ h_0(\omega) < \left( \frac{\alpha_3 + \alpha_4}{\beta_1 \alpha_3 \theta_0(\omega)} \right) \frac{1}{\nu - 1} \]

Figure 1 illustrates this inequality. Point A is the intersection of the human capital accumulation curve with the 45° line. It segments the population’s human capital into two groups. Families to the left of A face a permanent decline in human capital while those to the right of A experience a permanent increase. In this regard, note that increasing returns in parents’ human capital have been observed in China [see Knight and Shi (1996)].

\(^3\)In addition, when the economy operates without government we also find: (a) A change that affects \( \beta_1 \) has no effect on income inequality; (b) Assuming \( \nu < 1 \), an increase in \( \nu \) will increase inequality at all dates. Comparing two countries (or, alternatively, considering a technical change in a given country) the above results show that the two types of technical changes are asymmetric with respect to their effect on income inequality. When the process becomes more efficient (i.e., \( \beta_1 \) increases) it affects similarly all families. For example, if computers are used in each household the effect on income distribution is neutral; however, if parental skills play a more important role in the education process inequality will rise. As we shall see later, in the presence of public education, the former result is modified, while the later result is reinforced.
4.2 Inequality with Public Education

Now we introduce public education and assume that its level is determined by the government. Currently, we do not choose explicitly the social decision mechanism underlying its determination. The level at date $t$ is $e_{gt}$ and it is financed by taxing labor income at a fixed rate $\tau_t (= e_{gt})$. In the sequel we assume that $\upsilon \leq 1$ and that $\eta \leq 1$ and, to simplify our analysis, we also assume that $\upsilon \leq \eta$. Does public education reduce inequality in equilibrium?

**Proposition 3** In the above economy let $h_0(\omega)$ be any initial human capital distribution. Increasing the provision of public education results in a more equal intragenerational income distribution in each date.

This result may not be surprising since public education in our framework dampens family attributes as it is provided equally to all young individuals (of the same generation), while it is financed by a flat tax rate on wage income. However, its importance lies in the fact that it is proved in *equilibrium* and that it holds for *all periods*. In addition, if one compares two countries which are similar in all respects except for the level of public education, the country which invests less in public schooling will face a higher inequality along the equilibrium path.

Let us consider now the variation over time of inequality. We show that if the tax rate remains fixed intragenerational income inequality declines over time.

**Proposition 4** If the same tax rate applies to all levels of income and remains fixed over time, then income inequality declines; namely, income inequality at date $t + 1$ is smaller than the inequality at date $t$.

This proposition illustrates the basic property of public education in our framework; namely, its role in smoothing family attributes. Thus, when human capital formation is characterized by constant or decreasing returns to scale in the family’s human capital, the existing socioeconomic disparities are diminished over time in the presence of public education.

The contribution of public education under increasing returns in family’s human capital is illustrated in Figure 1. Because of a positive $\tau_t$, the human capital accumulation curve shifts upward whereby intersection points $B$ and $C$ are obtained. Whereas $B$ is stable, $C$ is not
all families’ human capital to the left of $C$ converges to the human capital given by $B$ while that of families to the right of $C$ grows forever. Public education contributes therefore to a larger intellectual elite but the poorer segment of the population converges now to a positive level of human capital.

4.3 Comparative Dynamics of Efficiency

The role of technological change in the aggregate production function can be ignored since all labor incomes vary in the same proportion and therefore it has no impact on the equilibrium distribution of income. In contrast, cross-country differences in processes describing human capital formation do matter to income inequality.

We study several technological variations assuming that the human capital is generated by (1). One way to represent such an improvement is by increasing the ‘efficiency’ of the education environment, namely via the introduction of more sophisticated teaching facilities (computors, for example), reducing class size, better organization of schools and so forth. This amounts to increasing the parameters $\beta_1$ and/or $\beta_2$. Another form of technological improvement in this process is to enhance the effectiveness of the ‘teachers’ or ‘tutors’ through, for example, better training for teachers and advising parents about tutoring their child. Such an improvement amounts to increasing the parameters $v$ and $\eta$, that bring into expression the effectiveness of the human capital of the parents and/or the ‘teachers’ in the public education system. We assume that $v \leq 1$ and $\eta \leq 1$ in the sequel, even though this assumption can be relaxed in most cases.

An improvement in one country (vs. the other) in the production of human capital may result in a more efficient home-education or more efficient public-education, or both. We say that the provision of public education is more efficient if either $\beta_2/\beta_1$ is larger (without lowering neither $\beta_1$ nor $\beta_2$) or $\eta$ is larger, or both. We say that the private provision of education becomes more efficient if $\beta_1/\beta_2$ becomes larger (while neither $\beta_1$ nor $\beta_2$ declines) or $v$ becomes larger, or both. It is called neutral in the case where both parameters $\beta_1$ and $\beta_2$ increase while the ratio $\beta_2/\beta_1$ remains unchanged. Let us consider now the effect of each type of technological gap in the education process on intragenerational income inequality.

**Proposition 5** Consider improvements in the production process of human capital, given by equation (1). Then: (a) If public provision of education becomes more efficient the inequality
in intragenerational distribution of income declines in all periods; (b) If the private provision of education becomes more efficient then inequality increases in all periods; (c) If the technological improvement is neutral inequality remains unchanged at period 1 but declines for all periods afterwards.

This result demonstrates the asymmetry between a technological gap which exists primarily in the public schooling system and the one which arises in the home environment of learning. The inequality in human capital distribution increases when the private-component of education/learning becomes more efficient because family attributes, namely the human capital of parents, are magnified. However, a more efficient public education reduces inequality because all children are exposed to instructors with the same average level of human capital: below-average families have a greater return to public schooling than above-average families. When the technological gap in education is neutral, then along the ‘better’ equilibrium inequality declines, except for the first date, since, after the first period, the effectiveness of the public schooling outweighs that of home education.

**Median-Voter Equilibrium**

Our analysis thus far has been carried out under the assumption that the tax rate that finances education and, hence, the level of public education, is exogenously given. This assumption regarding the exogeneity of $\tau_t$ is questionable since political candidates care about resources invested in public education and their economic implications. Also, as families are heterogenous, the choice of an ‘optimal’ level of public schooling should represent a political equilibrium. The equilibrium we consider here is an application of the median-voter theorem, widely used in economic theory [see, e.g., Persson and Tabellini (2000), Section 3.3].

Let us substitute the first order conditions (11)-(13) in (5) to obtain an expression for the lifetime utility of agent $\omega \in G_t$ in terms of the tax rate $\tau_t$:

$$U_t(\omega) = B_t[1 - \tau_t]^{\alpha_1+\alpha_2+\alpha_3}[\beta_1 h_t(\omega) + \beta_2 \tau_t \bar{h}_t]^{\alpha_3+\alpha_4}E[\hat{\theta}(\omega)]^{\alpha_3}$$

(30)

where $B_t$ groups parameters and variables given to this individual at the outset of date t (including $\tau_{t+1}$). Since $U_t(\omega)$ is concave in $\tau_t$ there is a unique maximum for each individual’s
lifetime utility denoted by $\tau_t(\omega)$. It is obtained directly from the first order (necessary and sufficient) condition:

$$(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\beta_2 \tau_t(\omega)\overline{h}_t = (\alpha_3 + \alpha_4)\beta_2 \overline{h}_t - (\alpha_1 + \alpha_2)\beta_1 h_t(\omega)$$

It is clear that the heterogeneity in voter’s optimal policy $\tau_t(\omega)$ results from the heterogeneity in their human capital $h_t(\omega)$. In particular, the median voter’s choice is:

$$\tau_t(m) = [\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4]^{-1}[(\alpha_3 + \alpha_4) - (\alpha_1 + \alpha_2)\frac{\beta_1 h_t^v(m)}{\beta_2 \overline{h}_t}]$$

(31)

Some monotonicity results can be verified from the expression in (31):

$$\frac{\partial \tau_t(m)}{\partial \alpha_1} = \frac{\partial \tau_t(m)}{\partial \alpha_2} < 0 \quad \text{and} \quad \frac{\partial \tau_t(m)}{\partial \alpha_3} = \frac{\partial \tau_t(m)}{\partial \alpha_4} > 0$$

Also,

$$\frac{\partial \tau_t(m)}{\partial \left(\frac{\beta_1}{\beta_2}\right)} < 0 \quad \text{and} \quad \frac{\partial \tau_t(m)}{\partial \left(\frac{h_t^v(m)}{\overline{h}_t}\right)} < 0$$

(32)

Observed cross-country differences in education expenditures can be explained by these derivatives. For example, as $h_t(m)$ drops relative to $\overline{h}_t$, $\tau_t(m)$ rises: A below-average median voter favors a higher tax rate than an above-average median voter. Also, an increase in $v$ and $\beta_1/\beta_2$ [or a decrease in $\eta$] imply a lower tax rate for financing education.

Majority voting strengthens the results regarding income inequality attained under exogenous tax rates. To show that, consider, for example, Propositions 3 and 5 when $\beta_1$ increases. This results in higher inequality by Proposition 5. In addition, following the increase in $\beta_1$, majority voting implies a lower tax rate $\tau_t(m)$, which according to Proposition 3 leads to a further increase in inequality.

on future political outcomes. Voters may induce the end of public education this period but a constituency for an education policy can regenerate next period. See Hassler et al. (2002) for a model of rational dynamic voting.
Inequality and Growth

In our framework the economy has no other source of income besides the one generated by the aggregate production in which human and physical capital are used. Thus educational investments are essential to creating growth. Let us consider the growth-inequality relationship issue in our framework and the causality linking them. To that end, we wish to compare two countries which differ in some parameters of the human capital formation process. As in the empirical findings it turns out that there is no explicit stance on the causality between inequality and growth.

Let us consider first the effect that a technological change in the production of human capital has on output in equilibrium. Consider (1) and note that we call the first term on the RHS, $\beta_1 e_t(\omega) h_t^\iota(\omega)$, the home-component, and the second term, $\beta_2 e_{gt} h_t^\iota$, the public-component. An improvement in the production of human capital which makes either the public provision more efficient or the private provision more efficient results in higher output at all dates [see, e.g. (26)]. Any improvement, either in the public-component or the home-component, implies higher human capital stock as of period 1 and on. Since, the initial capital stock is given this increases the output in date 1 and, hence, the aggregate savings in this period. Thus the output in date 2 is higher and hence the capital stock to be used as well. Does a technological progress, which results in higher growth, also mean less inequality? Let us combine our results to obtain:

**Proposition 6** Consider some technological differences in the production process of human capital (1): (a) If the technological gap occurs in the home-component, hence either $\beta_1$ is higher or $v$ is higher (or both), it results in higher growth coupled with higher income inequality in all dates; (b) When the technological gap occurs in the public-component, hence either $\beta_2$ is higher or $\eta$ is higher (or both), it results in higher growth accompanied by less inequality.

The proof of this result follows directly from Proposition 5 and it is omitted. Consider, for example, the computer-information revolution as a technological improvement in enhancing knowledge, then we ask whether the home-component benefits more than the public-component in the process of forming human capital. We believe that in most developed countries computers and internet have enhanced the home-education considerably, while schools benefited only in a limited manner. Part (a) may provide some explanation to the recent observation (mostly during the last decade) that in most OECD countries economic growth is accompanied by increasing inequality in the distribution of income.
4.4 The Role of Endowments and Trade

To what extent are the results obtained so far robust with respect to international trade and capital mobility between countries? To answer this question consider two similar economies that differ only in their initial endowments of human capital: one economy has higher levels of human capital but the measure of inequality in the initial human capital distributions is the same. The next proposition compares the equilibrium path of these two countries in autarky.

Proposition 7 Consider two economies which differ only in their initial human capital distributions, \( h_0(\omega) \) and \( h^*_0(\omega) \). Assume that \( h^*_0(\omega) > h_0(\omega) \) for all \( \omega \), but the initial distributions have the same level of inequality. Then, the equilibrium from \( h^*_0(\omega) \) will have less income inequality at all dates \( t, t = 1, 2, 3, \ldots \).

Thus the initial distribution of human capital matters, hence a country that starts with higher levels of human capital, not necessarily more equal, has a better chance to maintain less inequality in its future income distributions.

Given the different endowments of human capital let us now introduce international trade and mobility of physical capital between these two economies, keeping labor immobile internationally. These assumptions about trade and factor mobility guarantee factor price equalization. In this setting, we claim that if initial endowments of either human capital or physical capital differ between countries then opening markets does not affect our results regarding income inequality.

Proposition 8 Consider two countries which differ in their initial conditions. Trade in goods and physical capital mobility will not alter the results attained earlier regarding income inequality under autarky.

Clearly, trade has a significant impact on wages, interest rates and outputs of the two countries. However, it is not difficult to see, from equations (16)-(18), that in our framework, such variations in the equilibrium factor prices do not affect our results regarding income inequality since labor incomes vary in the same proportion. Hence, trade plays no role in explaining income inequality in our framework. Introducing direct intergenerational transfers (via physical capital) in our economy will modify some of our results since, in this case, changes in factor prices affect individuals differently [see Karni and Zilcha (1994), Viaene and Zilcha (1994)].
(2002)]. However, there is an ongoing debate as to the empirical importance of monetary transfers between generations [see Laitner (1997)].

5 Computation

The preceding propositions single out education systems as the main determinant of income inequality in equilibrium. Though processes of human capital accumulation differ substantially among countries are they large enough to matter for the observed dispersion of inequality? The objective is to quantify the response of income inequality to changes in the parameters of education technology. Our approach is to develop a dynamic computable general equilibrium model with heterogeneous agents based on our theory. This method traces the time path followed by each group of families and looks at how they respond to different education systems. Aggregation over all families provides variables like the Gini coefficient and growth rate of output.

The deterministic equilibrium, namely when random abilities are set to their mean, is calibrated on statistics from the Netherlands over the period 1975-2000. Data for the key variables are summarized in Table 1. In addition, a number of assumptions have been made in order to compute the calibrated parameters of Table 2:

Human capital. The stock of human capital at $t = -1$ is approximated in two steps. Total employment is first divided in 7 scholastic achievements ranging from primary school to university degree. Using the wage of each educational type relative to that of a worker with a primary school certificate as weight, the weighted sum over educational types provides our proxy for the stock of human capital. While actual employment in 2000 is 69.17 hundred thousand, our proxy $h_{-1}$ is 89.61 hundred thousand primary school equivalent workers.

The last proposition can be strengthened considerably if we consider the special case where all individuals in all generations choose to invest in their child’s education, i.e., that $e_t(\omega) > 0$ for all $\omega$. Consider two countries which differ in the parameters of the model (including initial parameters of the model (including initial conditions). Assume, for simplicity, that only private education exists; namely, $\tau_t = 0$. Again, trade in goods and physical capital mobility will not affect the results concerning income inequality attained under autarky. The main point to notice in verifying this claim is that, since all the sets $A_t$ are empty, the variations of the parameters will only change the incomes, in each date, by some multiplicative term which is equal to all $\omega \in G_t$. The variation in the human capital distribution [see (17) and (18)] will not affect the inequality in the human capital distribution.
Families. We consider 13 heterogenous families with a human capital at $t = -1$ taking the values 1, 1.5, 1.8, 2.48, 3, 4, 4.5, 7.55, 8, 9, 14.78, 15 and 17. Each family has an initial $\omega = A, B, \ldots L$ and $M$. The median-voter at $t = -1$ is therefore individual $G$ with human capital $h_{-1}(G) = 4.5$, clearly to the left of the mean $\bar{h}_{-1} = 6.89$. These fictitious families are chosen with two criteria in mind. First, the sum of individual endowments of human capital is 89.61. Second, they approximate the quartile distribution of income given in Table 1. This quartile distribution is however inconsistent with the Gini coefficient of 2000. The following formula for the Gini coefficient is used:

$$
g_t = \frac{1}{2n^2 \bar{y}_t} \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j|
$$

where $n = 13$ represents the number of families, $\bar{y}_t$ is average income, $y_i$ and $y_j$ are individual incomes.

Production technology. We replace (4) by the Cobb-Douglas production function $q_t = \phi h_t^\theta (1-\tau_t)^{1-\theta} h_t^{1-\theta}$, that is $w_t = \phi(1-\theta)(h_t/(1-\tau_t)h_t)\theta$ and $(1+\tau_t) = \phi(1-\tau_t)h_t/k_t)^{1-\theta}$. As the theoretiical model assumes full depreciation, the actual $k_t$ is transformed to equate past savings.

Preferences. We pick $\alpha_4 = 1.70$ that falls within the interval of available empirical estimates for the weight for leisure in the utility function. Parameter $\alpha_3$ is chosen such that the poorest family $A$ does not participate to the education process, namely $e_{-1}(A) = 0$. Also, $\alpha_1$ and $\alpha_2$ are selected to obtain net savings.

Human capital formation. Parameters $\beta_1$ and $\beta_2$ are constructed to obtain $e_{-1}(A) = 0$ and to calibrate the observed growth rate of the economy.
Table 1  The baseline economy: Netherlands (1975-2000)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Output level (10^9 Euro, 2000)</td>
<td>401.089</td>
</tr>
<tr>
<td>Growth rate, real GDP per capita (%), 1975-2000</td>
<td>56.94</td>
</tr>
<tr>
<td>Capital coefficient (Euro, 2000)</td>
<td>4.6</td>
</tr>
<tr>
<td>Net savings (% of GDP, 2000)</td>
<td>12.45</td>
</tr>
<tr>
<td>Employment (10^5, 2000)</td>
<td>69.17</td>
</tr>
<tr>
<td>Quartiles, distribution of income (% , 1998)</td>
<td>4.8, 15.6, 27.4, 52.2</td>
</tr>
<tr>
<td>Gini coefficient, disposable income (2000)</td>
<td>0.325</td>
</tr>
<tr>
<td>Education expenditures (% of GDP, 1999)</td>
<td>5.1</td>
</tr>
</tbody>
</table>

Source: Dutch Central Bureau of Statistics

Table 2  Parameterization of baseline economy

<table>
<thead>
<tr>
<th>θ</th>
<th>φ</th>
<th>β₁</th>
<th>β₂</th>
<th>v</th>
<th>η</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>4.599</td>
<td>3.440</td>
<td>6.908</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>α₁</th>
<th>α₂</th>
<th>α₃</th>
<th>α₄</th>
<th>τ₋₁</th>
<th>h₋₁</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.172</td>
<td>0.902</td>
<td>1.2</td>
<td>1.7</td>
<td>0.051</td>
<td>89.61</td>
</tr>
</tbody>
</table>

Given the parameters of the model, the equilibrium path of all variables pertaining to a particular family is obtained in five steps. (1) A random number generator draws an innate ability at each date \(t(t = -1, 0, \ldots)\) from a normal distribution with mean 1 and standard deviation 0.1. (2) The human capital of any individual at date \(t\) is given by \((17)\) or \((18)\). (3) A vector of individual preferences for education expenditure, namely \(\tau_t(\omega)\), is computed based on \((31)\) and the median-voter’s preference is selected. (4) Aggregating the levels of human capital across individuals and equating the aggregate capital stock at date \(t\) to past savings, we obtain \(q_t, w_t\) and \((1 + r_t)\). Upon this information, each individual derives \(y_t(\omega)\) and the Gini coefficient is computed. (5) Given the time path of wages, marginal returns to physical capital and income of each family, each individual can compute \(e_t(\omega), c_{1t}(\omega), c_{2t}(\omega)\) and \(u_t(\omega)\). Step 1 is skipped when we consider a deterministic solution of the model; step 3 is skipped when we assume exogenously given education expenditures.
Table 3 presents the solutions of our calibrated economy with and without random abilities, with fixed and endogenous education expenditures. Given the parameters of Table 2 and the initial conditions at $t = -1$, the economy starts at $t = 0$ and an equilibrium path is computed for 200 periods. As patterns emerge within 20 periods, we discard the last 180 periods and reproduce the relevant statistics for $t = 0$, the average over the first 10 periods and over the second 10 periods. The deterministic solution in column (1) is the closest to the actual data of Table 1; the equilibrium with random abilities in column (2) approximates this solution because of the discrete number of families.

A feature of both columns (1) and (2) is that, because of public education, income inequality among dynasties decreases over time, as shown in Proposition 4. Though families start with different endowments at date $t = -1$, they tend to be similar after 20 periods. The effect of random abilities is to reduce the speed of family convergence. Also, as Dutch education expenditures are low by international standards, Proposition 1 suggests the possibility of a poverty trap. Using our calibrated parameters, a direct computation of conditions (20) and (21) gives a negative sign for the RHS in both cases. Hence, a poverty trap is excluded but it is important to note that the economy would experience positive growth even in the absence of public education ($e_g = \tau = 0$). The reason for this result is that the set $A_t$ in (19) would then be empty as all families increase their participation to education. The contribution of majority voting in column (3) is to show that, given the initial distribution of income, a political equilibrium would select higher values for $\tau_t$ than the ones given by the data.

Table 4 characterizes the equilibrium path of our economy after $\tau_t, \beta_1, \beta_2, \nu$ and $\eta$ have been modified, one by one. The table reports the response elasticities of the Gini coefficient, the growth rate of output, the wage-rental ratio as direct measure of the capital intensity in production and parental tutoring of the poorest family at $t = -1$. Taking column (2) of Table 3 as baseline equilibrium path, column (1) of Table 4 shows the response with respect to $\tau_t$ and therefore compare economies with different exogenous levels of educational expenditures. This column confirms the results of Proposition 3 in that an economy with 10 percent more education expenditure than in the Netherlands will experience a 4 percent drop in the Gini coefficient during the first 10 periods, a 7.1 percent drop during the second 10 periods. As parental tutoring decreases, the net effect on the country stock of human capital is moderate.
### Table 3 The equilibrium paths$^a$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic</td>
<td>Random</td>
<td>Random</td>
<td></td>
</tr>
<tr>
<td>solution</td>
<td>abilities</td>
<td>abilities</td>
<td>(median-voter)</td>
</tr>
<tr>
<td>(fixed $\tau_t$)</td>
<td>(fixed $\tau_t$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax rate</td>
<td>5.1</td>
<td>5.1</td>
<td>8.6</td>
</tr>
<tr>
<td></td>
<td>5.1</td>
<td>5.1</td>
<td>5.8$^b$</td>
</tr>
<tr>
<td></td>
<td>5.1</td>
<td>5.1</td>
<td>6.2$^c$</td>
</tr>
<tr>
<td>Gini coefficient ($g_t$)</td>
<td>0.387</td>
<td>0.389</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td>0.259</td>
<td>0.277</td>
<td>0.170</td>
</tr>
<tr>
<td></td>
<td>0.098</td>
<td>0.184</td>
<td>0.112</td>
</tr>
<tr>
<td>Growth rate (output)</td>
<td>13.98</td>
<td>12.36</td>
<td>25.57</td>
</tr>
<tr>
<td></td>
<td>50.56</td>
<td>50.31</td>
<td>56.00</td>
</tr>
<tr>
<td></td>
<td>56.94</td>
<td>57.21</td>
<td>59.98</td>
</tr>
<tr>
<td>Wage-rental ratio</td>
<td>0.874</td>
<td>0.892</td>
<td>0.761</td>
</tr>
<tr>
<td>$w_t/(1 + r_t)$</td>
<td>0.596</td>
<td>0.599</td>
<td>0.563</td>
</tr>
<tr>
<td></td>
<td>0.553</td>
<td>0.553</td>
<td>0.538</td>
</tr>
<tr>
<td>Parental tutoring</td>
<td>0.146</td>
<td>0.180</td>
<td>0.163</td>
</tr>
<tr>
<td>(poorest family, $e(A)$)</td>
<td>0.273</td>
<td>0.272</td>
<td>0.304</td>
</tr>
<tr>
<td></td>
<td>0.339</td>
<td>0.325</td>
<td>0.340</td>
</tr>
</tbody>
</table>

Notes. (a) Column (1) reports the equilibrium achieved assuming a fixed $\tau_t$, the calibrated parameters of Table 2 and random abilities fixed at unity. The other columns assume random abilities. For each variable, the first entry is the solution at date $t = 0$; the second and third row reports the average over the first 10 periods and the average over the second 10 periods; (b) the ten median-voters belong to families G, H and J; (c) the ten median-voters belong to families G, H, J and K.
### Table 4 Hypothetical economies$^a$

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_t$</td>
<td></td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\nu$</td>
<td>$\eta$</td>
</tr>
<tr>
<td>Gini coefficient</td>
<td>-0.403</td>
<td>2.265</td>
<td>-1.754</td>
<td>4.537</td>
<td>-7.700</td>
</tr>
<tr>
<td>$(g_t)$</td>
<td>-0.707</td>
<td>3.321</td>
<td>-2.057</td>
<td>5.173</td>
<td>-16.029</td>
</tr>
<tr>
<td>Growth rate</td>
<td>0.082</td>
<td>0.533</td>
<td>0.548</td>
<td>1.290</td>
<td>0.897</td>
</tr>
<tr>
<td>(output)</td>
<td>0.089</td>
<td>0.538</td>
<td>0.482</td>
<td>1.815</td>
<td>1.006</td>
</tr>
<tr>
<td>Wage-rental ratio</td>
<td>-0.116</td>
<td>-0.706</td>
<td>-0.726</td>
<td>-2.192</td>
<td>-1.486</td>
</tr>
<tr>
<td>$w_t/(1 + r_t)$</td>
<td>-0.125</td>
<td>-0.713</td>
<td>-0.642</td>
<td>-3.350</td>
<td>-1.649</td>
</tr>
<tr>
<td>Parental tutoring</td>
<td>-0.311</td>
<td>1.492</td>
<td>-1.786</td>
<td>6.302</td>
<td>-4.050</td>
</tr>
<tr>
<td>$e(A)$</td>
<td>-0.165</td>
<td>0.683</td>
<td>-0.763</td>
<td>12.270</td>
<td>-2.217</td>
</tr>
</tbody>
</table>

Notes. (a) For each variable, the first row reports the average elasticity over the first 10 periods and the second row, the average elasticity over the second 10 periods.

As reflected by the small elasticities on growth and the wage-rental ratio. Taking the political equilibrium of column (3) in Table 3 as benchmark equilibrium path, columns (2) to (5) of Table 4 report the response elasticities with respect to the education parameters. These elasticities characterize economies with different contributions of home-education ($\beta_1$, $\nu$) and of public-education ($\beta_2$, $\eta$) to human capital formation. Elasticities in columns (4) and (5) have been computed for $\nu$ and $\eta$ taking the value of 0.9 instead of 1 in order to avoid multiple equilibria associated with increasing returns. It is clear from the table that, like in Propositions 4 and 5, higher economic growth is coupled with more inequality in the distribution of income when $\beta_1$ and $\nu$ rise; in contrast, a positive correlation between growth and inequality is obtained in the case of $\beta_2$ and $\eta$. In this regard, Table 4 adds that all variables are very sensitive to externalities $\nu$ and $\eta$ arising from the country and family stocks of human capital. For example, a mere 10 percent reduction of $\eta$ (from 1 to 0.9) leads to a 10 percent reduction in output growth and to an increase in income inequality by 160.3 percent. As a rule, the elasticities obtained for
the Gini coefficient are larger in absolute value than those of other variables. Hence, income inequality seems to be sensitive to processes of human capital accumulation.

6 Conclusion

We have studied the cross-country differences in income distribution in an overlapping-generations economy with heterogeneous households. Heterogeneity results from random innate abilities and the nondegenerate initial distribution of human capital across individuals. We derive a number of results which provide explanations for observed cross-country differences in income inequality based on variations in processes of human capital formation (which incorporate both parental, or private, education as well as public education). In particular, our results suggest testable hypotheses regarding a cross-country comparison of inequality and: (a) externalities of family’s (and society’s) human capital; (b) the effective level of public education; (c) the efficiency of public schooling and parental tutoring; (d) economic growth; (e) initial conditions, represented here by the initial level and distribution of human capital and physical capital; (f) the international environment, such as trade and physical capital mobility.

Our framework makes some specific assumptions and is therefore subject to the issue of robustness. First, the selection of our functional forms was strongly motivated by stylized facts. For example, incorporating parental role in the human capital formation process is justified due to the repeatedly reported evidence that it has an empirical relevance in a large number of countries. Second, the model assumes away taxation of the returns to savings but expanding the tax base to include this type of tax does not alter the results concerning income inequality. In contrast, this framework can be generalized to include an additional redistributive measure by the government, such as social security. Some of our results may vary in this situation where intergenerational transfers take place via both education and social security.

7 Appendix

Before we prove our Propositions let us bring the following Lemma.

**Lemma 9** Let $X(\omega)$ and $Y(\omega)$ be two non-negative random variables which assume values in a compact interval $[a, b]$ and satisfy: $EX = EY$. Let $Z(\omega)$ be a positive random
variable independent of $X$ and $Y$. If $X(\omega)$ is more equal than $Y(\omega)$ (in the SDSD sense), then $XZ$ is more equal than $YZ$.

**Proof:** Denote the cumulative distribution functions of $X, Y, Z$ by $F(\xi), G(\xi)$ and $H(\xi)$ correspondingly. Let $[\alpha, \beta]$ be the support of $Z$. Define,

$$W(m) = \Pr\{XZ \leq m\} = \Pr\{Z = \rho \text{ and } X \leq \frac{m}{\rho}, \rho \in [\alpha, \beta]\}$$

It is clear that $W(m) = \int_{\alpha}^{\beta} F(\frac{m}{\rho}) H(\rho) d\rho$. In the same way we define the c.d.f of $YZ$ as $W^*(m) = \int_{\alpha}^{\beta} G(\frac{m}{\rho}) H(\rho) d\rho$. Let the support of $W$ and $W^*$ be $[c, d]$. Now,

$$\Delta(t) = \int_{c}^{d} [W(m) - W^*(m)] dm = \int_{\alpha}^{\beta} \int_{c}^{d} [F(\frac{m}{\rho}) - G(\frac{m}{\rho})] dH(\rho) dm = \int_{\alpha}^{\beta} \int_{c}^{d} [F(\frac{m}{\rho}) - G(\frac{m}{\rho})] dm dH(\rho)$$

Now, by changing variables, for each fixed $\rho$ we obtain that:

$$\int_{c}^{d} [F(\frac{m}{\rho}) - G(\frac{m}{\rho})] dm = \rho \int_{c}^{d} [F(\frac{m}{\rho}) - G(\frac{m}{\rho})] d\frac{m}{\rho} = \rho \int_{c}^{d} [F(q) - G(q)] dq \leq 0$$

by our assumption about $X$ and $Y$. Thus, we obtain that $\Delta(t) \leq 0$ for all $t$ in $[c, d]$ and $\Delta(d) = 0$. This completes the proof. $\square$

This Lemma allows us to prove inequality between income distributions while ignoring the "mixing" effects of the random ability $\theta_t(\omega)$ since it is independent of the human capital levels of the parent or the given individual.

**Proof of Proposition 1:** Consider the inequality (19), which basically defines the set $A_t$, using (17), (18) and (A3) we derive:

$$\overline{h}_{t+1} = \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \int_{\sim A_t} h_t(\omega) + (1 - \mu(A_t)) \frac{\alpha_3 \beta_2 e_g}{\alpha_3 + \alpha_4} \overline{h}_t + \mu(A_t) \beta_2 e_g \overline{h}_t$$

$$+ \mu(A_t) \beta_2 e_g \overline{h}_t$$

(34)

Therefore,

$$\overline{h}_{t+1} < \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \int h_t(\omega) + \frac{\alpha_3 \beta_2 e_g}{\alpha_3 + \alpha_4} \overline{h}_t + \mu(A_t) \beta_2 e_g \overline{h}_t - \frac{\alpha_4}{\alpha_3 + \alpha_4}$$

Thus to assure that $\overline{h}_{t+1} < \overline{h}_t$, we need to show that:
\[
\frac{\alpha_3}{\alpha_3 + \alpha_4} [\beta_1 + \beta_2 e_g (1 + \mu(A_t) \frac{\alpha_4}{\alpha_3})] \leq 1 \tag{35}
\]

Thus, if we replace \( \mu(A_t) \) by 1 and take \( e_g \) to satisfy condition (20), then the above inequality holds. To prove part (b) let us reconsider equation (34) above. It is clear that:

\[
\bar{h}_{t+1} > \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \int h_t(\omega) - \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \int_{A_t} h_t(\omega) + \frac{\alpha_3 \beta_2 e_g}{\alpha_3 + \alpha_4} \bar{h}_t + \frac{\alpha_3 e_g}{\alpha_3 + \alpha_4} \mu(A_t) \beta_2 e_g h_t
\]

However, using inequality (19) we obtain that \( \int_{A_t} h_t(\omega) < \mu(A_t) \frac{\alpha_3 \beta_2 e_g}{\alpha_3 \beta_1} \bar{h}_t \). Hence:

\[
\bar{h}_{t+1} > \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} \bar{h}_t + \frac{\alpha_3}{\alpha_3 + \alpha_4} \beta_2 e_g \bar{h}_t
\]

Thus, \( \bar{h}_{t+1} > \bar{h}_t \) holds whenever \( \frac{\alpha_3 \beta_1}{\alpha_3 + \alpha_4} + \frac{\alpha_3}{\alpha_3 + \alpha_4} \beta_2 e_g \geq 1 \). Namely, it holds under condition (21).

\[\Box\]

**Proof of Proposition 2:** In fact we need to consider here the inequality in the distribution of human capital over time. Given the initial human capital distribution \( h_0(\omega) \), if \( \nu < 1 \) then the distribution \( h_1(\omega) \) is attained (up to a constant) from \( h_0(\omega) \) by applying a strictly concave transformation; hence [using Theorem 3.A.5 in Shaked and Shanthikumar (1994)], \( h_1(\omega) \) is more equally distributed. This process can be continued for all \( t = 2, 3, \ldots \). Now, when \( \nu > 1 \), \( h_1(\omega) \) is obtained (up to a constant) from \( h_0(\omega) \) via a strictly convex function, hence it is less equally distributed (by the above reference).

**Proof of Proposition 3:** As we see later it is sufficient to prove this result under the assumption that \( e_t(\omega) > 0 \) for all \( \omega \in G_t \). When this is not the case, raising \( e_{gt} \) entails higher income for all low income individuals \( \omega \in A_t \) which only reinforces the claim. Let us consider (1) for \( t = 0 \). Since \( h_0(\omega) \) is given, \( h_0^0(\omega) \) and \( \bar{h}_0 \) are fixed. By raising \( e_{g0} \) the distribution of the human capital for generation 1, \( h_1(\omega) \) becomes more equal. This follows from Lemma 2 in Karni and Zilcha (1994). Moreover, we claim from (17) that the average human capital in generation 1 increases as well. Increasing \( e_{g0} \) will result in higher \( h_1(\omega) \) for all \( \omega \) and higher level of \( \bar{h}_1 \). Moreover, it also implies that \( h_0^0(\omega) \) will have a more equal distribution [see, Shaked and Shanthikumar (1994), Theorem 3.A.5].
Now, let us consider $t = 1$. Increasing $e_{g1}$ will imply the following facts: $h^*_1(\omega)$ becomes more equal and $\beta_2 e_{g1} h^*_1$ is larger than its value before we increased the levels of public education. Using (17) and the same Lemma as before we obtain that $h_2(\omega)$ becomes more equal. This process can be continued for $t = 3, 4, \ldots$, which establishes our claim. Now let us consider the set of families with $e_1(\omega) = 0$. To simplify our argument assume that initially $e_{g0} = 0$, then as $e_{g0}$ increases $h_1(\omega)$ will be equal or larger than in the private provision case for all $\omega \in G_1$, where $\omega \in A_0$. Namely, we claim that:

\[
\beta_2 e_{g0} h^*_0 \geq \beta_1 e_0(\omega) h^*_0(\omega) \quad \text{for all } \omega \in A_0
\]  

Using (28) to substitute $e_0(\omega)$ and using the upper bound for $h^*_0(\omega)$ from (19), we see that this inequality always holds since, by assumption, $v \leq \eta$. This fact certainly reinforces the proof of our earlier case since at the lower tail of the distribution of income we raised and equalized the income for all $\omega \in G_1$, where $\omega \in A_0$. This process can be continued for all generations. \hfill \Box

**Proof of Proposition 4:** Let us show first that in each generation individuals with higher level of human capital choose at the optimum higher level of time to be allocated for private education of their offspring. To see this let us derive from the first order conditions, using some manipulation, the following equation:

\[
1 - [1 + \frac{\beta_1 \alpha_4}{\alpha_3}] e_t(\omega) = \frac{\alpha_4 \beta_2}{\alpha_3} e_{g_t} h^*_t [h^{-\nu}_t(\omega)] \quad \text{for } e_t(\omega) > 0
\]  

which demonstrates that higher $h_t(\omega)$ implies higher level of $e_t(\omega)$. Let us show that such a property generates less equality in the distribution of $y_{t+1}(\omega)$ compared to that of $y_t(\omega)$. It is useful however, to apply (16) for this issue. In fact it represents the period $t + 1$ income $y_{t+1}(\omega)$ as a function of the date $t$ income $y_t(\omega)$ via the human capital evolution. Define the function $Q : R \rightarrow R$ such that $Q[h_t(\omega)] = h_{t+1}(\omega)$ using (17) whenever $\omega \notin A_t$, and when $\omega \in A_t$ this function is defined by: $Q[h_t(\omega)] = \beta_2 e_{g_t} h^*_t$. As we have indicated earlier [see the proof of Proposition 3], this function is monotone nondecreasing and satisfies: $Q(x) > 0$ for any $x > 0$ and $Q(x) / x$ is decreasing in $x$. Therefore [see, Shaked and Shanthikumar (1994)], the human capital distribution $h_{t+1}(\omega)$ is more equal than the distribution in date $t$, $h_t(\omega)$. This implies that $y_{t+1}(\omega)$ is more equal than $y_t(\omega)$. \hfill \Box
Proof of Proposition 5: Let the initial distribution of human capital \( h_0(\omega) \) be given.

Compare the following two equilibria from the same initial conditions: One with the human capital formation process given by (1) and another with the same process but \( \beta_2 \) is replaced by a larger coefficient \( \beta_2^* > \beta_2 \). Clearly, we keep \( \beta_1 \) unchanged. Let us rewrite (16) as follows:

\[
\begin{align*}
    y_{t+1}(\omega) &= C_t[h_t^v(\omega) + \frac{\beta_2}{\beta_1}e_\omega h_t^\eta] & \text{for all } \omega \not\in A_t \\
    y_{t+1}(\omega) &= C_t[\frac{\beta_2}{\beta_1}e_\omega h_t^\eta] & \text{for all } \omega \in A_t \\
    y_{t+1}^*(\omega) &= C_t'[h_t^{v*}(\omega) + \frac{\beta_2^*}{\beta_1^*}e_\omega h_t^{\eta*}] & \text{for all } \omega \not\in A_t \\
    y_{t+1}^*(\omega) &= C_t'[\frac{\beta_2^*}{\beta_1^*}e_\omega h_t^{\eta*}] & \text{for all } \omega \in A_t 
\end{align*}
\]

where \( C_t \) and \( C_t' \) are some positive constants. Since \( h_0(\omega) \) is fixed at date \( t = 0 \) we find [using once again the Lemma from Karni and Zilcha (1994)] that \( \frac{\beta_2^*}{\beta_1^*} > \frac{\beta_2}{\beta_1} \) imply that \( y_t^*(\omega) \) is more equal to \( y_t(\omega) \). We also derive that \( h_1(\omega) \) are lower than \( h_1^*(\omega) \) for all \( \omega \) and, hence, \( \bar{h}_1 < \bar{h}_1^* \). This inequality reinforces the result when \( \mu(A_0) > 0 \). By (17), using the same argument as in the last proof, \( h_1^v(\omega) \) is more equal than \( h_1^v(\omega) \) and \( \frac{\beta_2^*}{\beta_1^*}e_\omega \bar{h}_1^\eta > \frac{\beta_2}{\beta_1}e_\omega \bar{h}_1^\eta \), hence \( h_2(\omega) \) is more equal than \( h_2^*(\omega) \). This same argument can be continued for all dates \( t = 3, 4, 5, … \)

Also note that \( A_t \subset A_t^* \) (where \( A_t^* \) is the set of families in \( G_t \) who choose \( e_t(\omega) = 0 \)) since \( \frac{\beta_2^*}{\beta_1^*}e_\omega \bar{h}_t^\eta > \frac{\beta_2}{\beta_1}e_\omega \bar{h}_t^\eta \) for all \( t \). This only contributes to the more equal distribution of \( y_{t+1}^*(\omega) \) since the left hand tail has been increased and equalized compared to the \( y_{t+1}^*(\omega) \) case.

To complete the proof of part (a) of this Proposition consider the case where we increase \( \eta \). When we increase the value of \( \eta \), keeping all other parameters constant, we are basically increasing the second term in (17), \( [\bar{h}_0]^\eta \), while \( [h_0(\omega)]^\eta \) remains unchanged. By Lemma 2 in Karni and Zilcha (1994) we obtain that the distribution of \( h_1(\omega) \) becomes more equal. Taking into account the families \( \omega \in G_1 \) who belong to \( A_0 \) (i.e., the lower tail of the distribution of income) only reinforces the higher equality since their incomes are uniformly increase to \( \beta_2^*e_\omega \bar{h}_0^\eta \), while for all other \( \omega \in G_1, \omega \not\in A_0 \) the proportional raise in their income is smaller. This can be continued for \( t = 2 \) as well since it is easy to verify that \( [\bar{h}_1]^\eta \) increases while \( [h_1(\omega)]^\eta \) becomes more equal. Now, this process can be extended to \( t = 2, 3, … \), which complete the proof of part (a).

The proof of part (b) follows from the same types of arguments using the fact that if \( \beta_1 < \beta_1^* \) then \( \frac{\beta_2^*}{\beta_1} > \frac{\beta_2}{\beta_1} \) and, hence, \( h_1(\omega) \) is more equal than \( h_1^*(\omega) \) and \( \bar{h}_1 > \bar{h}_1^* \). This process leads, using similar arguments as before, to \( y_t(\omega) \) more equal than \( y_t^*(\omega) \) for all periods \( t \).

\( \square \)

Claim: Comparing two economies which differ only in the parameter \( v \). The economy
with the higher \( v \) will have more inequality in the intragenerational income distribution in all periods.

Since the two economies have the same initial distribution of human capital \( h_0(\omega) \) the process that determines \( h_1(\omega) \) differs only in the parameter \( v \). Denote by \( v^* < v \leq 1 \) the parameters, then it is clear that \([h_0(\omega)]^{v^*} \) is more equal than \([h_0(\omega)]^v \) since it is attained by a strictly concave transformation [see, Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Likewise, the human capital distribution \( h_1^*(\omega) \) is more equal than the distribution \( h_1(\omega) \). This implies that \( y_1^*(\omega) \) is more equal than \( y_1(\omega) \). Now we can apply the same argument to date 1: the distribution of \([h_1^*(\omega)]^{v^*} \) is more equal than that of \([h_1(\omega)]^v \), hence, using (17) and the above reference, we derive that the distribution of \([h_2^*(\omega)]^{v^*} \) is more equal than that of \([h_2(\omega)]^v \).

This process can be continued for all \( t \).

Consider now the claim in part (c). From (17) we see that inequality in the distribution of \( h_1(\omega) \) remains unchanged even though all levels of \( h_1(\omega) \) increase due to this technological improvement. In particular, \( \overline{h}_1 \) increases. Now, since inequality of \( h_2^*(\omega) \) did not vary but the second term in the RHS of (17) has increased due to the higher value of \( \overline{h}_1 \), we obtain more equal distribution of \( h_2(\omega) \). When \( \mu(A_0) > 0 \) the higher \( \overline{h}_1 \) results in higher income to all \( \omega \in G_1 \) who belong to \( A_0 \), which only reinforces the more equality in \( y_2^*(\omega) \). Now, this argument can be used again at dates 3, 4, ..., which completes the proof. \( \square \)

**Proof of Proposition 7:** Observe the following two equations used in the proof of Proposition 5: 
\[
y_{t+1}(\omega) = C_t[h_1^*(\omega) + \frac{\beta_2}{\beta_1}e_{gt}\overline{h}_t] \quad \text{for all} \quad \omega \notin A_t.
\]
\[
y_{t+1}(\omega) = C_t[\beta_2e_{gt}\overline{h}_t] \quad \text{for all} \quad \omega \in A_t.
\]

Similarly,
\[
y^*_t(\omega) = C_t^*[h_1^*(\omega) + \frac{\beta_2}{\beta_1}e_{gt}\overline{h}_t] \quad \text{for all} \quad \omega \notin A_t^*.
\]
\[
y^*_t(\omega) = C_t^*[\beta_2e_{gt}\overline{h}_t] \quad \text{for all} \quad \omega \in A_t^*.
\]

Since \( h_0 \) and \( h_0^* \) are equally distributed, the same holds for \( h_0^*(\omega) \) and \([h_0^*(\omega)]^v \), since \( v \leq 1 \). Moreover, since \( \overline{h}_0 < \overline{h}_0^* \) we obtain that \( h_1^*(\omega) \) is more equal than \( h_1(\omega) \) [again, see Lemma 2 in Karni and Zilcha (1994)]. It is easy to verify from (17) that \( h_1(\omega) \) are lower than \( h_1^*(\omega) \) for all \( \omega \). Note that since \( y_1^*(\omega) = C_0^*\beta_2e_{gt}\overline{h}_t \) for all \( \omega \in A_0 \) and \( y_1(\omega) = C_0\beta_2e_{gt}\overline{h}_t \) for all \( \omega \in A_0 \) and on these sets \( y_1^*(\omega) > y_1(\omega) \) the above argument is not affected by the existence of \( A_0 \) and \( A_0^* \) with positive measure. In particular we obtain that \([h_1^*(\omega)]^{v^*} \) is more equal than \([h_1(\omega)]^v \) [see Theorem 3.A.5 in Shaked and Shanthikumar (1994)]. Also we
have $[\bar{h}_1]^\eta < [\bar{h}_1]^\eta$. This implies, using (17), that $h_2^*(\omega)$ is more equal than $h_2(\omega)$. As in our earlier proofs it is easy to see that this process can be continued to generalize this to all periods.

\[ \square \]

**Proof of Proposition 8:** Let us use the fact that in our model the inequality in incomes originates from the inequality in human capital distribution, since the same wage rate multiplies $h_t(\omega)$ [see (17)]. Now, the trade and physical capital flow will result in equal wages and rates of interest in both countries. Moreover, we claim that in such a case there is no effect on the optimal choices of parental investment in their children; namely, that $\epsilon_t(\omega)$ will not vary. This can be verified directly from (12) and (13), after substituting $y_{t+1}(\omega)$ by (16): given $h_t(\omega)$, $\epsilon_t(\omega)$ and hence $h_{t+1}(\omega)$ will not vary as we change $r_{t+1}$ and $w_{t+1}$ as well. Thus the human capital accumulation process will not vary and the sets $A_t$ as well [see inequality (19)]. Now, consider (17) and (18) to verify that the distribution of $h_{t+1}(\omega)$ will not change for $t=0,1,2,\ldots$. \[ \square \]

**References**


[42] Quah, D., 2002b, One third of the world’s growth and inequality, Working paper (April), London School of Economics.


Figure 1  IRS and Public Education

\[ h_{t+1}(\omega) \]

\[ h_t(\omega) \]

\[ \tau_t > 0 \]

\[ \tau_t = 0 \]

B 45° C A