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Exploring the Possibility of Detecting Rare Events in Models with Large Extra Spatial Dimensions

Master Thesis

of

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Chapter 1

Motivation For The Large Extra Dimensions Model

1.1 Success of The Standard Model

The standard model is a mathematically consistent renormalizable theory, which has successfully interpreted the observable fundamental particles and their electromagnetic, weak and strong interactions.

From a theoretical point of view, the standard model is a theory based on local gauge symmetries, the space-time symmetries of Einstein's special relativity, and quantum physics. This has been a great theoretical success, since any viable model, attempting to describe nature, should be based on the very fundamental principles of nature, as we know them from the evolution of physics in the past century. The complete standard model is an SU3$\bigotimes$SU2$\bigotimes$U1 theory which is manifestly broken to the SU2$\bigotimes$U1 electroweak model and the SU3 model of the strong interactions, that is quantum chromodynamics.

The great success of the standard model is that over the years it has been tested in collider experiments, up to the energy scale of almost 1 TeV. In all these tests it has been found to be in remarkable agreement with all the experimental data, and what's more important, almost all the predictions given by the model were confirmed. That is, it has successfully predicted the existence and form of the weak neutral current, the existence and masses of W and Z bosons, and the mass of c quark. The charged current weak interactions were successfully incorporated,as was quantum electrodynamics. As for the strong interactions, the developments in lattice field theory, along with experiments on heavy quarks and high energy $p\overline{p}$ and ep collisions indicate that quantum chromodynamics is the correct theory. Finally we have the discovery of the top quark, completing the third fermion family of the model and the verification , up to the 1 TeV energy scale of the structureless nature of quarks and leptons.

Seen from a strictly experimental point of view, there is almost no reason for moving beyond the standard model. One deviation is the recently discovered neutrino oscillations, implying that neutrino's have mass. Of course there is still the Higgs boson to be discovered, but that is not a problem since the energy scale at which it is expected to be found, has not yet been probed. Given the obvious success, why should we doubt the standard model? In section 1.3 we will see that despite all its success, there are still a lot of unsolved problems, that a proper fundamental theory should be able to answer. Before we go on examining the problems of the standard model, we will briefly describe the electroweak symmetry breaking, since this will help us to fully understand several of the problems we will be refereing to.

1.2 Electroweak symmetry breaking

One of the most important contents of the standard model is the symmetry breaking of the $SU(2) \times U(1)$ gauge symmetry of electroweak interactions. Electromagnetism and weak interactions are unified above an energy scale, that is the electroweak scale. The separation occurs because the gauge bosons responsible for the weak interactions, $W^{\pm}$ and Z, acquire mass via the spontaneous symmetry breaking, whilst the photon remains massless, mediating the electromagnetic interactions. At the same time, it is clear that the same mechanism is established as the origin of fermion masses.

In order to achieve the symmetry breaking, a scalar field should be introduced in the Lagrangian of the standard model.The scalar field has a lagrangian[1]:

$$L = (D^{\mu}\varphi)^{\dagger} D_{\mu}\varphi - V(\varphi) \tag{1.1}$$

where the Scalar field potential is given by:

$$V(\varphi) = \frac{1}{2}\mu^2 \varphi^{\dagger}\varphi + \frac{1}{4}\lambda(\varphi^{\dagger}\varphi)^2 \tag{1.2}$$

With μ^2 chosen to be less than zero, the scalar potential has a minimum

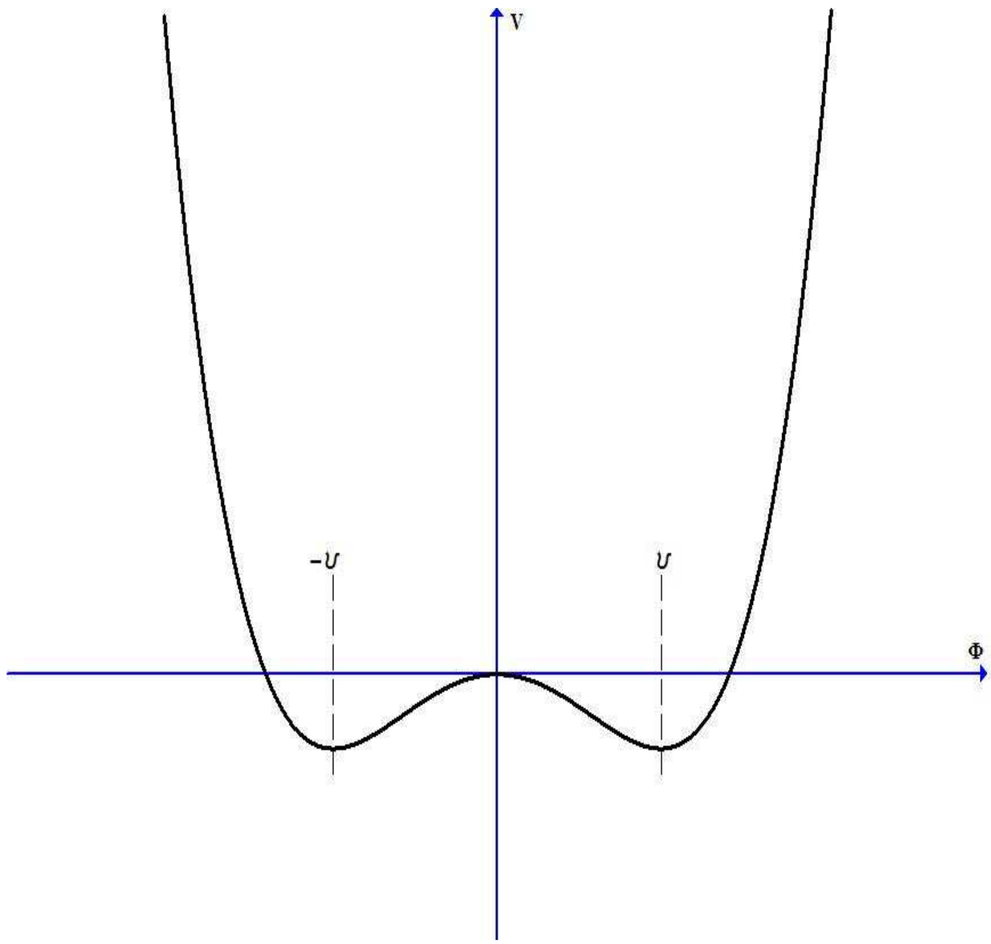

Figure 1.1: Higgs Potential.

value at the points $\pm v$ where v can be easily calculated to be $v = \sqrt{-\mu^2/\lambda}$. Thus the electroweak symmetry is spontaneously broken to the U(1) of electromagnetism and the SU(2) of weak interactions.

Choosing to expand around the minimum we introduce a new field H, which will prove to be the Higgs boson field. With the substitution $\varphi = v + H$ we get the Higgs potential after the symmetry is broken:

$$V = -\frac{\mu_4}{4\lambda} - \mu^2 H^2 + \lambda v H^3 + \frac{\lambda}{4} H^4 \qquad (1.3)$$

The first term is the vacuum expectation value of the Higgs potential

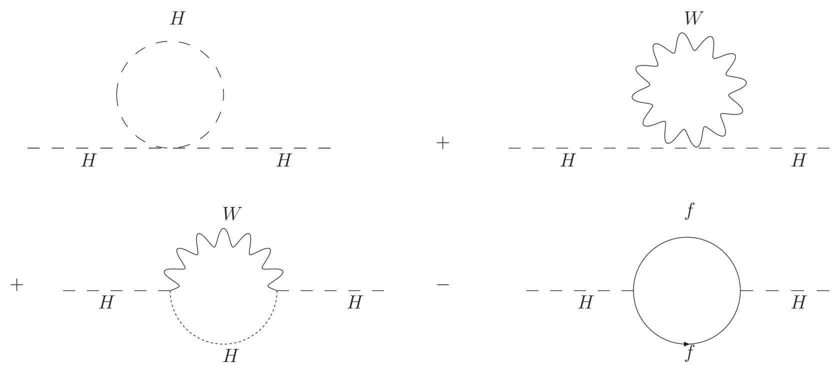

Figure 1.2: Quantum Corrections to Higgs bare mass

$$\langle |V| \rangle = -\frac{\mu_4}{4\lambda} \tag{1.4}$$

The third and fourth terms represent the cubic and quadratic interactions of the Higgs scalar. The second term represents a tree-level mass, that is the bare Higgs mass

$$M_{bare} = \sqrt{2}\mu \tag{1.5}$$

Of course this does not provide us with a prediction of the Higgs mass, since the parameter μ is not known.

Given the above mechanism and the correct choice of the vacuum expectation value of the scalar field φ, that is fixed by the low energy phenomenology of weak interactions:

$$\langle \varphi \rangle_0 = \sqrt{-\mu^2/2|\lambda|} \approx 174GeV \tag{1.6}$$

$W^{\pm}$ and Z bosons receive there required mass and the photon remains massless. Fermions mass will be given by the formula:

$$m_f = \lambda_f \langle \varphi \rangle \tag{1.7}$$

where λ_f is the Yukawa coupling to the Field.

1.3 Problems Of The Standard Model

In the following we will examine, some of the problems, regarding the standard model. We will emphasize on the Hierarchy problem which is the main motivation for theories with large extra dimensions. In the process we also list some of the observations for our universe, which the standard model fails to explain.

1. *arbitrariness of the standard model*

   The Standard Model is not considered to be satisfactory from a theoretical point of view because of the large number of free parameters.

   - three gauge couplings.
   - two parameters in the Higgs sector: μ and λ.
   - nine quarks(u,d,c,s,t,b and charged lepton e,μ, τ) masses.
   - three mixing angles and one CP-violating phase for the quark system.
   - the QCD parameter θ (coupling for the $F^{\alpha}_{\mu\nu}\widetilde{F}^{\alpha\mu\nu}$ term)

   which amounts to 19 free parameters. One may wish fewer parameters for a fundamental theory.

2. *Gauge Problem*

   The standard model is a complicated direct product of three sub-groups SU3$\bigotimes$SU2$\bigotimes$U1, with separate gauge couplings. There is no explanation for why only the electroweak part is chiral. Similarly, the standard model incorporates but does not explain another fundamental fact of nature: the electric charge quantization.

3. *Fermion Problem*

   All matter in the universe, under ordinary conditions, can be constructed out of the fermions of the first family(ν_e, e^-, u, d). Yet we know from laboratory studies in high energy colliders that there are at least two more fermion families:(ν_μ, μ^-, c, s) and (ν_τ, τ^-, t, b),which are heavier copies of the first family, with no obvious role in nature. The standard model gives no explanation for the existence of the heavier families and no prediction for their quantum numbers. Furthermore, there is no prediction for the fermion masses, which vary at least five orders of magnitude. As mentioned in section 1.2, fermions mass is described by equation 1.7. Since $\langle\varphi\rangle$ is fixed at 174GeV, the only fermion with a natural mass scale is the top quark with mass $m_t = 175GeV$. This does not account for the diversity of quark and lepton masses, especially from one family of quarks and leptons to another. This question must be addressed by a theory of Yukawa couplings yet to come. If complete, this theory should also predict the number of families.

4. *Hierarchy Problem*

   There are at least two seemingly fundamental energy scales in nature: the electroweak scale $m_{ew} \sim 10^3 GeV$ at which the electroweak symmetry breaking should occur and the Planck scale
   $M_{Pl} = G_N^{-\frac{1}{2}} \sim 10^{18}$,where gravity becomes as strong as the gauge interactions and a quantum theory of gravity is required. There is a difference of ~ 15 orders of magnitude between these two scales! The question *Why should nature choose two energy scales with such a big difference* , is known as the hierarchy problem. This is not just aesthetically wrong, but it also causes an important technical issue, regarding the Higgs boson mass and therefor it has been one of the greatest driving forces behind the construction of theories beyond the standard model.

   Although the Higgs boson has not been discovered yet, we can safely say, combining theoretical arguments and experimental limits, that it's mass cannot exceed the upper bound of 1TeV. However there is a complication. The mass predicted by 1.5 is only the bare Higgs mass.In order to find the effective measurable mass M_H which is bounded by 1TeV, we should consider quantum corrections to the 'bare' mass M_{bare}, coming from the loop diagrams of figure 1.2. If we try to calculate these corrections, we find that they are quadratically divergent. Hence we must consider a cutoff Λ, giving us the expression for M_h[2]

$$M_H^2 = (M_H^2)_{bare} + O(\lambda, g^2, h^2)\Lambda^2 \tag{1.8}$$

   If no new physics are expected between the electroweak scale and the Planck scale, the cutoff must be the Planck scale at which quantum gravity should appear. This means that a mass correction is of order

10^{19}! In itself this may not be a problem, since the bare Higgs mass can have any value, provided the physical Higgs mass M_H is around the electroweak scale. However, if there should exist a Higgs boson with mass close to 100 GeV, the bare Higgs mass would have a natural value of order the Planck scale. This means that there must be a cancelation at the level of 1 part in 10^{16}. A 'fine tuning' of this magnitude, is considered to be enormous and hence unnatural.

5. *Gravity Problem and Cosmological Constant Problem*

   Gravity, although it was the first of the fundamental interactions to be observed, it is certainly the most puzzling one. The only adequate theory to explain gravity, is the general theory of relativity. Although, this is a beautiful theory, it is not a quantum theory and therefor it should not be able to describe gravitational interactions at a very small scale. Gravitational quantum effects should appear at the Planck scale i.e. $10^{(19)} GeV$ which is far from what we have probed, however we are certain that this should be the case. Moreover if the goal of grand unification is to be met, there must be one fundamental theory explaining all four interactions. This is certainly not the standard model, since gravity cannot fit in it's framework.

   In addition to the fact that gravity is not unified and not quantized there is also the difficulty of the cosmological constant. The cosmological constant can be thought of as the energy of the vacuum. Spontaneous symmetry breaking, which is required by the standard model, induces an energy density via the vacuum expectation value of the Higgs potential 1.4. The cosmological constant then becomes[2]

$$\Lambda_{cosm} = \Lambda_{bare} + \Lambda_{SSB} \tag{1.9}$$

   where Λ_{bare} is the primordial cosmological constant, which can be thought of as the value of the energy of the vacuum in the absence of spontaneous symmetry breaking. Λ_{SSB} is the part generated by the Higgs mechanism:

$$|\Lambda_{SSB}| = 8\pi G_N |\langle 0|V|0\rangle| \sim 10^{50} |\Lambda_{obs}| \tag{1.10}$$

   It is some 10^{50} times larger than the observational upper limit Λ_{obs}. This implies the necessity of severe fine tuning which is not natural.

In the modern point of view, a given theory(e.g. The Standard Model), given the problems listed in this section, should be considered as the effective theory of a more complete underlying theory, which adequately describes physics at an energy scale higher than a threshold *M*. In the case of the standard model this should be the scale of electroweak symmetry breaking i.e. 1TeV. As we have seen if we try to set the cutoff at energy scales much higher than this, we encounter quadratic divergence. A complete theory should be able to deal with these divergences, include the standard model at the low energy scales were it is successful and include gravity.

1.4 Physics beyond the standard model

In order to solve the problems mentioned in the previous section, many theories of physics beyond the standard model have been proposed. This involve technicolor, supersymmetry, string theory, theories with extra dimensions and many more. Next we list the most important for us and some of there features.

- *Supersymmetry*

  One of the most popular choices for physics beyond the standard model is supersymmetry. The basic idea is the introduction of a new symmetry, so that the Lagrangian becomes invariant under transformations which convert fermions in to bosons and vice versa. This means that every fermion should have a boson partner and every boson should have a fermion partner and therefor the number of particles doubles. The hierarchy problem, at least the technical problem of the Higgs mass divergence, finds its solution since one loop diagrams causing the problem cancel with one loop diagrams involving their supersymmetric partners. In fact the cancelation is only between the terms quadratic in Λ but the remaining Higgs mass corrections are only logarithmic in Λ and hence do not cause the same fine-tuning problems. Of course this is the case only if the supersymmetric partners mass does not exceed 1TeV. If there masses are bigger than one order of magnitude from their partners masses, then we are back to fine tuning and the main motivation for introducing supersymmetry is lost. Hence if supersymmetry is true supersymmetric particles will appear at the LHC.

There are several other arguments in favor of supersymmetry.It can provide a dark matter candidate in the form of a lightest supersymmetric particle. Another advantage of supersymmetry is the unification of coupling constants. If we are to achieve the main goal of a unified theory then all the coupling constants must be unified at an energy scale. In the content of the standard model the couplings 'run' with energy and they almost unify at the energy scale of $\sim 10^{15} GeV$.If we apply supersymmetry then this unification is exact! This unification does not include gravity, although the unification scale is not very far from the Planck scale.

Since no supersymmetric particle has been discovered, it is obvious that supersymmetry, if it exists, it must be broken. We do not know the mechanism that breaks supersymmetry, although several proposals have been made. The problem with supersymmetry is that many new parameters ~ 103 are needed in order to describe a supersymmetric model which makes it much less attractive. Moreover a lot of physicists argue that a region of parameter space can always be found to ensure that supersymmetry cannot be ruled out by either experimental or cosmological constrains.

- *Superstring Theory*

  Superstring theory is a theory of elementary particles, quantum gravity and a framework for understanding black holes. It provides a framework for solving the problems that appear in the standard model and many physicists consider it to be the ultimate theory of everything. A short description is given because of the essential role some of it's ideas play in the construction of the large extra dimensions model.

  The energy scale of interest is the Planck scale, considered to be the true scale of elementary particles. Particles as we know them have far too many properties-such as spin, charge, color, parity and hypercharge-to be truly elementary. There is obviously some internal machinery responsible for all this, which should be revealed at the Planck scale.

  In superstring theory the world is described by one dimensional objects called superstrings and there interactions[3]. A superstring curries both fermionic and bosonic degrees of freedom, so it a priori includes supersymmetry.For the consistency of the theory the existence of extra compact dimensions,the size of Planks length is necessary. All the different particles we observe and there properties are different modes of the vibration of the string. Not only this provides the mechanism we mentioned, but one of the modes has the properties of the graviton, hence gravity is automatically included in the theory. Another, very important for us, element of string theory is D-branes. D-branes are not included arbitrarily in string theory but are essential for it's consistency. They can be points(D0-branes), curves(D1-branes), sheets(D2-branes) or higher dimensional objects. They are dynamical objects, which means that they can move and bend. Open strings must have their end-points attached to the brain, whereas the closed strings (loops) like the graviton can move freely.

The main problem for including gravity in the sense of the standard model was the inevitable infinities this would cause. The reason is because the gravitational constant G_N has dimensions of energy squared and therefor at high energies the dimensionless coupling $G_N E^2$, becomes arbitrarily strong, which implies divergence in the perturbation theory. A similar problem regarding weak interactions puzzled the scientists in the 60's. The Fermi's constant G_F has also dimensions of energy squared. In position space the divergence comes when the two weak occur at the same spacetime point. The solution to this was to consider that the interaction is resolved by the exchange of a W boson. This has the effect of smearing out the interaction in spacetime and softening the high energy behavior. The solution for gravitational divergence is similar. At the Planck length the graviton and other particles turn out to be not points but one dimensional objects,loops. Their spacetime histories are then two dimensional surfaces. The world-sheet is smooth. There is no distinguishable point at which the interaction occurs and therefor, like in the weak interaction case, the divergence is gone.

Of course string theory is not a theory without problems. For one the properties of strings are revealed at the Planck scale which is considered to be beyond our experimental reach. What good is a physics theory if it cannot be tested experimentally? Another problem is that there are far too many solutions to the equations of string theory.Many of these solutions resemble our world but many do not. Worse, there is no dynamical mechanism that prefers one solution to any of the others, so string theory provides no explanation of the detailed properties of our world. It also fails to explain why our world has three large space dimensions and not more, or why the size of the compactified extra dimensions is so small.

- *Large Extra Dimensions*

  Large extra dimension models propose a new framework for solving the hierarchy problem. These are models who address the problem from a geometrical point of view, rather than a gauge approach, whithout ruling out supersymmetry.By assuming the existence of extra dimensions the size of order $\sim mm$, the gravitational and gauge interactions unite at the electroweak scale. Thus in this framework, there is only one fundamental energy scale and the hierarchy problem is solved. Although the motivation is the solution of the hierarchy problem, there are possibilities for solving other problems of the standard model, mainly those regarding gravity. This is achieved because a viable model of large extra dimensions can involve superstrings and therefor it can include the advantages superstring theory has to offer. In the next chapter we will present the model of large extra dimensions.

Chapter 2

Large Extra Dimensions

We begin this chapter by introducing the basic idea of extra dimensions, that is Kaluza-Klein theories. This is useful not only in order to understand the large extra dimension theory, but also because a lot of the results of the next section will be essential in our further discussion. Next we will present the theory of large extra dimensions, introduced by Arkani-Hamet, Savvas Dimopoulos and Gia Dvali. This will be followed by a section concerning the experimental and theoretical bounds of this model.

2.1 Theories with extra dimensions

The basic goal of modern high energy physics is the unification of gravity (which is a purely geometrical theory) with electromagnetism,weak and strong interactions which are based on gauge principles. If this goal is to be achieved there are only two possibilities:

1. To realize gravity as a gauge theory(and thus geometry as a consequence of gauge principles)

2. To realize gauge theories as geometric

The first approach, although an attractive idea, failed to do so because of the inevitable divergences that appear in such a theory.

The second approach demands radical changes in the way we understand space-time. Combining quantum physics with general relativity means that space and time should be regarded as dynamical objects. This implies that the number of spatial dimensions at a given energy is a dynamical physical observable, not given a priori.

As we have already pointed out in the previous chapter, string theory which is the most promising unifying theory we have, needs extra dimensions in order to make sense. Modern approaches to the solving of the hierarchy problem, like the large extra dimensions model, also suggest the appearance of more than the three well known spatial dimensions.

All of these theories have there bases on the first attempt to unify gravity with electromagnetism by introducing one extra dimension, that is the Kaluza-Klein theory.

2.2 Kaluza Klein Theory

The first theory involving the concept of extra dimensions was proposed by Kaluza [4]in an attempt to unify electromagnetism and general relativity. His basic idea was to postulate an extra, fifth dimension, but with all fields being independent of this extra dimension. The starting point would then be a 5-dimensional pure gravity in which because of the independence of the fifth coordinate, the fields can be expressed in 4-dimensional fields. If this way is proceeded we end up with a 4-dimensional metric for gravity, a Maxwell field and a scalar.

Later on Oskar Klein [5]extended this idea making it more natural. Instead of assuming total independence of the extra dimension, he assumed it to be compact. This means that the fifth dimension would have the topology of a circle, with a radius of the order of the Planck length. Five dimensional space-time then has the topology $R^4 \times S^1$.

Let us first define our conventions: hatted quantities will be the 5-dimensional ones, capital Latin indices will run on five dimensions ($M = 0, 1, 2, 3, 4$) wile Greek indices will run on the normal 4-dimensional space-time ($\mu = 0, 1, 2, 3$). The compact fifth coordinate must satisfy the periodical condition $x^4 = x^4 + 2\pi R$, where R is the radius of the extra dimension.

We begin with the 5-dimensional line element:[6]

$$ds^2 = \widehat{G}_{MN} dx^M dx^N \tag{2.1}$$

which after expanding becomes:

$$ds^2 = \widetilde{G}_{\mu\nu} dx^\mu dx^\nu + G_{44}(dx^4 + A_\mu dx^\mu)^2 \tag{2.2}$$

where we have made the identifications

$$A_\mu \equiv \frac{G_{4\mu}}{G_{44}} \tag{2.3}$$

$$\widetilde{G}_{\mu\nu} \equiv G_{\mu\nu} - \frac{G_{4\mu}G_{4\nu}}{G_{44}} = G_{\mu\nu} - A_\mu A_\nu G_{44} \tag{2.4}$$

So in 4-dimensions we are left with the fields:

$$\left.\begin{array}{lcl} \widetilde{G}_{\mu\nu} & \leftarrow & \textit{4-dimensional metric} \\ A_\mu & \leftarrow & \textit{4-dimensional vector} \\ G_{44} & \leftarrow & \textit{4-dimensional scalar} \end{array}\right\} \textit{allow these to depend only on } x_\mu$$

<u>Left-over symmetry</u>:

1. general coordinate reparameterizations

$$x^\mu \leftarrow x'^\mu(x^\nu) \tag{2.5}$$

2. local transformations of x^4 of the form:

$$x'_4 = x^4 + \lambda(x^\mu) \tag{2.6}$$

   under which the 4-vector transforms as

$$A_\mu \leftarrow A'_\mu - \partial_\mu \lambda \tag{2.7}$$

   in order for the line element to remain invariant.

So in 4-dimensions we are left with the gravitational metric and a 4-dimensional vector which is a gauge field (like the electromagnetic field) and this gauge symmetry arises as part of the five dimensional coordinate reparameterization group. All these indicates that the unification of electromagnetism and gravity has been achieved. To make this more obvious we can calculate the 5-dimensional Einstein-Hilbert action. We allow the fields $\widetilde{G}_{\mu\nu}, A_\mu, G_{44}$ to depend only on x^μ.We also consider a new massless 5-dimensional scalar field called the dilaton which also depends only on x^μ.

$$\widehat{S} = \frac{1}{2K_0^2}\int d^5x\sqrt{-|\widehat{G_{MN}}|}e^{-2\Phi}(\widehat{R} + 4\partial_\mu\Phi\partial^\mu\Phi) \tag{2.8}$$

where $\widehat{R}$ is the 5-dimensional Ricci scalar and $\widehat{g}$ is the determinant of the five dimensional metric. Since nothing depends on the fifth dimension we can compute the integral over it, which will give us $2\pi R$. The determinant of the 5-dimensional metric and the Ricci scalar are easy to compute:

$$\begin{aligned}
\widehat{R} &= R - 2e^{-2\sigma}\partial_\mu\partial^\mu e^\sigma - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu} \\
|\widehat{G}| &= G_{44}^2|G_{\mu\nu} - G_{\mu 4}G_{\nu 4}| = e^{2\sigma}|\widetilde{G}_{\mu\nu}| \\
e^\sigma &\equiv G_{44}
\end{aligned}$$

It is then straightforward to compute the action:

$$S = \frac{\pi R}{K_0^2}\int d^5x\sqrt{-|\widetilde{G_{\mu\nu}}|}e^{-2\Phi_d}(R_4 - \partial_\mu\sigma\partial^\nu\sigma + 4\partial_\mu\Phi_d\partial^\mu\Phi_d - \frac{1}{4}e^{2\sigma}F_{\mu\nu}F^{\mu\nu}) \tag{2.9}$$

σ	$\longleftarrow$	*scalar or radion field*
$\Phi_d \equiv \Phi - \frac{\sigma}{2}$	$\longleftarrow$	*4-dimensional dilaton field*
$F_{\mu\nu}$	$\longleftarrow$	*Maxwell field strength*

We see that the Kaluza-Klein method indeed succeeded in unifying gravity with electromagnetism, but the scalar that appears in the action was a problem in the early 20's. It was much later in the 80's when people started to study higher dimensional effective actions, that the Kaluza Klein method experienced a big revival.

The dilaton and radion fields are massless fields. Furthermore, one can see that the equations of motion allow a flat metric solution, that is $\widetilde{G}_{\mu\nu} = n_{\mu\nu}$ for any values of Φ_d and σ. So the different values of these fields label distinct degenerate vacuum configurations. Such fields are called moduli and are very important in string theory. As mentioned before in section 1.4, one of the major problems of string theory is how to choose the proper vacuum that describes our universe and discard the rest.

Another big problem regarding all theories with extra dimensions is what physical mechanism fixes the size of the extra dimension. The proper size of the extra circle ρ is given by the relation

$$\rho = e^{<\sigma>}R \tag{2.10}$$

Since the expectation value of the radion field $<\sigma>$ is not fixed by the equations of motion, the proper size of the extra circle is not fixed by any natural mechanism.

2.2.1 Kaluza-Klein modes

. One of the most important predictions of the Kaluza Klein model is a tower of new particles known as Kaluza-Klein modes.Let us consider a massless 5-dimensional scalar field $\Phi = \Phi(x^M)$. Since the fifth dimension is considered to be compact and periodical, the scalar field can be expanded in a Fourier series, that is[7]

$$\begin{aligned} \Phi(x^M) &= \Phi(x^\mu, x^4) = \Phi(x^\mu, x^4 + 2\pi R) \\ \Phi(x^M) &= \sum_{n=-\infty}^{\infty} \Phi_n(x^\mu) e^{\frac{\imath n x^4}{R}} \end{aligned} \tag{2.11}$$

The equation of motion of such a field is a 5-dimensional Klein-Gordon equation which can be written in 4-dimensions using the Fourier expansion of the field:

$$\begin{gathered} \partial_M \partial^M \Phi(x^M) = 0 \\ \Rightarrow \partial_\mu \partial^\mu \Phi_n(x^\mu) - \frac{n^2}{R^2} \Phi_n(x^\mu) = 0 \end{gathered} \tag{2.12}$$

So the modes $\Phi_n(x^\mu)$ behave as 4-dimensional massive fields, known as the Kaluza-Klien modes, with there mass given by the relation:

$$m^2 = \frac{n^2}{R^2} = (P_4)^2 \tag{2.13}$$

where P_4 is the quantized momentum in the extra dimension. This is a quite remarkable result for two reasons.

1. There is a massless mode $\Phi_0(x^\mu)$ in 4-dimensions that can be identified as the graviton, if the 5-dimensional field is properly chosen.

2. There is an infinite tower of new massive particles. A question that immediately arises is why we do not observe all these new massive

particles. This is because the mass is set by the radius of the extra circle. If this is chosen to be sufficiently small then all $\Phi_n, n \neq 0$ are massive and decouple. Of course at energies above R^{-1} one sees the full tower of KK modes, which can be a direct experimental observation of extra dimensions.

2.3 Large Extra Dimensions Model

As noted in the first chapter the Hierarchy problem has been one of the main driving forces for physics beyond the Standard Model. Many theories have been proposed to solve this problem and all of them were based on the assumption that there are two fundamental energy scales(M_{ew} and M_{Pl}), which are separated by a vast desert of a 10^{16} magnitude. However there is an important difference between these two scales. While electroweak interactions have been probed at distances $\sim M_{ew}^{-1}$, gravitational forces have not been probed at distances $\sim M_{Pl}^{-1}$. In fact gravity has only been tested at distances $\sim mm$[8] which is 10^{33} orders of magnitude larger than the Planck length. Hence while the electroweak scale can be thought of as an experimental certainty, the Planck scale is just based on the assumption that Newton's law of gravity is unmodified over the 33 orders of magnitude that separates experimental data from the Planck scale.

In 1998 Nima Arkani-Hamed, Savas Dimopoulos and Gia Dvali proposed a new framework (the 'ADD model')[9] which includes large extra dimensions, for solving the Hierarchy problem. In this framework, the gravitational and gauge interactions become united at the weak scale, which is taken to be the only fundamental short distance scale in nature. Hence the usual problem with the quantum corrections of the Higgs mass is trivially resolved: the ultraviolet cutoff of the theory is M_{ew}. The observed weakness of gravity on distances $> mm$ is due to the existence of new compact spatial dimensions who's size is of the order of mm. This is the reason they are called large: they are 16 orders of magnitude larger than the fundamental length of the electroweak scale ($l_{ew} \sim 10^{-16} mm$). Gravitons can freely propagate throughout the whole (n+4)-dimensional space-time (where n is the number of extra dimensions, while the Standard Model (SM) fields must be localized to a 4-dimensional manifold, at least at energies bellow the electroweak scale. From an experimental point of view passing the electroweak scale means that we should observe new phenomena possibly even quantum gravity.

2.3.1 Newton's Gravitational Law in Extra dimensions

Newton's inverse square law of gravity, is a direct consequence of the dimensionality of our space-time. This is because it can be derived using Gauss's law. In order to find the gravitational force field created by a massive body m_1, we place a test mass m_2 at a distance r from m_1, and draw a sphere of radius r surrounding m_1. Then the field at any point on the surface will be equal to the mass M over the area of the surface A. Since the space is three dimensional, the area A will be proportional to r^2 and we get the inverse square law.

$$F = G_4 \frac{m_1 m_2}{r^2} \tag{2.14}$$

where G_4 is the four dimensional Newton's constant,
which is of order $\sim 10^{-36}$.

Although gravity is found to be so weak, macroscopically it is the force that dominates our universe. The inverse square law is essential because it is the only case that gives bound states. This means that the universe could not have the structure that we see today (galaxies, stars and planets orbiting around them) if it was not 3-dimensional! This is another argument in favor of the three spatial dimensions, aside the obvious one, that we only see three spatial dimensions. However we know that at microscopic scales things are very different. Gauge interactions are more important at these scales and bound states such us the atom are created because of the inverse square law of electromagnetism. Also because of the weakness of gravity compared to the gauge interactions, we have only been able to test gravity down to the mm. For all we know, the inverse square law needs to be valid only down to the mm. Below that it can change and so this is something we should look into.

We suppose that there are n new extra compact spatial dimensions with radius R. Following the same procedure as we did at the beginning of this section we place two test masses of mass m_1, m_2 within a distance r. If the distance is match smaller than the size of the extra dimensions ($r << R$) then the compact extra dimensions are effectively flat. Hence the gaussian sphere we should draw around mass m_1 is a (n+3) dimensional sphere with a surface A proportional to r^{n+2}. So the gravitational force will be:

$$F = G_{n+4} \frac{m_1 m_2}{r^{n+2}} \tag{2.15}$$

where G_{n+4} is the (4+n)dimensional gravitational constant, which is different from G_4. Of course if r is set to be larger then R then we are back to the inverse square law.

2.3.2 Effective Planck Scale

We have already noted that Plank's mass is the energy scale at which gravity becomes strong enough to be comparable with the gauge interactions, hence quantum effects must appear. If we look at Newton,s inverse square law, equation 2.15 and the value of the gravitational constant G_4, we are let to two important conclusions:

1. the gravitational constant $G_4 \sim 10^{-36}$ is very weak compared to the couplings of the gauge interactions for example the electromagnetic coupling constant is equal to $\frac{1}{137}$ and the strong interaction coupling constant is of order ~ 1, at low energies.

2. From dimensional analysis G_4 has dimensions of inverse squared energy, $([G_4] \sim \frac{1}{[E^2]})$. The gauge couplings are dimensionless.

The second conclusion means that the strength of gravity, will grow with energy square. Although gauge interactions also change with energy, because gauge coupling constants run with energy, their dependence is only logarithmic. Hence at a high enough energy, that is the Planck scale, gravity will become equivalently strong as the gauge interactions:

$$M_{Pl4} \sim G_4^{-1/2} \tag{2.16}$$

Of course the ADD model changes all this. The Planck scale derived from Newton's inverse square law is considered to be a phenomenological constant wile the true Planck scale is

$$M_{Pl(n+4)} \sim \left(\frac{1}{G_{(n+4)}}\right)^{\frac{1}{n+2}} \tag{2.17}$$

We have seen how extra dimensions modify Newton's law at short distances. If we go at large distances then we should be able to derive the inverse square law that is experimentally verified and that we conceder to be the low energy phenomenology. Furthermore we will be able to connect the phenomenological Planck scale M_{Pl4} with the true Planck scale $M_{Pl(n+4)}$. To do all this we will need the two postulates of the ADD model:

1. There are n extra spatial compact dimensions , each one with radius $R \sim mm$, which are accessible by gravitons only.

2. The true Planck scale $M_{Pl(n+4)}$ is set at the electroweak scale m_{ew} that is of the order of 1TeV: $M_{Pl(n+4)} \sim m_{ew} \sim 1TeV$

We put two test masses m_1, m_2 separated by a distance $r >> R$. Suppose that we conceder the first mass m_1 to be the source located at the point x^i, x^k, where index i runs over the three infinite dimensions of our universe, while $k = 1, 2, ..., n$ runs over the n extra spatial compact dimensions. Gravity sees the whole (n+3)-dimensional space, so gravitons from m_1 will propagate in each extra dimension periodically returning to our 4-dimensional space-time every $L = 2\pi R$. The mass m_2 will see an infinite line for each extra dimension with mass m_1 put periodically every L. We can apply Gauss's law by considering a (3+n)cylinder C centered around the n-dimensional line of mass, with side length ℓ and caps being 3-dimensional spheres of radius r[10].

$$\oint_{surfaceC} F ds = S_{3+n} \cdot G_{n+4} \cdot m_2 \cdot M_C \tag{2.18}$$

where S_D is the surface area of the unit sphere in D-dimensions and M_C is the mass in the cylinder, $M_C = m_1 \frac{l^n}{L^n}$. The surface of the cylinder is $S_{cylinder} = 4\pi r^2 l^n$. Substituting all this we get the gravitational law:

$$F = \frac{S_{3+n} G_{n+4}}{4\pi L^n} \cdot \frac{m_1 m_2}{r^2} \tag{2.19}$$

Equation 2.19, is an inverse square law as required. Equating this with our familiar Newton's law 2.14 will give us the relation between the effective gravitational constant G_4 with the true gravitational constant G_{n+4}.

$$G_4 = \frac{S_{3+n}}{4\pi L^n} G_{n+4} \tag{2.20}$$

It is now straightforward to connect the effective Planck scale M_{Pl4} with the true Planck scale $M_{Pl(n+4)}$.

$$M_{Pl4}^2 = \frac{4\pi L^n}{S_{3+n}} M_{Pl(n+4)}^{n+2} \tag{2.21}$$

If we apply the second postulate of the ADD model, that is set $M_{Pl(n+4)}$ at 1 TeV, then we get a relation between the number of the extra dimensions and there size.

$$R \sim 10^{\frac{30}{n}-17} cm \cdot \left(\frac{1TeV}{m_{ew}}\right)^{1+\frac{2}{n}} \tag{2.22}$$

For $n = 1$ we get $R \sim 10^{13} cm$, which can be safely excluded since it implies deviations from Newtonian gravity over solar system distances. However all the other cases can not be excluded by any experiment up to this day. The case $n = 2$ gives $R \sim 10^{-2} cm$, which is just bellow the distances down to which the inverse square law is been verified by Cavendish type experiments. All the other cases with larger n are very far from the direct experimental tests of gravity.

Ending this section we should note that all the above conclusions have been calculated using a (n+4)-dimensional point of view. Another approach is possible. From our 4-dimensional point of view the extra dimensions effect our universe because of the KK modes of the graviton. So when we write down the law for the gravitational force we should consider all the KK modes mediating the gravitational interaction. This means that we do not simply have the $1/r^2$ contribution coming from the massless graviton, but also Yukawa potentials that sum over all the KK modes:

$$F = Gm_1 m_2 \sum_n \frac{e^{-(\frac{n}{R})r}}{r^2} \tag{2.23}$$

This consideration will lead to the same conclusions for the gravitational law and the connection between the effective Planck scale with the true Planck scale, we have reached using a (n+4)-dimensional point of view.

2.3.3 Physics in the extra dimensions

The first theoretical issue we must answer is why the standard model fields do not realize the extra dimensional space that we will call the 'bulk'. We can be sure that all the standard model fields are localized in a three dimensional space at least up to energies close to the TeV scale, which means down to distances $10^{-16} mm$. The answer to this question can be given by superstring theory. In that theory the standard model fields, would be localized to a three dimensional membrane in the theory's higher dimensional space-time. Open strings, corresponding to photons, electrons and all the other fundamental particles of the standard model, have there free ends stuck to the 3-brane. Therefor they cannot directly probe the bulk space. Gravitons on the other hand are spin-2 particles which corresponds to a closed loop of string in superstring theory. Thus they are not attached to the brane and can freely propagate in the extra dimensional space. This has another advantage: It combines the ADD model with the best theory of quantum gravity we have and at the same time it offers the exciting possibility of observing superstrings in the near future.

It is very difficult to understand what physics govern the bulk. We do not even know the topology of the extra dimensional space or the fields that propagate in it. However there are many interesting possibilities. Firstly there in no reason for our universe to be unique. Many other branes each one of them forming a parallel universe can share the bulk with us[11]. Physics on those branes can look like the standard model of our universe or can be very different, with different fields localized on them and unknown to us interactions. Of course we can not see these other branes nor our standard model fields can interact with them since they are localized, but that does not mean that they do not influence our universe. Each brane can be considered as a source of bulk fields, like the graviton or other fields propagating in the bulk. Depending on our position in the bulk and the nature of these other branes and the bulk fields, they could have striking effects on us, like determining the value of the standard model parameters like the electron mass, the number of particle generations and the electro-weak mixing angle. For example the chiral symmetry could be broken on some other brane and so the effect on us would be small, leaving an approximate chiral symmetry on our brane resulting to the small, but not zero, mass of the electron.

It might even be that our brane is folded in the bulk[11]. So very distant regions of our universe can be separated only by a small distance in the bulk. This provides a possible solution for dark matter. It is possible that the astrophysical and cosmological anomalies we attribute on dark matter are actually results of the gravitational interactions of ordinary mass mediated through the bulk.

All these may sound promising but there are at least three important theoretical issues the ADD model has to account for. Firstly *What stabilizes the size of the extra dimensions*. This is a question that any theory of compact extra dimensions has to answer. Until today there has not been any satisfying proposal to address this question so it remains the major problem of the ADD theory. It is actually a rephrase of the hierarchy problem. The question "why is the Planck scale so large", in the ADD model became "why are the extra dimensions large".

The second issue is *How can we preserve the stability of the proton*. If gravitational forces become strong at the TeV scale, then virtual black holes can mediate instant proton decay. Of course this is unacceptable since the proton has a lifetime that exceeds 10^{32}sec. The problem is that virtual black holes destroy all quantum numbers not associated with a long range field. We know of no such field coupled with baryon number. So the proton should decay with gravitational strength which would be disastrous if gravity gets strong at the TeV scale. One obvious way to get out of this is to associate baryon number with a gauge boson field in the bulk.

The third issue is *How do we preserve the gauge coupling unification predicted by supersymmetry near the Planck scale*. One possible solution for this was proposed by Keith Dienes, Emilian Dudas and Tony Ghergheta. They assumed that in addition to the mm-size extra dimensions in which only gravity propagates, there are $10^{-17}cm$ sized extra compact dimensions(corresponding to the TeV scale) in which the standard model fields also propagate. These new dimensions would accelerate the evolution of the standard model interactions and make the couplings unify just above the TeV scale.

2.3.4 Experimental Tests of the ADD model

One of the most interesting characteristics of the ADD model is that it can be experimentally tested in the near future. The first and most obvious test is to try and measure variations from Newton's law. Experiments that directly measure gravity, like Cavendish type experiments, will show a variation of the inverse square law ($F \sim \frac{1}{r^2}$) at distances smaller than a mm. For example if we have the case of two extra dimensions then below a mm an inverse quadratic law, ($F \sim \frac{1}{r^4}$) will describe gravity. Of course this kind of experiments are very difficult to accomplish. At the mm scale gravity remains much weaker than electromagnetic interactions. So Van der Waals forces or other electromagnetic effects that make their appearance at small scales will have a large effect on the measurement.

More promising results will occur at the LHC. First of all it is possible that we will have the first observations of quantum gravity and strings. We can not really be sure how this will look like but it is certainly a striking result. Miniature black holes will be produced since the LHC will reach the energy at which gravity becomes strong. Extra dimensional black holes will decay rapidly in a democratic manner to all the standard model particles. An analytical description of the creation, the characteristics and decay of miniature extra dimensional black holes will be the subject of the next chapter.

Finally another interesting possibility at the LHC is the case of missing energy. Since the standard model fields are localized on the brane within m_{ew}^{-1}, in sufficiently hard collisions, energy can escape in the bulk. Depending on the topology of the extra dimensional space this amount of energy may return to our universe or it may be lost in the bulk. In the first case the particles carrying away the missing energy can periodically return to our 4-dimensional world colliding with SM particles and depositing energy. This will result to a signal of $\sim 2\pi R$ displaced vertices. In the second case we will have a violation of energy conservation which will be a clear signal of extra dimensions.

2.3.5 Phenomenological and Astrophysical Constraints

In this section we make a rough estimate of current constraints coming from laboratory and astrophysical bounds. Short scale gravity experiments, are very difficult because gravity is not the dominant force force at these scales. The electrostatic effects that make their appearance like Van der Waals forces, together with mechanical and thermal vibrations, make it very difficult to isolate and measure gravity. However Cavandish type experiments have measure gravity down to the $\sim mm$ scale and found that Newton's inverse square law still stands. So if extra dimensions do exist, they must be smaller than a mm. Of course this is satisfied even for the case of two large extra dimensions, since equation 2.22, gives a size of the order of a mm.

Regarding the phenomenology of gravity there is no problem. On large scales gravity remains the same as described by Newton's law. On a scale smaller than the size of extra dimensions $\sim$ mm, gravity becomes stronger so one may argue that it would modify the phenomenology of atoms. However this is not the case. The atom scale is $\sim 10^{-8}$ m, at which the electromagnetic force remains much stronger than gravity.

Next we consider astrophysical constrains[10]. In the ADD framework the gravitons are strongly coupled ($\sim 1/TeV$), so they can be produced copiously on stars. Once they are produced, they can curry away energy in the bulk, which could result to unacceptable modification of stellar dynamics and the acceleration of the cooling process. The emission of gravitons in the (4+n)-dimensional space with $\sim 1/TeV$ coupling, can be seen from a 4-dimensional point of view as the emission of a large number of KK modes, each one of them weakly coupled ($1/M_{(4)}$). If the temperature of a star is large compared to the scale of the extra dimensions ($T >> R_n^{-1}$), then a large number of KK modes ($\sim (TR_n)^n$) can be produced. This gives a rough estimation for the rate of graviton production:

$$GPR \sim \frac{(TR_n)^n}{M_{(4)}^2} \sim \frac{T^n}{M_{(4+n)}^{2+n}} \tag{2.24}$$

In order to consider the bounds set on the Planck scale $M_{(4+n)}$, we borrow the estimations on the bound of the axion production, which has similar effects as our graviton production. The rate for axion production is proportional to $\frac{1}{F^2}$, where F is set by the stringent bounds to be larger than $10^9 GeV$. Applying the same bound on graviton production rate we get:

$$\frac{T^n}{M_{(4+n)}^{2+n}} \geq 10^{-18} GeV^{-2} \tag{2.25}$$

We will examine the cases of the sun, Red Giants and SuperNovae.

- sun

  The sun temperature is $\sim 1KeV$. So equation 2.25, will give a bound on $M_{(4+n)}$:

  $$M_{(4+n)} \geq 10^{\frac{18-6n}{n+2}} GeV \tag{2.26}$$

  The most dangerous case is $n = 2$, which will give an estimation for the fundamental Planck scale $M_{(6)} \geq 10GeV$, which is safe for the ADD model since the lowest bound is smaller than the $\sim TeV$electroweak scale.

- Red Giants

  The temperature of a red giant is estimated at $\sim 100KeV$. equation 2.25, will give a bound on $M_{(4+n)}$:

  $$M_{(4+n)} \geq 10^{\frac{18-4n}{n+2}} GeV \tag{2.27}$$

  Again we examine the $n = 2$ case which will give a bound on the fundamental Planck scale, $M_{(6)} \geq 1TeV$. The lowest bound satisfies the ADD model postulate, which sets the fundamental Planck scale close to 1TeV

- SuperNovae

  The most stringent bound is set by SN since they are the hottest stellar objects we observe. With a temperature reaching $\sim 30MeV$, we have a bound:

  $$M_{(4+n)} \geq 10^{\frac{18-2n}{n+2}} GeV \tag{2.28}$$

  This time the case of $n = 2$, gives a lowest possible value on $M_{(6)}$ of the order $\sim 10TeV$. This is quite a strong bound, which however does not exclude the $n = 2$ case. The cases of more extra dimensions are completely safe since for $n = 3$, we get $M_{(7)} \geq 1TeV$.

A more careful calculation can be conducted taking in to consideration the different processes that can lead to the emission of a graviton. These are:

- Gravi-Compton scattering($\gamma + e \rightarrow e + g$)
- Gravi-brehmstrahlung($Z + e \rightarrow Z + e + g$)
- graviton production in Photon fusion($\gamma + \gamma \rightarrow g$)
- Gravi-Primakoff process($\gamma + EMField \rightarrow g$)
- Nucleon-Nucleon brehmstrahlung($N + N \rightarrow N + N + g$)

However all the above possible processes emitting gravitons, have cross sections dominated by the factor of equation 2.24, so the results will not be much different than those listed above.

2.3.6 Cosmological constrains

It is clear that the ADD model will have a dramatic effect on cosmology since it changes the dimensionality of the universe. The most obvious changes concern the early universe[10]. Standard cosmology assumes that the early universe temperature was close to the Plank scale. If the ADD model is true then the Planck scale is lowered 16 orders of magnitude which will also lower the early universe temperature. Even so not much can be said about the early universe cosmology. Since we do not know the mechanism that compactifies and stabilizes the size of the extra dimensions, there is nothing we can say about the universe before its temperature drops below the TeV scale. However this is not a problem because we do not know anything about the universe at the TeV scale, to begin with.

Our current knowledge goes as back as the beginning of Big Bang Nucleosynthesis (BBN), which began when the universe temperature was $\sim 1MeV$. At this point we have to assume that the extra dimensions were already compact and stabilized at a size smaller than a mm. The 4-dimensional universe on the other hand continued its expansion which is very well described by the 4-d Robertson-Walker metric. If the extra dimensions are not to have any significant effect on this expansion we have to assume that the extra dimensional space is effectively empty.

The beginning of BBN must remain unaffected by the extra dimensions. So we must assume that there had been a time in the universe, described by a temperature T_*, at which the extra dimensions stopped effecting the evolution of the universe. The BBN must start after the temperature of the universe dropped below T_*, because we know that the evolution of the universe is normal 4-dimensional, since the beginning of BBN. So T_*, must be close, but not larger than 1 MeV. a bound can be set on T_*, if we consider the cooling of the universe during the BBN epoch. Two mechanisms are involved in the cooling process:

- cooling because of the expansion of the 4-d universe

$$\frac{d\rho}{dt}|_{expansion} \sim -3H\rho \sim -3\frac{T^2}{M_{Pl(4)}}\rho \tag{2.29}$$

- cooling by evaporation in the extra dimensions.

  Gravitons can be produced and then escape in the bulk carrying away energy. Equation 2.24 gives the rate of graviton production which is proportional to $1/M_{(4+n)}^{n+2}$. From dimensional analysis we know that $\frac{d\rho}{dt}$ has dimensions of $[E^5]$, so we get:

$$\frac{d\rho}{dt}|_{evaporation} \sim -\frac{T^{n+7}}{M_{4+n}^{n+2}} \tag{2.30}$$

We know that a correct description of the cooling process is given by the expansion of the 4-d universe. So this must be the dominate one and we must set it to be larger than the evaporation process, at least up to the point of T_*. We know that $\rho \sim T^4$, so we get:

$$T_* \leq \left(\frac{M_{(4+n)}^{n+2}}{M_{Pl(4)}} \right)^{1/n+1} \tag{2.31}$$

If we set the fundamental Planck scale at 1 TeV ($M_{(4+n)} \sim 1$) TeV, then we get for the most dangerous case of n=2, a bound on $T_* \leq 10$ MeV. If we set the fundamental Planck scale at the more preferable, by astrophysical constrains, value ($M_{(4+n)} \sim 10$) TeV, we get $T_* \leq 100$ MeV for n=2 and $T_* \leq 10$ GeV for n=6. In any case $T_* \geq 1$ MeV, so the starting point of BBN is safe.

2.4 Conclusions

The ADD model offers a new and exiting way for solving the Hierarchy problem. By imposing large extra compact dimensions it manages to make gravity strong at the electroweak scale. A sufficient explanation to the question why we do not see the extra dimensions is provided by the fact that extra dimensions are accessible by the graviton only, while all the standard model fields are localized on a 3-brane, we realize as our universe. If the model is correct then amazing possibilities arise for the near future of experimental physics. Deviations from Newton's law in small scale measurements of gravity and what's more important the first observations of quantum gravity and the verification of string theory. Of course many questions remain unanswered. The most important of them is what stabilizes the size of the extra dimensions close to the mm. One may argue that the ADD model has simply rephrased the hierarchy problem from "Why is gravity so weak compared to the gauge interactions" to "Why are the extra dimensions so large". However the possibility of large extra dimensions is not excluded by any data so this is a model we should carefully look into.

Chapter 3

Black Holes

In the ADD scenario gravity is considered to be weak because of the large size of extra dimensions. If we probe smaller distances than the size of these extra dimensions, we will realize a modification of the Newtonian gravity depending on the number of the extra dimensions. In this chapter we will consider the modification of black hole properties caused by the ADD scenario. Miniature (4+n)-dimensional black holes will be much different than the 4-dimensional ones. What's more important, the lowering of the Planck scale down to the TeV, will cause the production of extra dimensional miniature black holes at the LHC, providing a clear signal for extra dimensions and an opportunity to study black hole properties and quantum gravity.

3.1 General Characteristics of Black Holes

A black hole is a high mass density object, predicted by the equations of general relativity. Because of the large mass density a very strong gravitational field surrounds the black hole, curving up space-time. The main characteristic of a black hole is the event horizon, a spherical surface of radius R_s called the Schwarzschild radius. If anything, pulled by the gravitational attraction of the black hole, passes the event horizon it can never escape and return back. This applies even to light hence the name black hole. It is very easy to compute the Schwarzschild radius, even using classical physics arguments. From Newtonian gravity we know that the minimum velocity one needs to escape the gravitational attraction of a massive body of mass M and radius R is given by the escape velocity, $v_{esc} = \sqrt{\frac{2GM}{R}}$. Since the largest possible speed in nature is the speed of light c, we just have to set the escape velocity to be smaller or equal to the speed of light:

$$R_s = \frac{2GM}{c^2} \qquad \textit{Schwarzschild radius} \tag{3.1}$$

To get a feeling about the size and density of a black hole, equation 3.1, estimates the Schwarzschild radius of earth to be the size of a mm with corresponding mass density $d \sim 10^{27} Kg/m^3$!

In the early years of general relativity black holes were considered to be completely black objects, continuously sucking up energy dew to there large gravitational attraction, becoming larger and larger all the time. However a more recent research conducted by S. Hawking, showed otherwise. According to quantum field theory vacuum is not empty at all, because of quantum fluctuations creating particles and antiparticles, of opposite energy $(+E, -E)$, living for a very short time interval, as long as allowed by the time-energy uncertainty principle. After that the particle-antiparticle pair meets again and annihilates back to nothingness. Suppose know that such a pair is created outside but very close to the event horizon. If the negative energy particle falls into the black hole it will be forever lost from its partner which because of its positive energy can travel away from the black hole. This means that the black hole will loose an amount of energy, and the same amount will travel away. So this mechanism allows black holes to radiate and decay. Hawking proceeded to prove that a black hole radiates like a black body. He calculated the thermal characteristics, like temperature and entropy[12]:

$$T = \frac{1}{4\pi GM} \qquad \textit{black hole temperature} \tag{3.2}$$

$$S = \frac{A}{l_{Pl}^2} \qquad \textit{black hole entropy} \tag{3.3}$$

where A is the surface of the event horizon $A \sim R_s^2$ and l_{Pl} is the Planck length ($l_{Pl} \sim 10^{-32} mm$). These two relations lead us to some important conclusions about black holes and there behavior:

- Large black holes are cooler than small ones, although they have larger energies. The importance of this is that it is very difficult to observe the Hawking radiation of a large astronomical black hole, because it has a very law temperature. So although most scientists agree that Hawking's radiation is correct, the verification is very difficult.

- A radiating black hole becomes hotter as it decays. This is an amazing result coming from the fact that black holes have negative heat capacity.
- The Hawking radiation originates from outside the black hole horizon. So the black hole decay is democratic to all the standard model particles and does not depend on any of the properties of the black hole besides its mass, charge and angular momentum. There is no connection between the Hawking radiation and the matter that the black hole is made up.

The last conclusion leads to the information paradox. Black holes are characterized by there energy, angular momentum and electric charge and these are the only quantities preserved. When a particle falls into a black hole all the information (baryon number, flavor etc.) is lost. This is a clear violation of quantum theory properties and has caused a lot of discussion. It is clear that for a complete description of black holes, we need quantum gravity. Superstring theorists G.'t Hooft, L.Susskind suggested holography as a solution for the information paradox.The holographic principle is a speculative conjecture about quantum gravity theories, claiming that all of the information contained in a volume of space can be represented by a theory that lives in the boundary of that region. Every quantum state corresponds to a Planck area ($A_{Pl} = l_{Pl}^2$) and so the event horizon surface can be filled up with a number of states equal to the entropy of the black hole.

3.2 Extra Dimensional Black Holes

In the ADD model gravity is altered at distances smaller than the size of extra dimensions which is assumed to be the order of a mm. Astrophysical black holes are very large to be effected by the change of the gravitational law. However black holes with a Schwarzschild radius smaller than the size of the extra dimensions will have there properties drastically altered than what we would expect if the world would remain 4-dimensional at distances smaller than a mm. A change in small black holes properties is very important for two reasons. Firstly small black holes play an essential role in cosmology since primordial black holes had been produced during the BBN epoch. Secondly there is a large possibility that small black holes will be produced in large numbers at the LHC. Next we list the characteristics of extra dimensional black holes (size, Hawking temperature, entropy and decay rate) and compare them with the ordinary 4-dimensional ones. We should note that in order to have an exact expression we should use higher dimensional general relativity. However the correction is only a numerical factor, and we will neglect it for the time being, since we are only interested in the physical properties of extra dimensional black holes.

1. Size of extra dimensional black holes

   To calculate the Schwarzschild radius of a (4+n)-dimensional black hole we equate the kinetic energy of a particle of mass m moving with the speed of light ($c = 1$), with the gravitational potential energy:

$$\frac{m}{2} \sim \frac{G_{(n+4)} M m}{r_{s(n+4)}^{n+1}} \tag{3.4}$$

   Using the definition of the fundamental Plank scale we get the expression of the (4+n)-dimensional Schwarzschild radius[13]:

$$r_{s(n+4)} \sim \frac{1}{M_{pl(n+4)}} \left(\frac{M}{M_{pl(n+4)}} \right)^{\frac{1}{n+1}} \tag{3.5}$$

   It is useful to compare the Schwarzschild radius of an (n+4)-dimensional black hole with a 4-dimensional one. To do so we use relations 3.1, 3.5 and the definition of the effective Planck scale $M_{Pl(4)} = G_4^{-2}$.

$$\left(\frac{r_{s(4)}}{r_{s(n+4)}} \right) \sim \left(\frac{r_{s(n+4)}}{R} \right)^{n} \tag{3.6}$$

   This relation is valid only for $r_{s(n+4)} < R$, so $r_{s(4)} < r_{s(n+4)}$. This leads to the conclusion that:

   A (n+4)-dimensional miniature black hole is larger than the respective 4-dimensional black hole of the same mass

2. Entropy

   Following Hawking,s argument, the entropy of a given black hole will be the surface of the event horizon over the surface of an area with sight length the fundamental Planck length[13].

$$S_{(n+4)} \sim \left(\frac{r_{s(n+4)}}{l_{Pl(n+4)}} \right)^{n+2} \tag{3.7}$$

In order to compare this result with the entropy of a same mass black hole living in a 4-dimensional universe we need a relation connecting the two. It is easy to find one, the definition for the 4-dimensional black hole entropy and the relation between the fundamental Planck scale and the effective Planck scale ($M^2_{Pl(4)} \sim R^n M^{n+2}_{Pl(n+4)}$):

$$S_{(n+4)} = S_{(4)} \left(\frac{R}{r_{s(4)}} \right)^{n/(n+1)} \tag{3.8}$$

Since the size of the extra dimensions R is larger than the Schwarzschild radius $r_{s(4)}$, because we are only considering miniature black holes, we are led to the conclusion:

A (n+4)-dimensional miniature black hole has a larger entropy than the respective 4-dimensional black hole of the same mass

Thus at distances smaller than R there are more quantum states available and the holography constrains are less restrictive. The question is what happens with larger black holes. A naive approach would suggest that since the fundamental Planck length is now 16 orders of magnitude larger than the usual 4-dimensional Planck length, this will lower the number of states available for a large black hole in a dramatic way, making the holographic principle more restrictive. However this is not the case. The entropy from a (n+4)-dimensional point of view is proportional to the area of the horizon in (n+4)-dimensional Planck units. So it will be

$$S \sim r_{s(4)} R^n M^{n+2}_{Pl(n+4)} \tag{3.9}$$

which gives back the same result as the one calculate from a 4-dimensional perspective. The reason is that although the unit area has increased, so has the available area since we have extra dimensions. These two exactly cancel each other.

In summary for large black holes the holography constraints are the same in any case, wile for small black holes extra dimensions make them less restrictive.

3. Hawking Temperature Now that we have the entropy of the black hole 3.7, it is easy to calculate the Hawking temperature, just by applying the first law of thermodynamics[13]:

$$T_{(n+4)} = \frac{dE}{dS} = \frac{dM}{M^{n+2}_{Pl(n+4)} dA} \tag{3.10}$$

which will give

$$T_{(n+4)} \sim M_{Pl(n+4)} \left(\frac{M_{Pl(n+4)}}{M} \right)^{\frac{1}{n+1}} \sim \frac{1}{r_{s(n+4)}} \tag{3.11}$$

Since we already know that the Schwarzschild radius of an extra dimensional black hole is larger than the respective 4-dimensional one we can say about the temperature:

A (n+4)-dimensional miniature black hole is cooler than the respective 4-dimensional black hole of the same mass

4. Lifetime A miniature black hole will evaporate because of the Hawking radiation. If we assume that the black hole radiates from the horizon like a perfect (n+4)-dimensional black body and that during it's evaporation, it does not absorb any energy then a good estimation for the decay rate is[13]:

$$\frac{dE}{dt} \sim (horizonarea) T^{n+4} \tag{3.12}$$

Using equation 3.11 for the temperature, 3.5 for the Schwarzschild radius and expressing all this in mass terms, we can integrate the above relation to get:

$$\tau_{(n+4)} \sim \frac{1}{M_{Pl(n+4)}} \left(\frac{M}{M_{Pl(n+4)}} \right)^{\frac{n+3}{n+1}} \tag{3.13}$$

A similar calculation for a 4-dimensional black hole of the same mass gives:

$$\tau_{(4)} \sim \frac{1}{M_{Pl}} \left(\frac{M}{M_{Pl}} \right)^{3} \tag{3.14}$$

Since the fundamental Planck scale $M_{Pl(n+4)}$ is much smaller than the phenomenological Planck scale $M_{Pl(4)}$ we reach the conclusion

A (n+4)-dimensional miniature black hole lives longer than the respective 4-dimensional black hole of the same mass

5. Average particle multiplicity.

3.2.1 Black holes radiate mainly on the brane

It has been argued that a small extra dimensional black hole would radiate mainly in the bulk. The argument was that a small black hole would have a typical mass at the TeV scale which would result to a Hawking temperature of the same magnitude. Such a temperature is higher than the masses of a very large number of KK modes. A black hole radiates democratically to all the possible particles, restricted only by the preservation of energy, charge and angular momentum. Thus a given black hole should radiate mainly KK modes and thus most of it's energy would escape in the bulk. However this is not the case. As shown by R.Emparan, G.Horowitz and R.Myers black holes radiate mainly on the brane[14]. The reason for this is that since the KK modes are excitations in the full space their overlap with a small black hole is suppressed by the geometric factor $(r_{s(n+4)}/R)^2$ relative to the standard model fields. This factor is exactly the same as the number of KK modes allowed by energy conservation to be emitted. So the geometric suppression precisely compensates for the enormous number of modes, hence the total contribution of all KK modes of a bulk field is the same as that of a single Standard Model field.

Chapter 4

Black Holes At The LHC

The Large Hadron Collider (LHC) is expected to reach energies above the fundamental Planck scale as this has been described by the ADD model. This will result in a very large production of miniature black holes which will evaporate almost immediately. Such black holes will decay to every particle of the Standard Model. In this chapter we begin with a short description of the LHC and the CMS detector and we continue describing the production of black holes at the LHC, their decay and experimental signals that can be detected at CMS.

4.1 The Large Hadron Collider

The Large Hadron Collider[15] is a high energy proton-proton accelerator, designed to explore physics at the TeV scale. The reason for doing so is to discover the last undiscovered component of the Standard Model, the Higgs boson and at the same time explore new physics beyond the Standard Model. As already noted in the first chapter new physics are expected to appear at the TeV scale in order to solve the theoretical issues the standard model failed to explain. The center of mass energy of the colliding beams will be 14 TeV, however the energy available for the inelastic collision is only a fraction of the total c.o.m energy of the protons. The reason for this is that protons are composite objects. At such high energies the inelastic collision occurs between the constituent particles of the protons (quarks and gluons), which curry only a fraction of the total proton momentum. The fractions are statistically distributed according to the parton distribution functions (pdf).

There are two reasons for using proton beams at the LHC. Firstly since we are not really sure for the exact energies at which new physics will appear, a wide spectrum of effective collision energies serves better the goal of discovering new physics. Future colliders using electron-positron beams will serve the purpose of precise measurement of whatever new physics the LHC discovers. Secondly the interaction rate needs to be sufficiently high to curry out the search for new and rare phenomena. The LHC is designed to reach a luminosity $L = 2 \times 10^{33} cm^{-2} s^{-1}$, which means that about 10^{11} particles are collided every 25 ns with the beam transverse size $16 \mu m$ at the collision point. It is not possible to produce the large amount of antiprotons needed for such a large luminosity. The only way to overcome this is to use protons in both beams. Of course the two beams must be accelerated in separated beam pipes which intersect at the collision point.

Before entering the LHC the two beams will pass through several stages of acceleration passing through linear accelerators and Synchrotron. In the LHC beam pipes 1232 superconducting dipole magnets of a magnetic field at $8.33T$ will accelerate the beams at the required momentum to reach the energy of 14 TeV. The coils of the superconductors will be kept at superconductive temperature with the use of superfluid helium at 1.9 K.

Three detectors will be used to curry out the experiments at the LHC. Two of them , the CMS and the ATLAS are general purpose detectors aiming to the discovery of new physics. A third detector called ALICE will explore the properties of quark-gluon plasma studying heavy ion collisions.

4.2 The CMS detector

The extrapolation of rich and diverse physics at the LHC requires large scale particle detectors of great complexity in order to be able to reconstruct the proton collisions in as much detail as possible. The CMS[16] is a general purpose detector so it consists of four different detector layers, cylindrically structured around the beampipe like a barrel with complementary layers in two endcaps covering the forward direction. From the inside to the outside a central tracking device, an electromagnetic and a hadronic calorimeter, and a muon spectrometer are placed.

The CMS detector has a compact and dense design with 15m diameter and a length of 21.5m. Because the precision for the momentum measurement is inversely proportional to the magnetic field B and the size L of the detector,

$\frac{\sigma_{p_T}}{p_T} \propto \frac{p_T}{BL^2}$, a strong solenoidal magnetic field of 4T is chosen. The tracker at CMS, uses a combination of silicon pixels at the inner part and strips at the outer part.

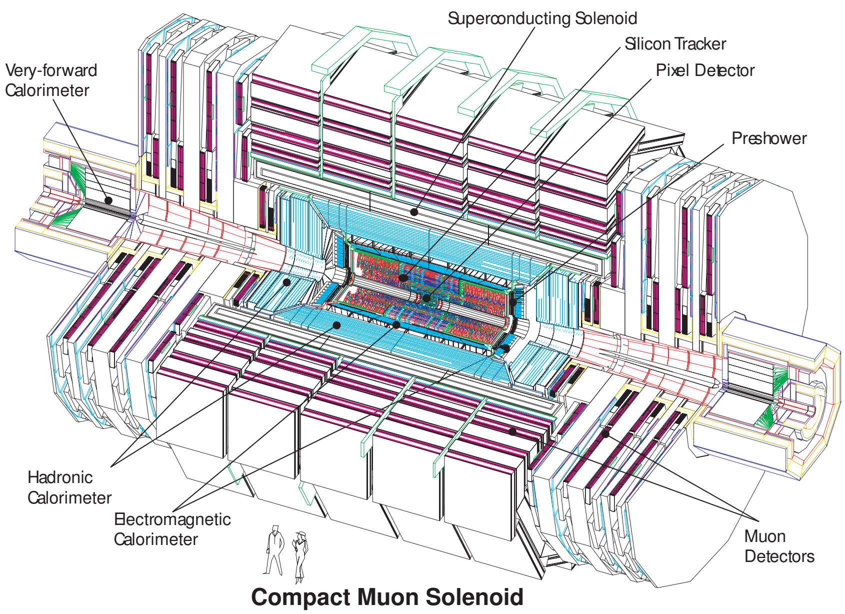

Figure 4.1: The CMS Detector

4.2.1 The CMS calorimeter system

In order to detect the signatures of new physics processes, the CMS detector will need to identify and precisely measure the energies and location of electrons and photons. This is the task of a sub-detector called the Electromagnetic Calorimeter (ECAL)[17] which is split into barrel $|n| \leq 1.479$ and end-cap $1,479 \leq |n| \leq 3$ sections to surround the collision point of the interacting protons.The ECAL consists of around 118,000 tapered lead tungstate crystals with about 25,000 in the end-caps. Due to the density of the lead tungstate ($8.3g/cm^3$), electrons and photons entering the crystals interact and produce "showers" of secondary particles. Each shower itself consists of many electrons and photons, but these possess less and less energy as they travel through the calorimeter and are absorbed. By measuring the scintillation light produced by all of the secondary electrons, it is possible to determine the energy and position of the primary electron or photon which first hit the calorimeter. The scintillation light is collected within each crystal by total internal reflection and finally detected at the end of the crystal by a photodetector. For this reason, the crystals have to have good light transmission in the wavelengths concerned, but at the same time be dense enough to absorb the many hundreds of secondary particles that can be produced in each shower. The electromagnetic calorimeter (ECAL) is situated outside the tracker. It uses $PbWO_4$ crystal to collect the energy from photons and electrons which provides an excellent energy resolution for energies ranging between 1 GeV to 1 TeV. For precise data taking, the pseudorapidity needs to be less than 2.6.

The hadronic calorimeter allows to measure energy and position of charged hadrons. By measuring the energy and direction of jets, this calorimeter plays an essential role in the identification of quarks and gluons. Moreover, it can detect particles that not interact with the detector such as the neutrino ν (by conservation of the momentum in transverse plan). The barrel, or central detector, is a cylindrical structure which surrounds the collision region and measures the energy of quarks and jets emerging at large angles relative to the beam direction. The endcap calorimeters look like end-plugs and enclose the ends of the barrel region. There are two of these, one at each end of the central detector. The forward calorimeters measure energy at very small angles relative to the beam and are located outside the muon system along the beam line. There are two of these detectors, one at each end of CMS.

4.3 Black hole production at the LHC

The black hole production in colliders can be estimated using semiclassical arguments. All is needed to create a black hole of mass M_{BH} is two partons colliding with center of mass energy $\sqrt{\hat{s}} = M_{BH}$ and an impact parameter smaller than the respective Schwarzschild radius. At a first estimate the cross section for black hole production will be of the order of the area within the collision must take place, that is the black hole horizon[18].

$$\sigma(M_{BH}) \approx \pi R_s^2 \tag{4.1}$$

With the center of mass energy reaching 14 TeV, black holes are expected to be produced copiously at the LHC. Of course the energy will be distributed to the partons of the accelerating protons and at such high energies, the collision will occur at the parton level. The longitudinal momentum carried by each parton can be calculated using the parton distribution functions (pdf). To get an estimation of the black hole production less suppose that we have a deep inelastic collision of two partons carrying the one third of the total available energy to the proton, which is a quite large possibility. The cross section for a black hole production can be estimated using equation 4.1, which gives about $250pb$ if the Planck scale is set at $1TeV$. With the Planck scale set at $10TeV$, the cross section drops to the femptobarn scale but still is very large. Hence a large number of black holes with masses close to 5 TeV are expected to be produced, wile more massive black hole creation will be extremely rare because collisions with a much larger than $5TeV$ center of mass energy are unlikely.

To get a more precise formula for calculating the cross section for the creation of black holes at the LHC we must sum over all the possible parton pairs colliding, each one carrying a fraction x of the total momentum of the protons. The statistical distribution of the momentum for each parton is given by the parton distribution function $f(x)$ and the cross section for black hole creation at the parton level is given by equation 4.1[19].

$$\sigma_{pp\rightarrow bh}(\tau_m, s) = \sum_{ij} \int_{\tau_m}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_i(x) f_j(\tau/x) \sigma_{ij\rightarrow bh}(\tau s) \tag{4.2}$$

where $\sqrt{s}$ is the collider center of mass energy, x is the parton momentum fraction, $\tau = x_i x_j$ is the parton-parton center of mass energy squared function and the black hole mass is assumed to be $M_{BH} = \sqrt{\tau s}$. In any case the cross section will be proportional to the Schwarzschild radius of the black hole, which grows with energy. So unlike the cross section for standard model particle production which falls with energy, the black hole cross section grows with energy. The following table shows the estimated cross section for various cases for the total number of dimensions and for Planck Mass. It is obvious that the number of dimensions does not effect the production cross section, at least not as much as where we set the fundamental Planck scale. For the ADD model the production and study of black holes at the LHC is somewhat problematic. For any prediction regarding black holes to be valid, the black hole mass must be well above the fundamental Planck scale $M_{BH} >> M_{Pl}$. If the mass of a given black hole is close to the Planck mass then quantum gravity effects take over and any semiclassical treatment is at best just an approximation. In the case of the ADD model the Planck scale is definitely

N.Dimensions	M^{bh}_{min}(TeV)	$M_{Pl}(TeV)$	Cross Section (pb)
6	2	1	3071
6	4	1	225.2
6	4	2.5	12.05
7	2	1	2388
7	4	1	156.4
7	4	2.5	12.52
9	2	1	2145
9	4	1	125.8
9	4	2.5	1193

larger than a TeV with the most preferable cases close to 10TeV (see chapter 2). So the Black holes at the LHC will have masses very close to what is considered to be the fundamental Planck scale.

4.4 Black hole decay at the LHC

Once produced, these miniature black holes are expected to decay almost instantaneously with a typical lifetime $\sim 10^{-26} sec$ as can be found using equation3.13. The decay will pass three major phases:

- Balding Phase

  During this first phase the black hole will lose the asymmetry and moments due to the violent production. At the end it will be a spherical semiclassical black hole.

- Hawking Evaporation Phase

  This will be the major phase of the Black hole during which it will lose most of it's energy to Hawking radiation behaving like a Black body of temperature the Hawking temperature given by equation 3.11. The black hole temperature will be rising as expected by the black hole thermodynamics and at the same time it's size will become smaller.

- Planck Phase

  At the end of the Hawking evaporation phase the black hole will be so small and hot that it will enter into the Planck phase. This is the phase at which quantum gravity effects become very strong and we do not really know much about this phase.

The most interesting phase is of course the Hawking evaporation phase. It is expected that during this phase the black hole will be radiating democratically to all the standard model particles because the black hole evaporation respects only energy, charge and angular momentum conservation. Of course at the LHC the colliding beams will consist of protons so charge conservation dictates that more positive charged particles will be emitted. A fraction of the energy can be lost to gravitons escaping in the bulk, however the possibility for emitting a graviton is the same as any standard model particle, so only a fraction $\sim \frac{1}{60}$, of the total black hole energy will be lost in the bulk. Although the black hole becomes hotter and hotter as it evaporates, we can assume a constant temperature during the black hole evaporation, justified by the very short lifetime.

The Black hole are expected to be created almost at rest at the lab frame. So the events coming from black radiation will be characterized by high sphericity and high multiplicity. Finally since the mass of the black hole will be large compared to the masses of the standard model particles that will be emitted, a high transverse momentum is expected.

Chapter 5

Analysis

In this last we try to reconstruct the Higgs boson mass created from miniature black hole decay, through the Hawking radiation. We consider the $b\bar{b}$ channel for Higgs decay ($H \rightarrow b+\bar{b}$), which is expected to be the dominant channel for a light standard model Higgs boson,$M_H \sim 130GeV$ and we try to reconstruct Higgs mass using b-jets. For the black hole production we use the charybdis event generator, created by Christopher Michael Harris for his phd thesis at the Cambridge university.

5.1 Charybdis Event Generator

The Charybdis[20] event generator was constructed in order to study the production and decay of miniature black holes at the LHC, predicted by theoretical models with large extra dimensions, mainly the ADD model. The total number of dimensions can be anything between 6 and 11. Six is the smallest number allowed by phenomenological bounds (see chapter 2,section 2.3.5), and anything more than eleven ruins the whole idea of large extra dimensions. Following the ADD model the Planck mass is set at the TeV scale. The Generator allows a range of values for the Planck mass, between 1 TeV and 5 Tev. ($1TeV \leq M_{Pl} \leq 5TeV$). The black holes are created with a geometric parton level cross section $\sigma = \pi R_s^2$, as described in chapter 4. The black hole mass is always larger than the Planck mass, in order to avoid quantum gravity effects, and it must be smaller than the center of mass energy available in the collision. The temperature of the black hole is calculated using equation 3.11, and it can be set to be constant throughout

Name	Description	Values
TOPDIM	Total number of Dimensions (n+4)	6-11
MPLNCK	Planck Mass (GeV)	1000-5000
MINMSS	Minimum Mass of Black Holes (GeV)	<MAXMSS
MAXMSS	Maximum Mass of Black Holes (GeV)	$<E_{c.o.m.}$
NBODY	Number of Particles in Remnant decay	2-5

the decay process, or it can be set to change as the black hole decay's. In any case it must be assumed that when a black hole emits a particle, it reaches a thermodynamical equilibrium, before it emits the next particle. Once created, the black hole decays via the Hawking radiation mechanism until it's mass reduces to the Planck scale. After this the remnant decays to any two standard model particles allowed by energy conservation. This whole process is valid only for black holes masses much larger than the Planck mass ($M_{BH} >> M_{Pl}$). If this is the case then the semiclassical approximation is valid and the Hawking radiation is the dominant mechanism for black hole decay. The black hole decay respects only energy, charge and angular momentum conservation, as indicated by the theoretical models. The case of graviton emission in the bulk is excluded since the dominant radiation will be to standard model particles on the brane as shown in chapter 3. CHARYBDIS respects the conservation of energy, angular momentum and charge as it should, but it also assumes the conservation of baryon number.Althought this is not justified by the theory for black holes, of the ADD model is to be viable then somehow we must deal with proton decay which could be mediated by virtual black holes. One way to do this is to assume that the baryon number is a conserved quantity.

The generator itself only performs the production and parton level decay of the black hole. After that the parton shower evolution, hadronization and particle decays are performed with PYTHIA.

5.2 PYTHIA Simulation Program

The PYTHIA[21] simulation program was mainly developed by Torbjorn Sjostrand as a general-purpose generator which uses Monte-Carlo techniques to simulate the distributions and parametrizations of the models involved in the description of high energy collisions.PYTHIA is a program for the generation of high-energy physics events, i.e. for the description of collisions at high energies between elementary particles such as e+, e-, p and $\bar{p}$ in various combinations. Together they contain theory and models for a number of physics aspects, including hard and soft interactions, parton distributions, initial and final state parton showers, multiple interactions, fragmentation and decay.

5.3 Exploring rare events

Black hole production at the LHC can offer the possibility for exploring rare events which can not be detected from Standard Model procedures.

- Easy to distinguish Black Hole events from standard model events.

  1. Democratic Decay to all the Standard Model Particles. Black hole can decay to anyone of the 138 degrees of freedom available from standard model. Almost 100 d.o.f. are quarks and gluons, which result in hadronic activity, while $\sim$ 20 d.o.f. are leptons 1 and 1 photon. This results to a characteristic signal in the detector of hadronic to leptonic activity $5 : 1$ and hadronic to photonic activity $100 : 1$

  2. High multiplicity of decay products.

  3. High sphericity.
     The black holes are expected to be produced at rest at the collision point. So the particles emitted from the black holes will travel to all the directions resulting to high sphericity events.

  4. Large visible transverse energy. The mass of the black holes produced at the LHC can be anything between 1TeV and ∼8TeV, depending on the fundamental Planck mass.(See table 4.3)

Using the above characteristic signals to detect black hole productions gives as the opportunity to isolate the respecting events and study them in an environment with minimized Standard Model background. At the LHC with a c.o.m. energy $\sqrt{\hat{s}} = 14TeV$, the QCD background is expected to be enormous, so it will be very difficult to study rare events, unless we can somehow cut the QCD background. A characteristic example is the Higgs boson. As we can see in figure 5.2, the Higgs boson is expected to decay through the $b\bar{b}$ channel, if it's mass is $\sim 123GeV$, or through the W^+W^- channel, for a higher Higgs mass. Both these decay modes are impossible to see in Standard model events, since the latest will involve a large creation of $b\bar{b}$ coming from other events. So in the best case the Higgs boson, using standard model events will be detected through the photon channel, although the Higgs branching ratio to photons is very small small for a higgs mass below $\sim 150GeV$ and zero for $M_H > 150GeV$. However if we cut the Standard model events using the black hole characteristics, then we can have clear signals and maybe we can detect rare events.

In the next section we will try to reconstruct the Higgs boson using the $b\bar{b}$ channel, in Black Hole events.

5.4 Higgs Boson

The discovery of the higgs particle is one of the primary goals at the LHC. It is the last and probably the most important ingredient of the standard model, since the Higgs mechanism through the spontaneous symmetry breaking is responsible for giving mass to all the standard model particles. At the same time the Higgs field acquires a mass who's value depends on the parameters of the model.(see 1.5). We assume $M_H = 127GeV$.

5.4.1 Bounds on Higgs Mass

As we saw in the first chapter the tree level Higgs mass is given by the formula $M_{bare} = \sqrt{2}\mu$, where μ is an unknown parameter of the standard model, determining the scalar potential responsible for the spontaneous breaking of the electroweak symmetry. This bare mass receives corrections from higher order diagrams and these corrections depend on the physics that determine the universe in energies larger than the TeV scale where the electroweak symmetry breaking is expected to occur. Depending on the physics beyond the standard model, various modifications of the original Higgs boson have been proposed, including little higgs, charged higgs etc. In this thesis we only discuss the original standard model higgs boson with the cutoff for the quantum corrections set at the TeV scale as proposed by the ADD model.

Although the Higgs mass is not determined by theory or experiment, several bounds on its mass can be set, both by theoretical arguments and from experimental data. The LEP collider , with center of mass energy $\sqrt{s} \geq 208GeV$ did not observe clear signals for the creation of the Higgs boson. Combining the final results from the four LEP experiments, ALEPH, DELPHI, L3 and OPAL, leads to an experimental lower bound for the Higgs boson mass of 114.4 GeV at 0.95 confidence level[22]. Theoretically an upper bound of 1 TeV is set by unitarity[23]. However M_H is not expected to be that high, with the favored values not exciting 200GeV. In this thesis we use $M_H = 127GeV$.

5.4.2 Higgs Creation

There are several procedures for the creation of the Higgs boson at hadron colliders. Since we are not really sure about all the characteristics of the Higgs boson, we cannot be sure for the production cross sections for each process. In figure 5.1, we see the four major processes we expect at the LHC from Standard Model physics. These are Gluon fusion (a), vector boson fusion (b), Associative production with W (c) and an example of the diagrams having associative production with a top pair (d).

The dominant procedure is the gluon fusion which gives a cross section for Higgs production $\sigma_{gg \to H} \sim 50pb$, for a center of mass energy of the pp collision at 14TeV and the Higgs mass being bellow 200GeV. The other three procedures have a much lower cross section, $\sigma_{other} \to \leq 1pb$

If the ADD model is correct then we expect that Higgs bosons will also be created via the Hawking radiation mechanism of black holes. The possibility to get a Higgs boson from black hole decay is expected to be about 0.02, since the Higgs boson is one of the Standard Model modes, each one of them produced with the same possibility at black hole decay. As we saw in the previous chapter miniature black holes will be produced with a large cross section $\sigma_{BH} \sim 250pb$. Combining the two estimations the cross section for Higgs boson production from black hole decay will be of the same size as the gluon fusion procedure ($\sigma_{BH \to H} \sim 50pb$).

5.4.3 Higgs Decay Modes

Once the Higgs boson is created, it will directly decay into pairs of all massive particles, respecting all quantum numbers. The branching ratios for Higgs

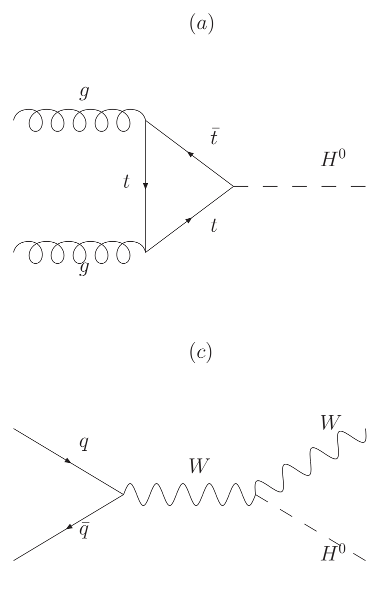

Figure 5.1: Higgs Production

decay modes depend highly on it's mass(see figure 5.2). For $M_H \sim 123GeV$ the dominant decay mode is the $b\bar{b}$, while for a higher Higgs mass, the decay to vector bosons becomes dominant. Since in this thesis we will be examining low mass Standard Model Higgs, we will look for associated production with two b-jets from the Higgs decay.

5.5 b-tagging

In order to decide which jets to use we must tag them as b-jets. There are several technics for b-tagging. The b-tagging performed for the jets we will use was done using the secondary vertex technic[24].

The b-quarks have a long lifetime $\tau_b \sim 10^{-12} sec$and this results at a very long B-Hadron lifetime. This means that hadrons containing b-quarks have sufficient lifetime that they travel some distance before decaying. On the other hand, their lifetimes are not so high as those of light quark hadrons,

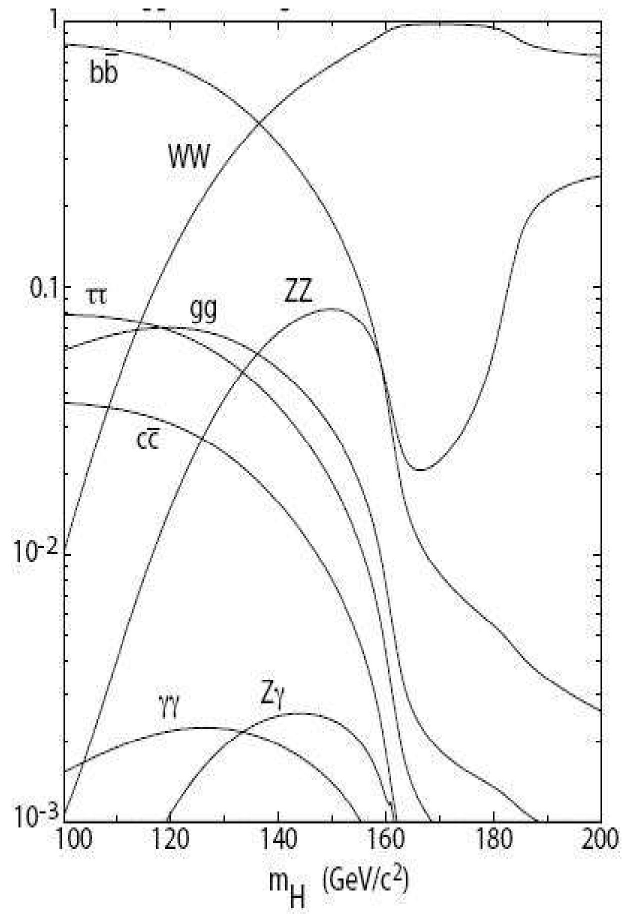

Figure 5.2: SM Higgs Decay Branching Ratios

so they decay inside the detector rather than escape. So when b-quarks are produced at the collision point, which is the primary vertex, travel a relatively long distance before forming a b-jet, at the secondary vertex (see figure5.3). By associating two or more tracks of a jet with a single secondary vertex, the jet can be tagged as b-jet, with a discriminator value giving the confidence level that a jet tagged as b is truly a b-jet.

5.6 Data Analysis

After running the event generator for several cases involving different number of dimensions, Planck scale and black hole mass, and performing the

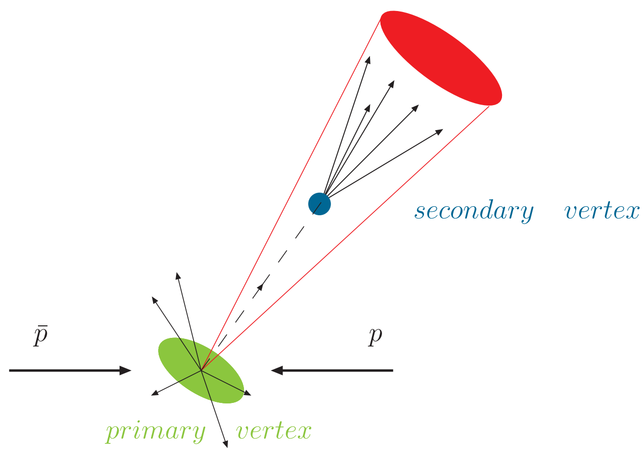

Figure 5.3: secondary vertex after b-creation

hadronization using PYTHIA, we ended up with data files each one of them containing 100000 black hole events, their decay products at the generator level, the products of hadronization, the resulting jets and the b-tagged jets, each one of them assigned with a discriminator value. We used these files to see the characteristics of the black hole decay, the kinematic properties of the generated particles (mainly the Higgs boson and the b quark that interest us in this thesis) and jets, and finally to try and reconstruct the Higgs boson mass using b-tagged jets.

5.6.1 Black Hole Decay and Higgs Decay Multiplicity

In figure 5.4, we can see the decay products of black holes with mass $M_{BH} = 4TeV$ for Planck mass $M_{Pl} = 2TeV$ and total number of dimensions n=7.

At a first glance this histogram seems to violate the democratic decay of the black hole. However this is not the case, since the original collision is proton-proton collision so the preservation of charge dictates that a larger number of positive charged particles will occur. It is obvious that the most preferable decay modes are quarks and gluons as expected (hadronic to leptonic activity $5:1$).

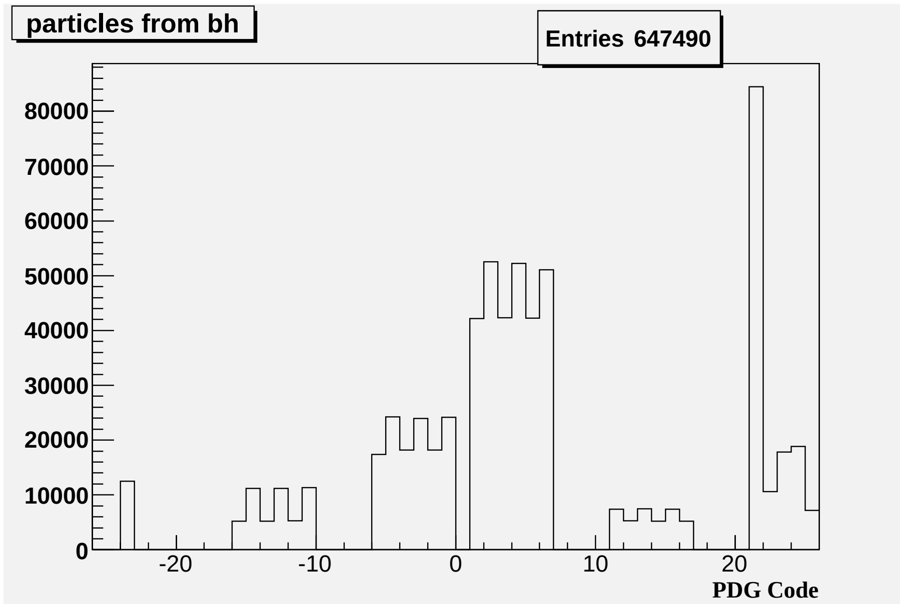

Figure 5.4: Black hole decay products($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 1TeV$)

The decay of 100000 black holes results to 647490 Standard model particles so the average multiplicity for a black hole decay is approximately 6 particles.From a total number of 647490 of standard model particles, the 7208 are Higgs bosons. This means that the probability that a black hole emits a Higgs boson is roughly 1% as expected by the theoretical argument of democratic decay.

In figure 5.5 we can see the decay products of the Higgs boson at the generator level. It is obvious that the dominant channel for Higgs decay is the $b\bar{b}$ chanel as expected by theory for a light Higgs boson of mass $M_H = 123GeV$(see figure 5.2). A much smaller number of Higgs goes to $\tau\bar{\tau}$, W^+W^-, $c\bar{c}$ and ZZ.

5.6.2 Kinematics

Next we view the kinematic properties of the particles and jets in black hole events. In figure 5.6 we can see the transverse momentum that the Higgs boson carries after it is emitted from a black hole. It is easy to see that the higgs P_T is large as would be expected for any particle emitted from a black hole of mass 4TeV and a Hawking temperature of $T_H \sim 0.7TeV$. This is

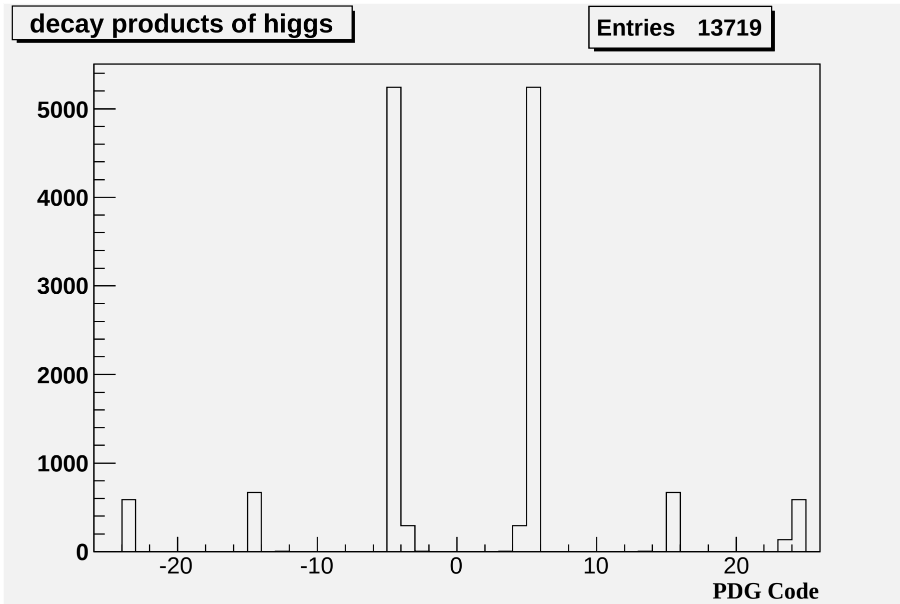

Figure 5.5: Higgs boson decay products($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 1TeV$)

because a 4 TeV object decays to $\sim$ 6 standard model particles, so each one of them has to carry a large amount of energy.

The major problem caused by the large Higgs transverse momentum is that the $b\bar{b}$ quarks in which Higgs decays, are not well separated as seen in figure 5.7. The resulting b-jets will be very forward, and many of them can not be distinguished since each cone formed by a jet has ΔR equal to 0.5. Of course this same fact can be used in our advantage to cut the qcd background since in qcd events we do not expect $b\bar{b}$ quarks to come out so close.

The last kinematic plot 5.9 describes the scalar sum of energy $\sum E_T$ in black hole events. As expected the distribution of $\sum E_T$ appears to have a maximum $\sim 4TeV$ close to the mass of the black hole. This can provide a cut for qcd events, mainly qcd events with small transverse momentum which have a large cross section.

5.7 Reconstruction of Higgs mass.

In order to test if our goal of reconstructing the Higgs boson mass using b-tagged jets we first examine how the mass plot looks like if we follow the

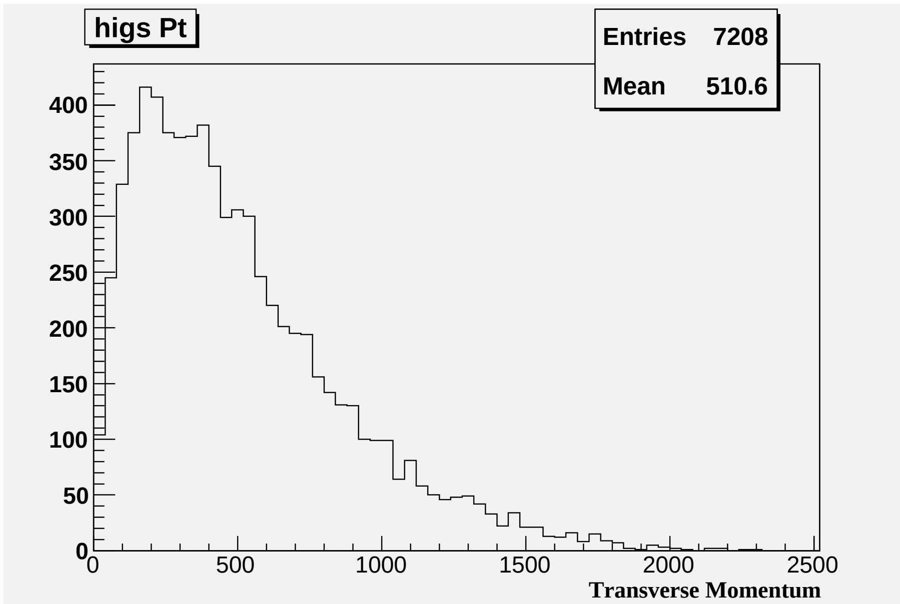

Figure 5.6: Higgs boson $P_T(M_{min}^{BH} = 4TeV,\, M_{Pl3} = 1TeV)$

Higgs boson at the generator level and identify the $b\bar{b}$ quarks in which it decays. These $b\bar{b}$ quarks are then matched with the calibrated jets (JetIC5), provided that their transverse momentum is above 40GeV, by examining for each b-quark track, which jet is the closest i.e. $\Delta R_{b-jet} = minimum$. We then combine the matched jets in order to calculate the invariant mass. The result, seen in figure 5.10, is very promising.There is a very clear peak around the assumed value of the Higgs mass $M_H = 127GeV$. Of course it is more than obvious that it is not possible in a real experiment to find the Higgs boson and follow it to see to what it decays! In a real experiment all we will be able to see is energy deposited in the hadronic and electromagnetic calorimeter of our detector. So in order for our work to have some practical meaning we must try to reconstruct the Higgs mass just by using the jets which deposit their energy on the calorimeters. All we know is the amount of energy, the transverse momentum and the track of the jets. The jets can be identified as b-jets by using the b-tagging technic described earlier, which associates a discriminator value for each jet. The larger the discriminator value, the larger the possibility that the jet is truly a b-jet. However at the same time, demanding a large discriminator value will limit the statistics.

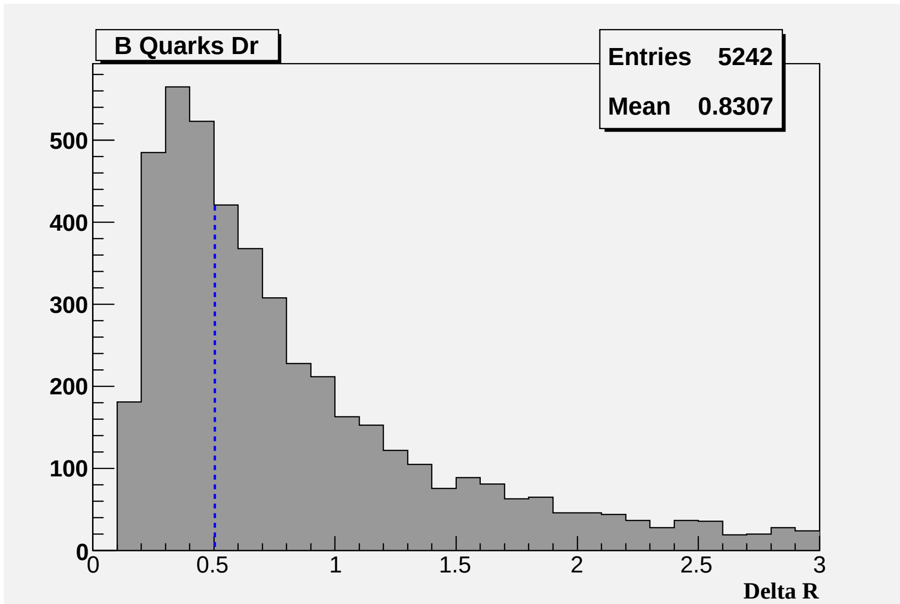

Figure 5.7: Δr for $b\bar{b}$ from Higgs($M^{BH}_{min} = 4TeV$, $M_{Pl3} = 1TeV$)

This is because a large discriminator value will not only cut jets that are not false b-jets, but also true b-jets.

Taking into account the kinematic characteristics of Higgs events, described in the previous section, we demand:

1. discriminator value larger then 2 ($D > 2$).

2. Minimum jet transverse energy 40 GeV (Jett$P_T > 40GeV$).

3. The two jets used to calculate the invariant mass must be close. $0.5 < \Delta R < 1.5$.

The result is not as good as expected. As we can see in figure 5.11, the peak around the value of Higgs mass remains, however the background is quite large so we cannot safely say that we identify the Higgs boson. One attempt that makes things look a little better is shown in figure 5.12, were we also demand that the chosen event is characterized by a small value of missing transverse energy($ME_T < 100GeV$). We use this final result in order to fit the parameters and see in what confidence we can claim that we have successfully recontracted Higgs events. The fitted diagram appears in figure

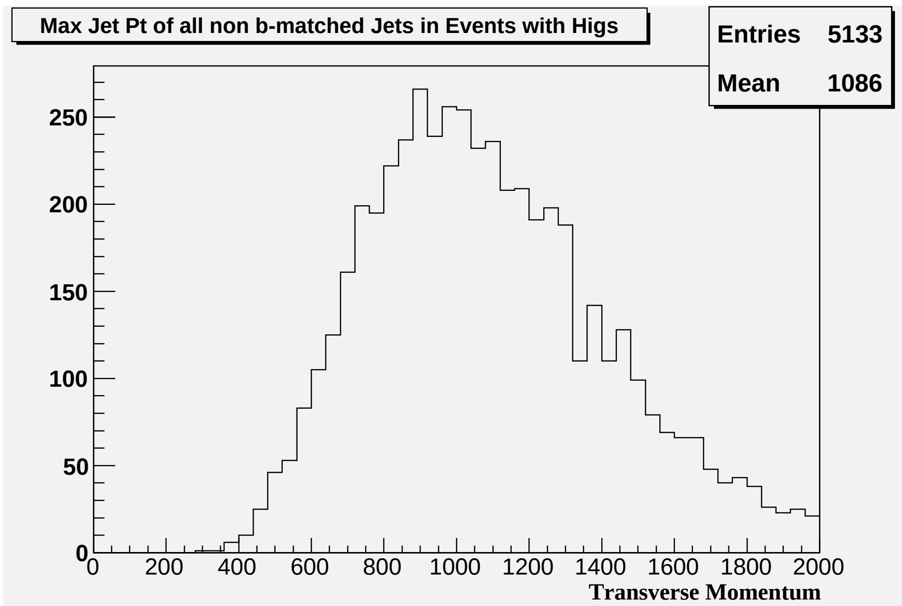

Figure 5.8: $P_{T(max)}$ of non b jet in Higgs event($M^{BH}_{min} = 4TeV$, $M_{Pl3} = 1TeV$)

5.13. For the fitting we have used a 4th order polynomial plus a gaussian. This resulted in a 61 ± 13 number of Higgs events for $\sigma = 5$.

Finally we examine the case of a different Planck mass $M_{Pl} = 2.5TeV$, to see how things change. Firstly we should note that a bigger Planck mass has a dramatic effect ont the black hole production cross section, which for the case of $M_{BH} = 4TeV$ and number of extra dimensions n=3, drops from $\sigma_{(n=3,M_{BH}=4,M_{Pl}=1)} = 156.4pb$ to $\sigma_{(n=3,M_{BH}=4,M_{Pl}=2.5)} = 12.52pb$, see table 4.3.

As expected the case of a larger Planck mass effects the Hawking temperature, which rises to $T_H \sim 2.2TeV$, three times hotter that the respective Hawking temperature for $M_{Pl} = 1TeV$. This results in a larger transverse momentum carried out by the decay products of the Black hole and so the $b\bar{b}$ quarks in which the Higgs boson decays have their tracks even closer then before, see figure 5.14. The b-jets falling into not distinguishable cones ($\Delta R < 2.5$) reach the amount of 63%, so the statistics of the remaining events are very small.

Ending the analysis of the data we should note that if black holes at the LHC are created then we will be offered with the possibility of studying rare events otherwise hidden by standard model procedures. As we have seen the

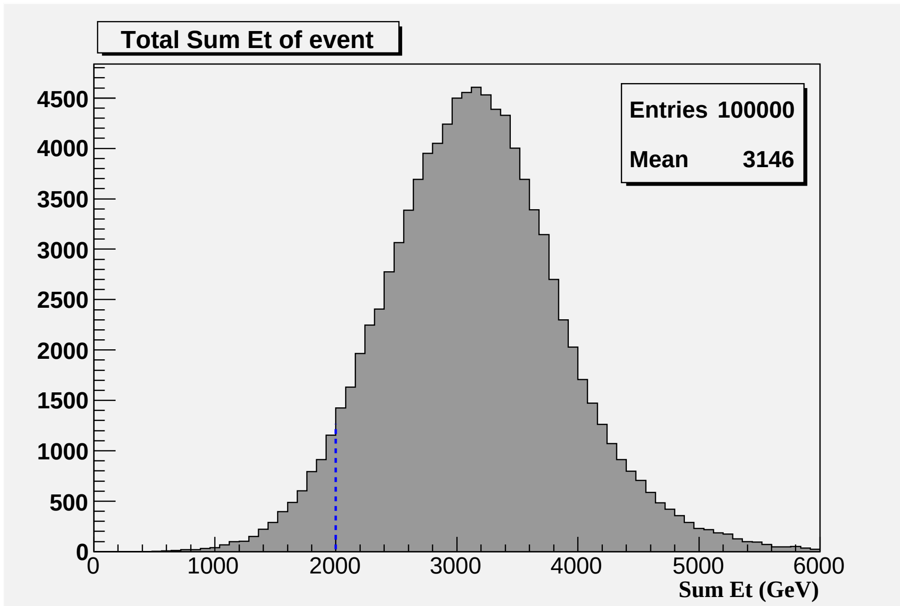

Figure 5.9: $\sum E_T$ in Higgs events($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 1TeV$)

black hole events will be very characteristic and they can be isolated and studied separately. A work to be done would be to look for a more massive higgs boson which would mainly decay to W^+W^-. It is possible that such a case would give a more clear signal than the one we got here from $b\bar{b}$, since in the case of W's, one can look for a signal of leptons in which the W bosons can decay. A lepton signal is more clear since it is fairly easier to identify them than the b-jets, and also other procedures resulting to leptons (directly from black hole emission or other), are more suppressed than the b-quark production. Also in the case of leptons their is no problem with distinguishing jets, which in the case of $H \to b\bar{b}$ resulted in loosing a large number of events with Higgs.

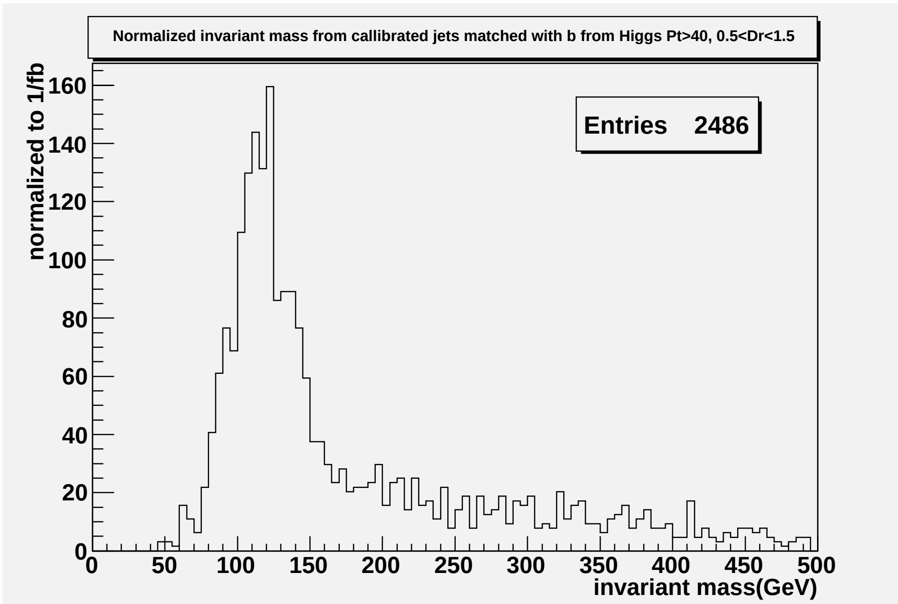

Figure 5.10: Invariant mass from jets matched with $b\bar{b}$ coming from Higgs($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 1TeV$)

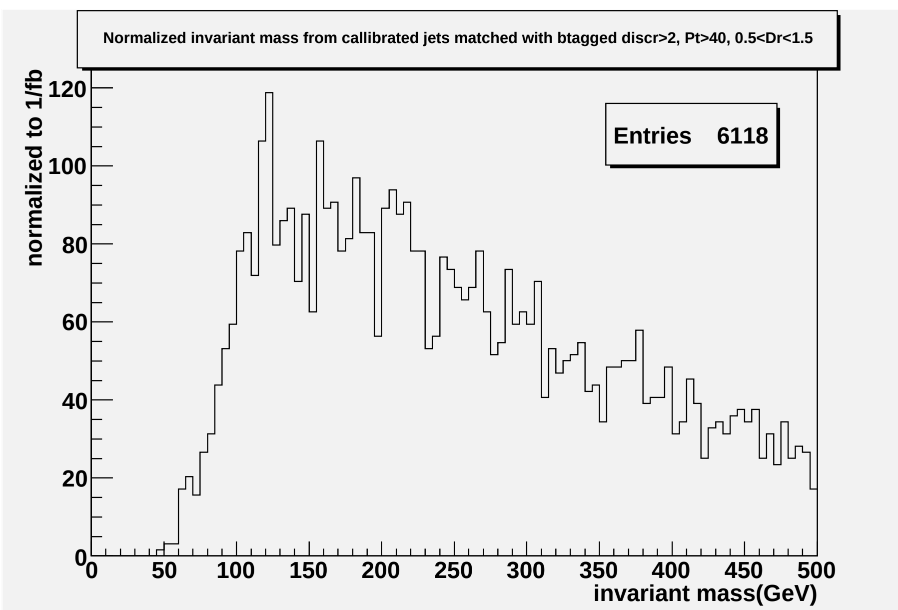

Figure 5.11: Invariant mass from b-tagged jets ($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 1TeV$)

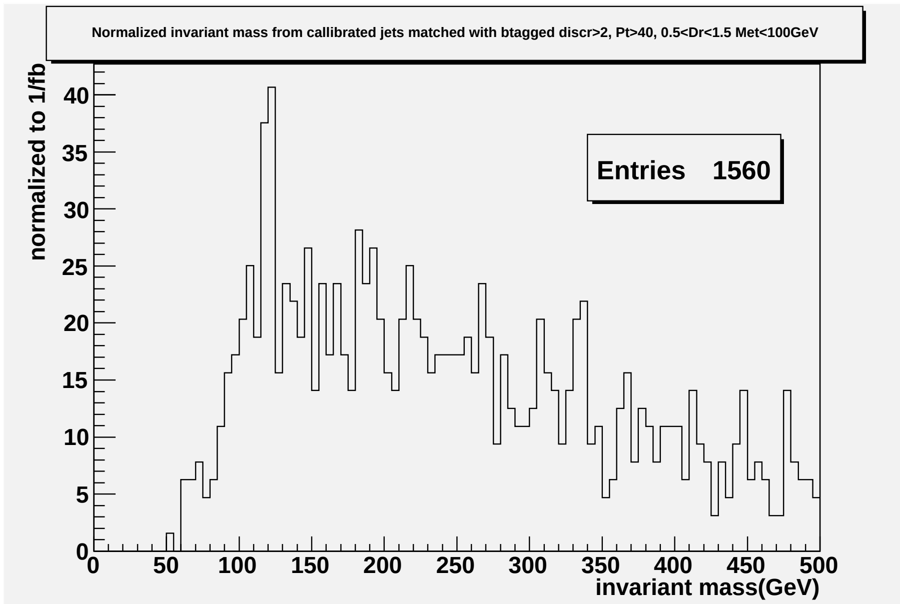

Figure 5.12: Invariant mass from b-tagged jets, $ME_T < 100GeV$ ($M^{BH}_{min} = 4TeV$, $M_{Pl3} = 1TeV$)

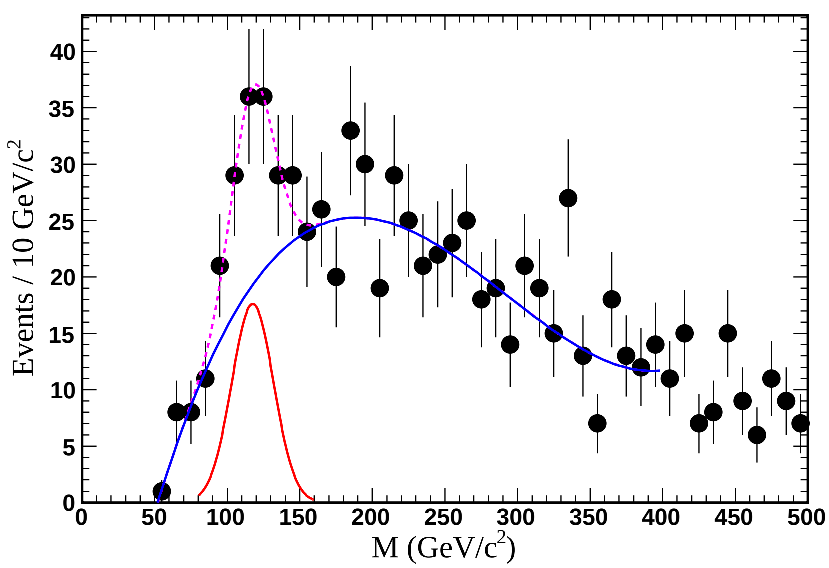

Figure 5.13: Higgs reconstruction fitted data($M^{BH}_{min} = 4TeV$, $M_{Pl3} = 2.5TeV$)

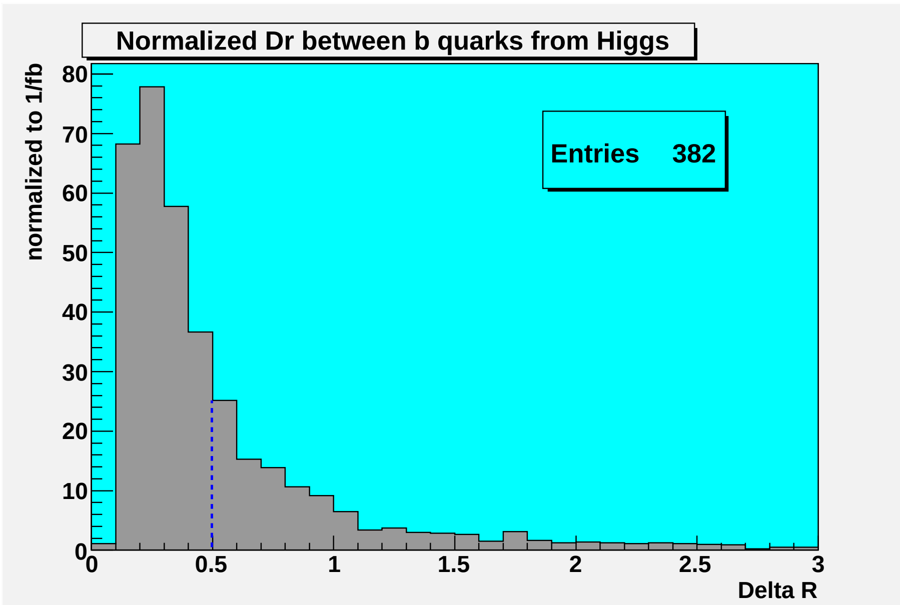

Figure 5.14: ΔR Between $b\bar{b}$ from Higgs ($M_{min}^{BH} = 4TeV$, $M_{Pl3} = 2.5TeV$)

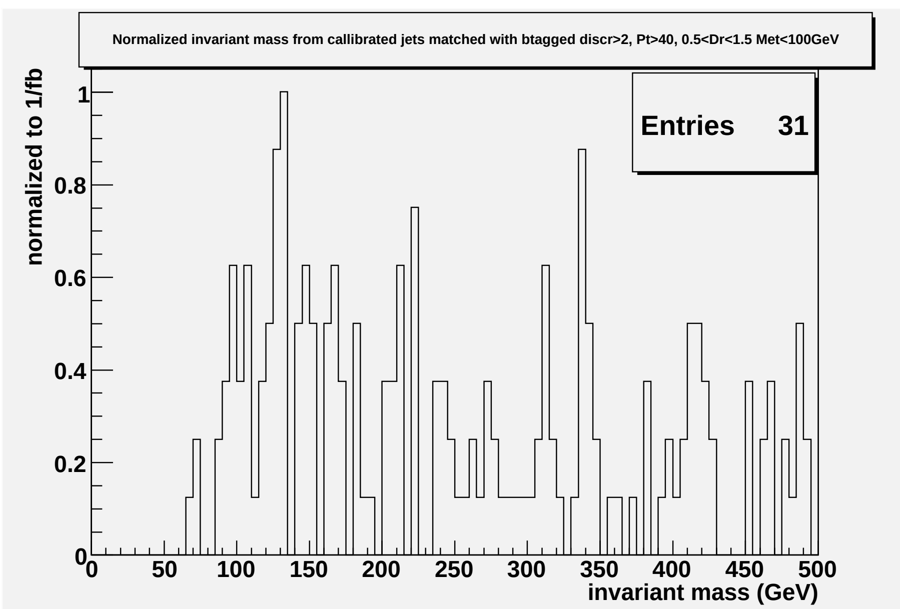

Figure 5.15: Invariant mass from b-tagged jets, $ME_T < 100GeV(M_{min}^{BH} = 4TeV$, $M_{Pl3} = 2.5TeV)$

Chapter 6

Conclusions

The model of large extra dimensions (ADD model) [9] is for the time being one of the candidates for solving one of the most puzzling problems of the Standard Model, that is the hierarchy problem, set by the vast difference between the two fundamental energy scales(M_{ew} and M_{Pl}) of particle physics. In the framework of the ADD model, the Planck scale is set at the same order of magnitude as the Electroweak energy scale, by imposing large extra dimensions, the size of a mm, which are accessible by gravitons only, while all the standard model particles are considered to be localized on a 3-dimensional brane that is our universe. Gravity then is considered to be weak not because of a large Planck scale but because of the size of extra dimensions, so all we currently know about gravity is low energy phenomenology.

Introducing new physics in this manner offers the possibility to address to a number of other unsolved problems like the cosmological constant problem and dark matter. The main advantage of the ADD model is that it is compatible with the other major theories for physics beyond the Standard model, that is supersymmetry and Superstring theory. It actually borrows ideas from superstring theory, which also needs the existence of extra spatial dimensions, in order to localize the standard model fields on a 3-brane.

To our current knowledge the ADD model is not excluded by any experimental or observational data. Near future experiments will be able to test the model. Cavendish type experiments will measure any possible deviations from the inverse square Newton law, and high energy collider experiments like the LHC will probe energy scales above the TeV scale which is considered to be the fundamental Planck scale. So if the proposed model is correct we will be able to observe the creation of miniature Black Holes at the LHC, or even have the first observations of quantum gravity!

Of course the ADD model is not a model without problems. The major problem is what stabilizes the size of the extra dimensions, which is a problem for any theory which imposes extra dimensions. Other problems are proton decay and the preservation of gauge coupling unification close to the Planck scale, one of the most promising theoretical predictions of supersymmetry.

Assuming that miniature Black Holes will be created at the LHC, we considered the possibility of observing rare events, otherwise concealed by Standard model events. The reason for this is that miniature black holes once created, they will decay, mainly via the Hawking radiation procedure, giving characteristic signals which will allow to isolate them from other events. The main signals for Black hole production are high multiplicity, high sphericity, large transverse momentum and large scalar $\sum E_T$. By cutting Standard model background, we were able to reconstruct the Higgs boson mass using the $b\bar{b}$ channel which is the favorable decay mode if the Higgs boson mass is close to 120 GeV. Although we were not able to get a clear Higgs signal, the fact that a peak in the mass plot appeared around the assumed value of the Higgs mass, is very promising since it would not be possible to see such a peak using standard model events. A future research could try to reconstruct Higgs using events with a lower minimum black hole mass, in order to achieve a larger black hole creation cross section. Of course from a theoretical point of view this is considered risky since the black hole mass must be much larger than the Planck mass in order to avoid quantum gravity. A more promising direction is to look for a Higgs boson with a larger mass were the dominant decay modes are W^+W^- and ZZ bosons.

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