Bulat N. Khabibullin (Russia, Ufa) ZERO SUBSEQUENCES FOR A CLASS OF ENTIRE FUNCTIONS AND CRITERIONS OF COMPLETENESS FOR EXPONENTIAL SYSTEM

Denote by \( \mathbb{N}, \mathbb{R}, \) and \( \mathbb{C} \) the sets of natural, real and complex numbers.

For a segment \( I_d \subset \mathbb{R} \) of length \( d \), we denote by \( C(I_d) \) and \( L^p(I_d) \) the space of continuous functions on \( I_d \) with sup-norm and the space of functions \( f \) with finite norm

\[
\|f\|_p := \left( \int_{I_d} |f(x)|^p \, dx \right)^{1/p}, \quad p \geq 1.
\]

For \( \sigma \in (0, +\infty) \), denote by \( B^\infty_\sigma \) the Bernstein space (of type \( \sigma \)) of all entire (holomorphic on \( \mathbb{C} \)) functions of exponential type \( f \) bounded on \( \mathbb{R} \), i.e.

\[
|f(z)| \leq C_f \exp(\sigma |\text{Im} \, z|), \quad z \in \mathbb{C},
\]

where \( C_f \) is a constant. Let \( \Lambda = \{\lambda_k\}_{k \in \mathbb{N}} \) be a point sequence on \( \mathbb{C} \) without limit points in \( \mathbb{C} \), and all points \( \lambda_k \) are pairwise various.

The sequence \( \Lambda \) is a zero subsequence for \( B^\infty_\sigma \) iff there exists a nonzero function \( f_\Lambda \in B^\infty_\sigma \) such that \( f_\Lambda \) vanish on \( \Lambda \), i.e. \( f_\Lambda(\lambda_k) = 0 \) for \( k \in \mathbb{N} \).

The exponential system \( \{e^{i\lambda_k z}\}_{k \in \mathbb{N}}, \, z \in I_d \), is complete in \( C(I_d) \) (\( L^p(I_d) \) resp.) iff the closure of its linear span coincides with \( C(I_d) \) (\( L^p(I_d) \) resp.). We solve the following problems.

- **Complete description of zero subsequences for \( B^\infty_\sigma \).**

- **Criterions of completeness of exponential system \( \{e^{i\lambda_k z}\}_{k \in \mathbb{N}} \) in \( C(I_d) \) or \( L^p(I_d) \) to within one or two exponential functions.**

We formulate these new results in terms of Poisson and Hilbert transforms. Their proofs are unpublished (2011, May).

From the criterions of completeness we can obtain all basic old results on completeness of exponential systems in spaces \( C(I_d) \) and in \( L^p(I_d) \) (for example, famous Berling–Malliavin Theorem on the radius of completeness), and also new results.