The Impact of International Trade on Institutions and Infrastructure

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Abstract

We develop a theoretical model that explores the impact of international trade on both institutions and infrastructure, while explicitly addressing the correlation between institutional quality and infrastructure investment. We show that trade leads to higher infrastructure investment so that domestic firms become more productive and can thus better compete internationally. However, infrastructure investment also has a detrimental effect on firms, as it is financed through firm taxation. As a result, when some firms have stronger political ties than others, trade leads to weaker institutions and more cronyism as the government attempts to lower the tax burden on the politically connected firms. Moreover, we show that trade with a partner characterized by high aggregate firm productivity or lower firm fixed costs induces a country to invest more heavily in both its infrastructure and its institutional framework.

JEL classification: F12; H54; P48

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1 Introduction

Crony capitalism is prevalent in many developing countries, where firms with close ties to the government receive favors of significant economic value. Such preferential treatment includes tax breaks, investment credits, subsidies, subsidized loans, or relaxed regulatory oversight by the authorities. For example, Sorj (1998, p. 28) argues that Brazil’s state land policies have been traditionally characterized by the following pattern: “the state takes responsibility for the onus, the bonus is distributed among the dominant classes, and the crumbs [migalhas] are left over for the subaltern groups.”\(^1\) In Indonesia, during Suharto’s 32-year rule, there was well-documented deep-seated cronyism and nepotism; as a result, firms connected to the Suharto family experienced a negative shock to their stock values once rumors started circulating that Suharto was facing serious health problems (Fisman, 2001).\(^2\) South Korea is another such example of an Asian economy with a high degree of cronyism. Since its independence in 1948, it has seen a seemingly endless flow of corruption scandals bring down scores of its elites, including members of many presidential staffs, numerous military officers, politicians, bureaucrats, bankers, businessmen, and tax collectors (Kang, 2002). Moreover, in Russia in the 1990s, as the economy was integrating in the world economy, there is evidence that politically strong governors were successfully assisting (some) enterprises in their regions to avoid paying federal taxes (Ponomareva and Zhuravskaya, 2004).

According to existing theory, cronyism is stronger in the presence of high rents, as the latter create contests for their capture by the different elite groups, resulting in severe distortions in the economy (e.g., Auty, 2010). At the same time, international trade is often proposed as a remedy for improving economic and political institutions (e.g., IMF, 2005; Johnson et al., 2009), and thus reducing cronyism. However, opening an economy to trade potentially results in precisely such high rents. A question that then naturally arises is whether international trade is in fact the appropriate remedy for cronyism. In this paper, we propose a novel mechanism

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\(^1\) Pereira (2003) argues that the recent comprehensive agrarian reform of the Cardoso government (1995–2002) was characterized by a similar pattern, as it disproportionately benefitted a small number of politically powerful, large producers.

\(^2\) Similar outcomes have been observed in Malaysia following the Asian financial crisis of 1997 (Johnson and Mitton, 2003).
explaining how international trade could lead to more cronyism, although it still results in overall efficiency gains for the economy. The key point is that the government can use public investment to increase the international competitiveness and thus the profitability of domestic firms (e.g., by investing in trade-related infrastructure). We then argue that in the presence of politically connected firms, the ability of the government to implement such policies might lead, under trade, to a deterioration of the institutions in place.

More specifically, we model an economy where consumer preferences are characterized by love for variety. Furthermore, we assume that some firms have political connections to the government, allowing them to receive preferential tax treatment as compared with the rest of the firms (e.g., allowing them to secure larger tax breaks or evade their tax obligations more extensively than nonconnected firms). The government performs two main tasks in this economy: It carries out trade-related infrastructure investment and determines the quality of institutions, financing both through firm taxation. The former task has important ramifications for the production side of the economy, as it affects firms’ productivity, and thus their ability to compete against foreign firms both domestically and abroad. The latter has a significant impact on the distribution of the tax burden among firms. The reason is that stronger institutions mitigate the importance of political connections, leading to a more equal tax treatment across firms. On the other hand, weaker institutions enable politically connected firms to better exploit their connections in order to receive preferential tax treatment at the nonconnected firms’ expense.

We find that in equilibrium only the firms that are relatively politically connected find it optimal to engage in production. Furthermore, under trade, only the most connected of them export.\(^3\) We then show that trade results in political favoritism permeating the economy further in comparison with autarky. Intuitively, once the economy opens up to trade, the government has a strong incentive to invest in trade-related infrastructure, so that firms’ productivity and thus their international competitiveness increase. However, greater infrastructure investment implies a higher firm tax burden. Therefore, the government caring about the politically

\(^3\)This result is consistent, at a broad level, with the findings of a number of empirical studies that show that political connectedness and firm size are strongly and positively correlated (e.g., Agrawal and Knoeber, 2001; Johnson and Mitton, 2003; Faccio, 2006).
connected firms, “compensates” them by distorting institutions in their favor so that they receive a more favorable tax treatment. In other words, our findings illustrate that in the presence of politically connected firms, international trade is not a remedy for weak institutions, as an open economy ends up with more extensive cronyism than a closed one (but it still attains higher national welfare). At a more general level, our findings suggest that a reform involving extensive trade liberalization might give rise to an oligarchic regime.

Another important finding that emerges from our analysis is that the domestic economy’s benefits from international trade are a function of its trading partner. The higher the aggregate firm productivity or the lower the firm fixed costs in the foreign country, the higher the infrastructure investment and the institutional quality in equilibrium in the home country. Institutional quality rises in this case because the government needs a larger tax base in order to finance the higher spending on domestic infrastructure.

Historically there are many instances where international trade has gone hand in hand with cronyism. In the 1700s, for example, the economies of the Caribbean concurrently experienced substantial trade growth, the proliferation of slavery, and the emergence of oligarchic societies (Engerman and Sokoloff, 2002). Another example is Germany and other European states in the 16th and early 17th century, where high economic and trade growth coincided with elevated taxation and the redistribution of resources to favored groups in the economy (Ogilvie, 1992). Our predictions, therefore, are consistent at a broad level with a number of historical experiences.

Our paper relates to a broad literature on international trade, infrastructure, and institutions. One strand of this literature addresses the effect of various aspects of institutional quality on trade flows. Specifically, Levchenko (2007) and Ranjan and Lee (2007) look at contract enforcement, Anderson and Marcouiller (2002) employ a broad measure of institutional quality in their analysis, whereas Dutt and Traca (forthcoming) focus on corruption. A second strand investigates the impact of infrastructure on international trade, including Bougheas et al. (1999) and Limao and Venables (2001). Moreover, a few papers, such as Francois and Manchin (2007), look at both infrastructure and institutions and study empirically their joint impact on trade flows.
Another strand of this literature explores how trade liberalization affects the quality of domestic institutions, either empirically or theoretically. Empirical papers include Cheptea (2007) and Ades and Di Tella (1997), whereas theoretical works, to which this paper belongs, include Segura-Cayuela (2006) and Do and Levchenko (2009). Segura-Cayuela (2006) argues that given a set of weak political institutions, trade might increase the inefficiency of domestic economic policies, whereas Do and Levchenko (2009) argue that trade openness may be detrimental to institutional quality when firms differ with regard to their productivity, which in their model directly affects their political power. These papers, however, examine neither infrastructure investment nor the interaction between firms’ political ties and institutional quality. Both are key for economic performance as the former affects countries’ export performance, while the latter determines firms’ political power, and thus influences the efficiency of government policies.

The setup of our model is provided in Section 2. The autarky equilibrium is derived in Section 3, and the trade equilibrium is obtained in Section 4. The equilibrium level of infrastructure investment and institutional quality is numerically derived and characterized in Section 5. Section 6 discusses some possible avenues for future research and concludes. All proofs are relegated to the Appendix.

2 The Model

Our modeling of the economy relies heavily on Dixit and Stiglitz’s (1977) model of monopolistic competition, and is clearly inspired by the Melitz (2003) model of trade with heterogeneous firms, as well as by Helpman et al. (2004). We now outline our model. In particular, we first describe the consumption and production decisions faced by the economic agents. We then define the country’s institutional quality and its government’s objective function. We conclude this section by specifying the timing of the different actions that will be undertaken in this economy.
2.1 Consumption

Consider an economy consisting of two sectors. The first sector produces a homogeneous good $z$, while the second one produces a continuum of differentiated goods indexed by $v$. The preferences of a representative consumer are captured by the following utility function:

$$U = \left[ \int_{v \in V} x(v)^\alpha \, dv \right]^{\frac{\beta}{\alpha}} \frac{\beta}{1-\beta},$$

where $V$ is the set of available varieties of the differentiated good, $x(v)$ is the quantity consumed of variety $v$, and $\alpha, \beta \in (0, 1)$, with $\beta$ measuring the share of total expenditure spent on the differentiated good. Standard utility maximization results in the following demand functions:

$$x(v) = Ap(v)^{-\varepsilon} \quad \text{and}$$

$$z = \frac{(1-\beta)E}{p_z},$$

where $A \equiv \frac{\beta E}{\int_{v \in V} p(v)^{1-\varepsilon} \, dv}$ is the shift parameter of the demand for any variety of the differentiated good that is exogenous from the point of view of an individual firm, $E$ is total expenditure, $p(v)$ and $p_z$ are the goods prices, and $\varepsilon = \frac{1}{1-\alpha} > 1$ measures the elasticity of substitution between different varieties of the differentiated good. The economy’s aggregate price level is then given by:

$$P = \left( \int_{v \in V} p(v)^{1-\varepsilon} \, dv \right)^{\frac{\beta}{1-\varepsilon}}.$$ 

2.2 Production

Labor is the only factor of production and is inelastically supplied at its aggregate level $L$. The homogeneous good $z$ is produced with a linear technology of the form $l_z = z$. We normalize $p_z$ to 1, implying that the wage in the economy also equals 1 in equilibrium.
In the differentiated sector, there is a continuum of firms, each of which can produce only a single variety. The mass of the potentially produced varieties is fixed and equals $N$. Variety $v$ can be supplied according to the following production technology:

$$l_v = a(v) f(F, \theta) + \frac{x(v)}{F},$$

where $F$ is the level of infrastructure in the economy, $\theta \in (0, 1)$ is a parameter representing institutional quality, with higher $\theta$ signifying higher institutional quality, and $a \geq 0$ is a parameter representing the discriminatory treatment the government applies across firms. Firms share the same $f(F, \theta) > f > 0$, with $\partial f / \partial F > 0$ and $\partial f / \partial \theta > 0$, but have a different $a$ and thus, a heterogeneous fixed cost of operation, $a f(F, \theta)$. Of course, each firm is free not to operate and therefore, not to incur this fixed cost. The technology specified in (4) reflects two key assumptions. First, in order to operate, firms have to incur a fixed cost that consists of (i) the costs levied by bureaucracy, such as regulatory compliance costs; and (ii) a lump-sum tax imposed by the government that is used to finance infrastructure spending and institution building, meaning that this tax is increasing in $F$ and $\theta$. Since firms receive differential treatment (in accordance with their $a$), some of them avoid part of these costs and thereby incur a lower fixed cost of operation, while others bear a disproportionately heavier financial burden. Second, active firms also face a variable cost of operation including production as well as distribution costs. This cost is homogeneous across firms. In addition, it is decreasing in the infrastructure level $F$ since, for example, a more developed transportation system reduces

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4 This assumption simplifies the analysis considerably. It has also been used by Chaney (2008) and Arkolakis (2008), among others.

5 We could alternatively assume that infrastructure investment and institution building are funded through a profit tax. However, this would not affect the qualitative nature of our findings. On a different note, our assumption that firm taxation—as opposed to other forms of taxation such as personal income taxes—finances infrastructure investment and institution building seems particularly appropriate for developing countries. For instance, according to the World Bank (WB, 2004), more than 70% of government revenues in low-income countries are raised through corporate taxes and taxes on commercial transactions.

6 See, for example, Pereira (2003) and Anassi (2004) for evidence on the link between political connectedness and tax evasion in Brazil and Kenya, respectively.

7 Note that all firms in the differentiated sector within our economy are characterized by the same productivity (although as will become evident below, firm productivity does differ across countries). We maintain this assumption because we choose to remain agnostic on the relation between firm productivity and political connectedness.
distribution costs, while a more reliable electricity grid has a dampening effect on the firms’ production costs.\footnote{See, for instance, Fernald (1999) and Esfahani and Ramírez (2003) for, respectively, US and cross-country evidence on the positive impact of infrastructure on productivity and economic growth.}

We can then index firms in the differentiated sector by $a$. Any firm deciding to produce faces a residual demand curve given by (1). Profit maximization yields the following pricing strategy for all firms:

$$p(a) = \frac{1}{F^\alpha}.$$ 

Thus, the equilibrium price is decreasing in $F$ as well as in the elasticity of substitution $\varepsilon$ (since $\varepsilon = \frac{1}{1-\alpha}$). Firm profit is then:

$$\pi(a) = p(a) x(a) - l(a) = A \left( \frac{1}{F^\alpha} \right)^{1-\varepsilon} (1-\alpha) - af(F, \theta),$$

implying that firms facing a lower discriminatory-treatment parameter $a$ enjoy higher profitability.

### 2.3 The Institutional-Quality Coefficient

In this economy, some firm owners in the differentiated sector have stronger political ties than others, allowing them to receive preferential treatment from the government. This preferential treatment could translate into lighter taxation, relaxed regulatory oversight by the authorities, or stiffer regulatory supervision of their competitors. By contrast, firms with weak political connections get exploited. The political ties of a given firm are represented by $a_0$, with a lower $a_0$ signifying that the firm is more politically connected. We assume that $a_0$ is uniformly distributed on $[0, 2]$. However, the ability of the government to discriminate among firms hinges on the checks and balances of the economy, summarized in our model by the institutional-quality coefficient $\theta$. As institutional quality improves (i.e., as $\theta$ rises), the impact of a firm’s political ties on its profits is mitigated.
More specifically, the effect of institutional quality on the significance of political ties is captured by the discriminatory-treatment parameter $a$:

$$a = a_0 (1 - \theta) + \theta,$$

with a cumulative distribution function $G(a) = \frac{a - \theta}{2(1 - \theta)}$. Therefore, for given institutional quality, a firm with closer ties to the government (i.e., a firm with a lower $a_0$) has to incur a lower fixed cost in order to supply the differentiated good. Intuitively, such a firm enjoys lower regulatory compliance costs and/or lighter taxation. As institutional quality improves, however, political ties to the current government matter less and firms’ fixed cost of operation converges to $f(F, \theta)$. It is apparent that firms with $a_0 < 1$ prefer poor institutions as they can then take advantage of their political connections, whereas firms with $a_0 > 1$ prefer institutions of high quality in order to avoid government exploitation.

It is important to note here that some empirical support for (6) is provided by Faccio (2006). In particular, she shows that in countries with high corruption (i.e., low institutional quality), a public announcement revealing a new political connection for a firm is associated on average with positive 5-day cumulative abnormal returns in the stock market of 4.32%. On the other hand, in countries with low corruption (i.e., high institutional quality), such an announcement results on average in almost zero abnormal returns.

### 2.4 The Government Objective Function

The government maximizes an objective function $\Gamma$ subject to a balanced-budget constraint, where $\Gamma$ comprises two terms: (i) labor real income; and (ii) real firm profits weighted by the discriminatory-treatment parameter $a$, such that firms with closer ties to the government receive a larger weight. Formally:

$$\Gamma = \frac{L}{P} + \frac{N \int_{a \in \Delta} \lambda(a) \pi(a) dG(a)}{P},$$

where $\Delta$ denotes the set of firms operating in equilibrium, and $\lambda(a)$ is the weight that the government places on firm $a$, with $\frac{\partial \lambda}{\partial a} < 0$. Observe that by setting $\lambda(a) = 1$ in $\Gamma$, national welfare $W$ is obtained:

$$W = \frac{L}{P} + \frac{N \int_{a \in \Delta} \pi(a) dG(a)}{P}.$$
2.5 The Multistage Game

In order to explore the implications of international trade for crony capitalism, two scenarios are considered for this economy: autarky and trade. For both, we consider the following multistage game:

- **Stage 1:** The government picks the institutional-quality coefficient $\theta$ so as to maximize its objective function $\Gamma$ defined by (7) subject to a balanced-budget constraint.
- **Stage 2:** The government selects the level of infrastructure $F$ in order to maximize $\Gamma$ subject once again to a balanced-budget constraint.
- **Stage 3:** Firms choose whether to actually engage in production or not. Production then takes place, and profits are realized.

This outlines the basic structure of our model. We solve our game recursively. Specifically, we first derive the closed- and open-economy equilibria for a given $\theta$ and $F$. Subsequently, we turn to Stage 2, and solve numerically for the optimal $F$ in autarky and under trade: $F^*_A$ and $F^*_T$, respectively. Last, we look at Stage 1, and obtain numerically the optimal $\theta$ in autarky and under trade: $\theta^*_A$ and $\theta^*_T$, correspondingly.

We should stress here that if we assumed simultaneous rather than sequential timing for the government decisions, all of our main findings would carry through. However, the analysis would be substantially more cumbersome. Therefore, we have chosen to present a sequential game structure.

3 Production Equilibrium in a Closed Economy

In this section, we assume the economy is closed, and that both $\theta_A$ and $F_A$ are given, where subscript $A$ refers to the autarky values. To characterize the autarky production equilibrium, we need to derive the cutoff level of $a$, $a_A$, such that all firms above this $a$ (i.e., firms facing a relatively high fixed cost of operation) decide not to supply the differentiated good.
In equilibrium, since only firms with \( a \leq a_A \) find it optimal to operate, we have that:

\[
\int_{v \in V} p_A(v)^{1-\varepsilon} dv = \frac{N}{(F_A \alpha)^{1-\varepsilon}} G(a_A) \Rightarrow A_A = \frac{(F_A \alpha)^{1-\varepsilon} \beta E_A}{NG(a_A)}.
\]  

(9)

Moreover, the cutoff firm \( a_A \) makes zero profit by definition, i.e., \( \pi_A(a_A) = 0 \), a condition that can be rewritten using (9) as:

\[
\frac{\beta E_A}{NG(a_A)} (1 - \alpha) = a_A f_A,
\]

(10)

where \( f_A \equiv f(F_A) \). We can now use equation (10) to determine \( a_A \), but first the equilibrium value of total expenditure \( E_A \) is required. To obtain the latter, we impose the goods market-clearing condition that total expenditure must equal national income, which is the sum of labor income plus the profits accruing to all active firms:

\[
E_A = L + N \int_{\theta_A}^{a_A} \pi_A(a) dG(a).
\]

Using equation (10), it is straightforward to show that:

\[
E_A = L + N f_A \frac{(a_A - \theta_A)^2}{4(1 - \alpha)}.
\]

(11)

Finally, plugging (11) into (10), we derive \( a_A \).

**Lemma 1** The cutoff value \( a_A \) in the autarky production equilibrium equals:

\[
a_A = \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4L \beta (1 - \alpha)(1 - \theta_A)(2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]},
\]

such that \( a_A > \theta_A \).

Lemma 1 shows that a strictly positive mass of firms operate in equilibrium.\(^9\) This mass decreases with the level of public spending on infrastructure, since a higher \( F_A \) results in an increased tax burden across firms.

**Lemma 2** The cutoff value \( a_A \) decreases with \( F_A \), i.e., \( \frac{\partial a_A}{\partial F_A} < 0 \).

\(^9\) As we show in the Appendix, a second equilibrium exists in which no firm produces, but we choose to focus on the more interesting (and standard) case where the firm with the lowest fixed cost of operation does produce.
Using equation (3) and Lemma 2, we obtain Proposition 1.

**Proposition 1**  Higher infrastructure investment results in a lower aggregate price index provided that \( \varepsilon - 1 > 0 \). This condition is satisfied if \( \frac{\partial F_A}{\partial P_A} \) is sufficiently small.

Intuitively, two offsetting forces are at work here. On the one hand, a higher \( F_A \) lowers the firms’ marginal cost of operation (i.e., \( \frac{1}{F_A} \)) and therefore prices, decreasing \( P_A \). On the other hand, it raises the firms’ fixed cost of operation (i.e., \( a f_A \)) and hence reduces the number of active firms (as demonstrated by Lemma 2), which acts to increase \( P_A \). In other words, infrastructure investment has two rival effects on firms: a beneficial productivity effect and a detrimental fixed-cost effect. If the latter is sufficiently weak (i.e., if \( \frac{\partial f_A}{\partial F_A} \) is sufficiently small), then \( \left| \frac{\partial a_A}{\partial F_A} \right| \) is sufficiently small and the inequality stated in the proposition is satisfied (recall that \( \varepsilon > 1 \) and \( \theta_A < a_A \)).

4  Production Equilibrium in an Open Economy

We next examine the production equilibrium that would arise in our economy under trade with a country characterized by strong institutions. To this end, we model international trade between the home economy (\( H \)) described above and a foreign one (\( F \)) that differs in only two respects: (i) its institutional-quality coefficient \( \theta \) equals 1, i.e., the foreign country is a perfect market economy; and (ii) there is a large (unbounded) mass of potential entrants into its differentiated sector. It is important to emphasize that the two countries are assumed to have the same endowment of labor \( L \), i.e., they are of equal size, which simplifies the comparison between the two economies.

Moreover, we assume that \( z \) can be traded costlessly, implying that as long as both countries produce some \( z \), wages in the two countries equal 1. Nevertheless, firms in the differentiated sector must incur two additional costs in order to export: a fixed per-period cost \( a f_X > 0 \), where \( i \in \{ H, F \} \) and \( f_X \equiv f_X (F^i) \), and a per-unit transportation cost \( \tau \). The former consists of the bureaucratic costs imposed on exporting firms (e.g., the cost of obtaining an export permit) as well as of a government lump-sum tax levied on them in order to finance infrastructure
investment and institution building, is a function of the discriminatory-treatment parameter $a$, and is measured in terms of units of labor. The latter is of the iceberg type, is common across firms, and is denoted by $\tau > 1$. $\tau$ represents the number of units of a particular variety that need to be shipped so that 1 unit arrives at its destination. Without loss of generality, we assume that home and foreign firms in the differentiated sector face the same transportation costs. Note that regardless of whether an exporting firm sells domestically or not, it still incurs the fixed cost $af^i$. A straightforward implication of this assumption is that for all exporting firms, it is optimal to also produce for their domestic market. Furthermore, observe that $\theta = 1$ in the foreign country means that all firms in its differentiated sector have the same fixed cost of operation $f^F$ and the same fixed cost of exporting $f^F_\times$ (i.e., $a = 1$ across country-$F$ firms).

Let us begin our analysis with the home country. To characterize its open-economy equilibrium, we need to determine the cutoff values of $a$ for production and exporting: $a_D$ and $a_X$, respectively. Each firm’s domestic pricing strategy is still $p^H_D(a) = \frac{1}{f^{H\alpha}}$, resulting in the following (domestic) profit:

$$\pi^H_D(a) = A^H \left( \frac{1}{f^{H\alpha}} \right)^{1-\varepsilon} (1 - \alpha) - af^H.$$  

(12)

For the exporting firms, the prices in the foreign market are higher due to the transportation costs: $p^H_X(a) = \frac{\tau}{f^{H\alpha}}$. Their export profits are then equal to:

$$\pi^H_X(a) = A^F \left( \frac{\tau}{f^{H\alpha}} \right)^{1-\varepsilon} (1 - \alpha) - af^H_X.$$  

(13)

By definition, we have that for the cutoff firms $a_D$ and $a_X$: $\pi^H_D(a_D) = 0$ and $\pi^H_X(a_X) = 0$, correspondingly. We also have that in equilibrium, total expenditure must equal national income:

$$E^H = L + N^H \left( \int^a_{a_D} \pi^H_D(a) dG(a) + \int^a_{a_X} \pi^H_X(a) dG(a) \right) =$$

$$= L + N^H \left( f^H(a_D - \theta)^2 + f^H_X(a_X - \theta)^2 \right),$$  

(14)

where the second equality uses the zero-cutoff-profit conditions.

\footnote{It could be argued that an exporting firm also faces a fixed per-period cost associated with the importing-country bureaucracy. However, introducing such a cost that were not too high would not affect the qualitative nature of our findings, and therefore, it is omitted for expositional simplicity.}
We next turn to the foreign country. Here, in the differentiated sector, all firms have an identical cost structure. In addition, unlike in the home country, there is an unbounded mass of potential entrants.\textsuperscript{11} Therefore, to characterize the trade production equilibrium of the foreign country, we need to obtain the equilibrium number of firms selling domestically and exporting: \( n^F_D \) and \( n^F_X \), correspondingly. The domestic and export prices are analogous to those of the home country: \( p^F_D = \frac{1}{F^F\alpha} \) and \( p^F_X = \frac{\tau}{F^F\alpha} \), respectively. The corresponding profits from domestic sales and exports are then:

\[
\pi^F_D = A^F \left( \frac{1}{F^F\alpha} \right)^{1-\epsilon} (1 - \alpha) - f^F \quad \text{and} \quad \pi^F_X = A^H \left( \frac{\tau}{F^F\alpha} \right)^{1-\epsilon} (1 - \alpha) - f^F_X. \tag{15}
\]

These profits are identical across firms because in a perfect market economy, all firms face the same fixed costs of operation and exporting. However, given that there is free entry, equilibrium firm profits are equal to zero, implying that the income-equals-expenditure condition is simply:

\[
E^F = L. \tag{17}
\]

Let us now look at the home and foreign aggregate price indices. In equilibrium, since only firms with \( a \leq a_D \) operate in country \( H \), its aggregate price index equals to:

\[
P^H = \left( N^H \left( \frac{1}{F^H\alpha} \right)^{1-\epsilon} + n^F \left( \frac{\tau}{F^F\alpha} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{18}
\]

At the same time, since only country-\( H \) firms with \( a \leq a_X \) export, the equilibrium aggregate price level of country \( F \) is given by:

\[
P^F = \left( N^H \left( \frac{\tau}{F^H\alpha} \right)^{1-\epsilon} + n^F \left( \frac{1}{F^F\alpha} \right)^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}. \tag{19}
\]

Straightforward algebra then reveals that:

\[
a_D = \frac{f^F_X}{f^H} \left( \frac{F^H\tau}{F^F} \right)^{\epsilon-1}, \tag{20}
\]

\textsuperscript{11}This assumption is made for analytical convenience given that foreign-country firms have a homogeneous cost structure.
\[ a_X = \frac{f^F}{f_X^H} \left( \frac{F^H}{F^F \tau} \right)^{\varepsilon - 1}, \tag{21} \]

\[ n_D^F = \frac{(1 - \alpha) \beta E^F}{f^F} - N^H \left( \frac{F^H}{F^F \tau} \right)^{\varepsilon - 1} \frac{a_X - \theta}{2(1 - \theta)}, \quad \text{and} \tag{22} \]

\[ n_X^F = \frac{(1 - \alpha) \beta E^H}{f_X^F} - N^H \left( \frac{F^H}{F^F \tau} \right)^{\varepsilon - 1} \frac{a_D - \theta}{2(1 - \theta)}, \tag{23} \]

where \( E^F \) and \( E^H \) are given by equations (17) and (14), respectively. A few remarks are here in order. First, infrastructure investment \( F^H \) has a direct positive impact on the number of country-\( H \) firms selling domestically (i.e., on \( a_D \)) as well as on the number of those exporting (i.e., on \( a_X \)). This is due to its productivity effect: As country-\( H \) firms become more efficient, they can better compete against the country-\( F \) firms both domestically and abroad. Nonetheless, as equations (20) and (21) demonstrate, investing in infrastructure also has a detrimental fixed-cost effect on country-\( H \) firms that is reflected in the rise of both \( f^H \) and \( f_X^H \), which acts to reduce the home-country cutoff values for production and exporting. In other words, the overall effect of an increase in \( F^H \) on the number of country-\( H \) firms being active in their domestic or export market is ambiguous. Of course, infrastructure investment in the foreign country \( F^F \) has the exact opposite ramifications for \( a_D \) and \( a_X \). Furthermore, note that \( a_D > a_X \iff f_X^F f_X^H \tau^{2(\varepsilon - 1)} > f^F f^H \), which is a standard assumption in the literature on trade with heterogeneous firms.\(^{12} \) This condition is clearly satisfied when both \( f_X^H \) and \( f_X^F \) are sufficiently large. Last, observe that the level of foreign (home) consumer expenditure on the differentiated good \( \beta E^F (\beta E^H) \) affects directly in a positive manner the number of country-\( F \) firms that are active in their domestic (export) market.

Let now \( \eta^H \equiv \frac{\partial f^H}{\partial F^H} \frac{F^H}{F^F \tau} \) and \( \eta_X^H \equiv \frac{\partial f^H}{\partial F_X^H} \frac{F^H}{F^F \tau} \). In the remainder of this section, we do comparative statics.

**Lemma 3** If the elasticity of substitution is sufficiently large, then the country-\( H \) cutoff values for production and exporting increase with infrastructure investment \( F^H \). In particular, if \((\varepsilon - 1) > \eta^H\), then \( \frac{\partial a_D}{\partial F^H} > 0 \); moreover, if \((\varepsilon - 1) > \eta_X^H\), then \( \frac{\partial a_X}{\partial F^H} > 0 \).

\(^{12} \)See, for instance, Melitz (2003) and Helpman et al. (2004).
In the trade regime, unlike the autarky regime, the home-country cutoff value for production $a_D$ might increase with infrastructure investment $F^H$. Similarly, the cutoff value for exporting $a_X$ might also rise with $F^H$. Intuitively, as we discussed above, an increase in $F^H$ has two offsetting effects on $a_D$ and $a_X$: a positive productivity effect and a negative fixed-cost effect. If the elasticity of substitution is sufficiently large, the former effect dominates because consumers are then highly responsive to price changes. The productivity effect outweighs the fixed-cost effect also when $\eta^H$ and $\eta^H_X$ are sufficiently low, since then raising infrastructure investment does not entail a heavy additional tax burden for country-$H$ firms.

Lemma 4 If the elasticity of substitution is sufficiently large, then the number of country-$F$ firms selling domestically decreases with infrastructure investment $F^H$. In particular, if
\[ (\varepsilon - 1) > \eta^H_X, \text{ then } \frac{\partial n^F_D}{\partial F^H} < 0. \]

The intuition underlying Lemma 4 is straightforward. If the elasticity of substitution is large enough (or $\eta^H_X$ is low enough), the productivity effect of infrastructure investment $F^H$ on country-$H$ firms outweighs its fixed-cost effect, resulting in fewer country-$F$ firms being active in their domestic market in equilibrium (i.e., a lower $n^F_D$). However, it is important to note that in this case, the number of foreign firms exporting (i.e., $n^F_X$) might still rise. This is due to an income effect. More specifically, if $\varepsilon$ is sufficiently large, income increases in the home country as $F^H$ rises due to larger aggregate firm profits, which might potentially lead to a higher demand for imports (despite the lower domestic-firm prices).

Proposition 2 One set of conditions guaranteeing that the aggregate price index in country $H$ decreases as $F^H$ rises is that (i) the elasticity of substitution is sufficiently large; and (ii) infrastructure investment in country $F$ is relatively large compared with that in country $H$. In particular, if:
\[ (\varepsilon - 1) > \max\{\eta^H, \eta^H_X\} \text{ and } 1 > \frac{F^H \partial a_D}{\partial F^H} + (a_D - \theta)(\varepsilon - 1) \left( \frac{F^H \tau}{F^F} \right)^{\varepsilon-1}, \]
then $\partial P^H / \partial F^H < 0$. 

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To gain some insight for Proposition 2, recall that as $F^H$ increases, the domestic prices of the home firms decrease. At the same time, the conditions stated in Proposition 2 ensure that product variety in the home country rises. As a result, the country-$H$ aggregate price index declines and real income increases.

**Proposition 3** If the elasticity of substitution is sufficiently large and infrastructure investment $F^H$ increases (decreases) with the institutional-quality coefficient $\theta$, then the country-$H$ cutoff values for production and exporting increase (decrease) with $\theta$. In particular, if $(\varepsilon - 1) > \eta^H$ and $\frac{\partial F^H}{\partial \theta} > (\leq)0$, then $\frac{\partial a_D}{\partial \theta} > (\leq)0$. Moreover, if $(\varepsilon - 1) > \eta^H_X$ and $\frac{\partial F^H}{\partial \theta} > (\leq)0$, then $\frac{\partial a_X}{\partial \theta} > (\leq)0$.

A high elasticity of substitution implies that both $a_D$ and $a_X$ increase with $F^H$. Proposition 3 then follows trivially when the second condition is also met.

## 5 Infrastructure and Institutions Equilibrium

We next turn to Stages 1 and 2 of our game. To complete our analysis, we need to derive the optimal institutional-quality coefficient $\theta$ and infrastructure investment $F^H$ in autarky and under trade, utilizing the production equilibria characterized above. To this end, we resort to numerical analysis.\(^{13}\) Let us now define the following two relationships. First, for simplicity, we assume that country-$H$ firms’ fixed costs are a linear function of infrastructure investment and of the cost of institution building:

\[
a f^H_j = a \left( f^H_j + b^H_j F^H + c^H_j \theta \right),
\]

where $f^H_j, b^H_j, c^H_j \geq 0$ and $j = X$ when the fixed cost of exporting is considered. At the same time, given that $F^F$ and the institutional-quality coefficient in the foreign country are fixed throughout our analysis, we simply set $f^F = f^F_j$ and $f^F_X = f^F_X$. Second, recall that firm owners in the differentiated sector differ with respect to their political ties to the government as captured by their parameter $a_0$. This affects firms in two ways. On the one hand, they share

\(^{13}\)The numerical analysis was carried out using Matlab. The file is available from the authors upon request.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
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</tr>
<tr>
<td>$\alpha$</td>
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<td>$f^F$</td>
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</tr>
<tr>
<td>$t$</td>
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<td>$c^H$</td>
<td>300</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.15</td>
<td>$c_X^H$</td>
<td>300/8</td>
</tr>
<tr>
<td>$F^F$</td>
<td>2,550</td>
<td></td>
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</tr>
</tbody>
</table>

Table 1: The benchmark model

unequally the infrastructure and institutions-building burden as equation (24) demonstrates. On the other hand, the government weights them differently when choosing infrastructure investment and institutional quality. In particular, the government places weight $\lambda(a)$ on firm $a$’s profits, where $\lambda(a)$ is decreasing in the discriminatory-treatment parameter $a$. This is captured in a simple manner by the following equation:

$$\lambda(a) = (2 - a) \omega,$$

(25)

with $\omega \geq 0$.

The parameters we initially use in order to obtain the numerical solution for the benchmark case are listed in Table 1. A number of assumptions underlie these parameters. First, $\omega = 2$, meaning that the home government faces some domestic political-economy pressures. Second, consumers spend 80\% of their income on the differentiated good, i.e., $\beta = 0.8$. Third, we assume that the cost of infrastructure and institution building is spread out over a 15-year time horizon, with the interest rate $r = 10\%$. Finally, the elasticity of substitution equals 5, which is within the range suggested by the literature (e.g., Anderson and van Wincoop, 2004).
5.1 Trade versus Autarky

We begin by deriving the autarky solution. Specifically, we obtain numerically the optimal level of infrastructure investment $F^*_A$ and the optimal institutional-quality coefficient $\theta^*_A$, utilizing the relationships and the parameters specified above. We find that the former equals 2,103 and the latter 0.2146. Moreover, national welfare $W^*_A$ (given by (8)) equals 8,068,377.

In comparison with autarky, under trade, infrastructure investment in country $H$ is higher, but the institutional-quality coefficient is set at a lower level. In particular, infrastructure investment $F^*_T = 2,216$ and the institutional-quality coefficient $\theta^*_T = 0.2024$. Intuitively, the home government chooses a higher $F^H$ under trade so that domestic firms can better compete against their foreign counterparts both in the home market and abroad. However, although a higher $F^H$ raises firms’ productivity, it also implies, for a given $\theta$, higher fixed costs of operation and exporting. The home government therefore now chooses a lower $\theta$ in order to benefit the politically connected firms by reducing their lump-sum-tax burden. Interestingly, infrastructure investment and institutional quality emerge as substitutes in our setting.

Note that as compared with autarky, under trade, infrastructure investment in country $H$ is 5.4% higher but the institutional-quality coefficient is 5.7% lower. Furthermore, a welfare comparison between autarky and trade reveals that in the home country, national welfare $W$ and the government objective function $\Gamma$ (given by (7)) are higher by, respectively, 23% and 11% under trade. This is mainly due to the fact that when its economy is open, its aggregate price index is 17% lower.

5.2 Trade Comparative Statics

Last, in order to obtain a deeper insight into the effects of trade, we do comparative statics with respect to the trade-related parameters. In particular, we look at the foreign firms’ fixed cost of operation $f^F$ and fixed cost of exporting $f^F_X$, as well as at country $F$’s infrastructure level $F^F$ and at the transportation-cost parameter $\tau$.
We begin with the parameters \( f^F \) and \( f^F_X \). A 5% decrease in either of them results in a trade equilibrium characterized by both higher infrastructure investment \( F^*_T \) and a higher institutional-quality coefficient \( \theta^*_T \). Intuitively, as \( f^F \) and \( f^F_X \) decrease, the home firms face stiffer competition in their foreign and domestic markets, respectively. As a result, the government of country \( H \) finds it optimal to invest more in infrastructure in order to raise the competitiveness of domestic firms. Remember now that the home government finances infrastructure investment by taxing active firms. Therefore, as there is need for higher infrastructure investment, the country-\( H \) government chooses a higher \( \theta \), reducing cronyism in the home country and allowing more firms to operate in equilibrium.

Next, we investigate how a reduction in country \( F \)'s aggregate productivity affects the open-economy equilibrium. To this end, we reduce \( F^F \) by 5% to 2,422.5. This results in a reduction in both \( F^*_T \) and \( \theta^*_T \). As \( F^F \) decreases and foreign firms become less competitive, the government of country \( H \) finds it optimal to invest less in infrastructure. Moreover, a lower \( F^F \) implies, ceteris paribus, a greater number of active firms in equilibrium in country \( H \) (given a sufficiently high \( \varepsilon \)). Therefore, as the financing of \( F^H \) is now easier, the country-\( H \) government chooses a lower \( \theta \), benefitting the most politically connected firms.

Finally, we look at the transportation-cost parameter \( \tau \). A 5% increase in \( \tau \) results in both a lower \( F^*_T \) and a lower \( \theta^*_T \). Intuitively, a higher \( \tau \) insulates the country-\( H \) (and country-\( F \)) firms from import competition. This reduces the need for infrastructure investment in the home country as productivity considerations become less important. Thus, as there is now need for less infrastructure investment, the country-\( H \) government chooses a lower \( \theta \), increasing cronyism in the home country.

Observe here that although infrastructure investment and institutional quality have emerged as substitutes when country \( H \) moves from autarky to trade, they exhibit a complementary relationship under the trade regime when the underlying trade-related parameters change. The intuition is straightforward. Even though the country-\( H \) government wishes to favor the politically connected firms by lowering \( \theta \), its policies are subject to a budget constraint, which is a function of firm taxation. When the economy moves from autarky to trade, a new source of tax revenue is at the government’s disposal, as it can now tax the firms’ export profits,
enabling it to both invest more in infrastructure and lower the quality of domestic institutions. However, under the trade regime, the budget constraint is binding, forcing the government to raise $\theta$ whenever higher infrastructure investment is required as a result of a change in the trade-related parameters.

6 Conclusion

This paper has investigated the impact of international trade on both institutional quality and infrastructure investment. Our analysis has rested on the assumptions that (i) some firm owners have stronger political ties than others, allowing them to receive preferential treatment from the government; (ii) a high institutional quality mitigates the significance of political connections; and (iii) infrastructure investment has two rival effects on firms: a detrimental fixed-cost effect since it is financed through lump-sum firm taxation and a beneficial productivity effect. We have demonstrated that trade leads to higher infrastructure investment so that domestic firms become more efficient and can therefore better compete internationally. However, trade also leads to weaker institutions and more extensive cronyism as the government attempts to lower the heightened tax burden on the politically connected firms. In other words, infrastructure investment and institutional quality have emerged as substitutes within our setting. Moreover, we have shown that trade with a partner characterized by high aggregate firm productivity or lower firm fixed costs induces a country to invest more in both its infrastructure and its institutional framework, even though the latter is against the interests of the firms with the stronger political ties.

In future work we plan to extend our analysis in a number of dimensions. In particular, we intend to develop a dynamic multicountry trade model in order to study whether trade could have a domino effect on institutional quality and infrastructure investment across countries. For instance, we wish to address the following question: If a developing country engaged in trade with a developed market economy and as a result invested heavily in its infrastructure as our model predicts, would this be a catalyst for infrastructure development and institutional reform for the rest of its trading partners? The answer to such a question would have important
policy implications for regions such as Africa or South America. Another avenue we plan to pursue is to empirically test our findings. Of course, the definition of institutions in our paper has been quite broad. This was deliberate, as we chose to abstract from institutional details for expository simplicity. Nevertheless, a number of predictions our model generates could be readily tested. In particular, does trade liberalization lead to higher trade-related infrastructure investment? Does it, however, simultaneously lead to more extensive cronyism (or increased inequality)? Such an exercise would provide us with significant insights regarding the overall gains and losses stemming from an extensive trade reform.

References


7 Appendix

7.1 Production Equilibrium in a Closed Economy

7.1.1 Proof of Lemma 1

Plugging (11) into (10), we obtain the following solutions for $a_A$:

$$ a_{A,1,2} = \begin{cases} 
\frac{N f_A \theta_A [1 - \beta (1 - \alpha)] - \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} & \\
\frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]}
\end{cases} $$

Let us look first at $a_{A_1}$:

$$ a_{A_1} = \frac{\theta_A [1 - \beta (1 - \alpha)]}{2 - \beta (1 - \alpha)} - \frac{\sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} < \frac{1 - \beta (1 - \alpha)}{2 - \beta (1 - \alpha)} < \theta_A. $$

We now turn to $a_{A_2}$:

$$ a_{A_2} = \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} > \theta_A \iff 4 N f_A L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) > 0, $$

which clearly holds. Therefore, we have a unique closed-economy equilibrium in which at least some of the potential producers in the differentiated sector do produce:

$$ a_A = \frac{N f_A \theta_A [1 - \beta (1 - \alpha)] + \sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}}{N f_A [2 - \beta (1 - \alpha)]} > \theta_A. $$

This concludes the proof of Lemma 1.

7.1.2 Proof of Lemma 2

Taking the first-order derivative of $a_A$ with respect to $F_A$, we obtain:

$$ \frac{\partial a_A}{\partial F_A} = -\frac{1}{f_A \theta F_A} \frac{2 L \beta (1 - \alpha) (1 - \theta_A)}{\sqrt{N f_A [4 L \beta (1 - \alpha) (1 - \theta_A) (2 - \beta (1 - \alpha)) + N f_A \theta_A^2]}} < 0, $$

since $\frac{\partial f_A}{\partial F_A} > 0$ by assumption. This concludes the proof of Lemma 2.
7.1.3 Proof of Proposition 1

Straightforward algebra reveals:

\[
\frac{\partial P_A}{\partial F_A} = \frac{\beta}{2(\varepsilon - 1)(1 - \theta_A)} \left( \frac{a_A - \theta_A}{2(1 - \theta_A)} \right)^{\beta + \frac{1}{1 - \varepsilon}} N^{\frac{\delta}{1 - \varepsilon}} (F_A \alpha)^{-\beta} \left[ \frac{(\varepsilon - 1)(\theta_A - a_A)}{F_A} - \frac{\partial a_A}{\partial F_A} \right] < 0 \Leftrightarrow \\
\Leftrightarrow \frac{(\varepsilon - 1)(\theta_A - a_A)}{F_A} - \frac{\partial a_A}{\partial F_A} < 0,
\]

where the “\(\Leftrightarrow\)” follows since the terms multiplying the square brackets are all strictly positive. The first term in (27) is strictly negative since \(a_A > \theta_A\) by Lemma 1, whereas the second one is strictly positive since \(\frac{\partial a_A}{\partial F_A} < 0\) by Lemma 2. However, if \(\frac{\partial f_A}{\partial F_A}\) is sufficiently small, then \(\frac{\partial a_A}{\partial F_A}\) is sufficiently small by (26), and the inequality in (27) is satisfied. This concludes the proof of Proposition 1.

7.2 Production Equilibrium in an Open Economy

7.2.1 Proof of Lemma 3

Straightforward calculations yield:

\[
\frac{\partial a_D}{\partial F^H} = a_D \left( \frac{\varepsilon - 1}{F^H} - \frac{1}{f^H \frac{\partial f^H}{\partial F^H}} \right) > 0 \Leftrightarrow \\
\Leftrightarrow \frac{\varepsilon - 1}{F^H} - \frac{1}{f^H \frac{\partial f^H}{\partial F^H}} > 0 \Leftrightarrow \varepsilon - 1 > \frac{f^H \frac{\partial f^H}{\partial F^H}}{f^H \frac{\partial f^H}{\partial F^H}} = \eta^H,
\]

where the first “\(\Leftrightarrow\)” follows since \(a_D > \theta > 0\). Moreover, we have:

\[
\frac{\partial a_X}{\partial F^H} = a_X \left( \frac{\varepsilon - 1}{F^H} - \frac{1}{f^X \frac{\partial f^H}{\partial F^H}} \right) > 0 \Leftrightarrow \\
\Leftrightarrow \frac{\varepsilon - 1}{F^H} - \frac{1}{f^X \frac{\partial f^H}{\partial F^H}} > 0 \Leftrightarrow \varepsilon - 1 > \frac{f^H \frac{\partial f^H}{\partial F^H}}{f^X \frac{\partial f^H}{\partial F^H}} = \eta^H,
\]

where the first “\(\Leftrightarrow\)” follows due to \(a_X > \theta > 0\). This concludes the proof of Lemma 3.

7.2.2 Proof of Lemma 4

Simple algebra reveals:

\[
\frac{\partial \eta^D}{\partial F^H} = -\frac{N^H}{2(1 - \theta)} \left( \frac{F^H}{F^H} \right)^{\varepsilon - 1} \left[ \frac{(\varepsilon - 1)(a_X - \theta)}{F^H} + \frac{\partial a_X}{\partial F^H} \right].
\]
If \( \varepsilon - 1 > \eta_X^H \), then \( \frac{\partial a_X}{\partial F^H} > 0 \) by Lemma 3, implying that the sum in the square brackets is strictly positive, and thus, \( \frac{\partial n_X^F}{\partial F^H} < 0 \). This concludes the proof of Lemma 4.

### 7.2.3 Proof of Proposition 2

We know from (18) that:

\[
p^H = \left( N^H \left( \frac{1}{F^H} \right)^{1-\varepsilon} \frac{a_D - \theta}{2(1-\theta)} + n_X^F \left( \frac{\tau}{F^F} \right)^{1-\varepsilon} \right)^{\frac{\beta}{1-\varepsilon}}.
\]

As \( F^H \) rises, the domestic prices of the home firms (i.e., \( p^H_D = \frac{1}{F^H} \alpha \)) decrease, whereas the export prices of the foreign firms (i.e., \( p^F_X = \frac{\tau}{F^F} \alpha \)) remain unchanged. At the same time, it is direct to show that total product variety in the home country equals:

\[
n_X^F + N^H \frac{a_D - \theta}{2(1-\theta)} = \left( 1 - \frac{\alpha}{F^F} \right) \left[ L + N^H \left( \frac{f^H (a_D - \theta)^2}{4(1-\theta)} + f_X^H (a_X - \theta)^2 \right) \right]
\]

\[
+ N^H \frac{a_D - \theta}{2(1-\theta)} \left[ 1 - \left( \frac{F^H \tau}{F^F} \right)^{\varepsilon-1} \right].
\]

It can be readily shown that \( \frac{\partial \Phi}{\partial F^H} > 0 \) if \( \frac{\partial a_D}{\partial F^H} > 0 \) and \( \frac{\partial a_X}{\partial F^H} > 0 \). These conditions are true (i.e., \( \frac{\partial a_D}{\partial F^H} > 0 \) and \( \frac{\partial a_X}{\partial F^H} > 0 \)) as long as \( (\varepsilon - 1) > \max \{ \eta^H, \eta_X^H \} \) by Lemma 3. Moreover, we have that:

\[
\frac{\partial \Lambda}{\partial F^H} > 0 \iff \frac{\partial a_D}{\partial F^H} > \left( \frac{F^H \tau}{F^F} \right)^{\varepsilon-1} \left( \frac{\partial a_D}{\partial F^H} + \frac{(a_D - \theta)(\varepsilon - 1)}{F^H} \right).
\]

The proposition then follows.

### 7.2.4 Proof of Proposition 3

Using the chain rule, we obtain:

\[
\frac{da_D}{d\theta} = \frac{\partial a_D}{\partial \theta} + \frac{\partial a_D}{\partial F^H} \frac{\partial F^H}{\partial \theta}
\]

\[
\text{and}
\]

\[
\frac{da_X}{d\theta} = \frac{\partial a_X}{\partial \theta} + \frac{\partial a_X}{\partial F^H} \frac{\partial F^H}{\partial \theta}
\]

The proposition then follows directly from Lemma 3.