Constrains on Supersymmetry from Cosmology
Constrains on Supersymmetry from Cosmology

Master Thesis
of
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May 2008
To my wife Eleni ...
vi
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Chapter 1

The Standard Model and Beyond

One of the most remarkable achievements of modern theoretical physics has been the construction of the Standard Model (S.M.) for weak, electromagnetic and strong interactions. It is a gauge theory based on the group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ which is spontaneously broken to $SU(3)_c \otimes U(1)_{EM}$. This relatively simple model epitomizes our present knowledge of quarks and leptons interactions at energies up to about hundred GeV. The underlying principles of the S.M. theory are:

I. Local gauge invariance

II. Spontaneous symmetry breaking

III. Renormalizability

IV. Locality

V. Unitarity

Despite that success of this theory it is widely expected that there is physics beyond the Standard Model, with new characteristic mass scale(s), perhaps up to, ultimately, a string scale.

The expectation is motivated by several fundamental questions that remain unanswered by the Standard Model. The most pressing one is better understanding of the mechanism of the electroweak symmetry breaking. The origin of flavour and of the pattern of fermion masses and of CP violation
also remain beyond its scope. Moreover, we know now that the physics of the Standard Model cannot explain the baryon asymmetry in the Universe. And on the top of all that come two recent strong experimental hints for physics beyond the Standard Model, that is very small neutrino masses and the presence of dark matter in the Universe. The list can be continued by including dark energy and inflation.

However indirect probes of physics up to multi-TeV scales, using rare decays, or using the sensitivity of electroweak precision data to virtual processes, show no significant deviations. Thus, in a strict experimental sense, we have no guidance for moving beyond the Standard Model, other than the clear anomalies of neutrinos oscillations, dark matter, and dark energy. So the experimental data in addition with the success of the Standard Model indicates that fundamental physics is closely tied to the basic principles of quantum mechanics and to symmetry principles, of which the most notable are relativity and local gauge invariance. These provide a very constrained set of rules for extensions of the Standard Model, and a great deal of theoretical investigation in the past twenty five years has been devoted to mapping out the possible scenarios consistent with these rules. Such scenarios are:

- Supersymmetry
- Grand Unification
- Gravity and String theory
- The physics of Extra Dimensions

Let us turn now to the way the quark-lepton-gauge-symmetry revolution has taught us to view the world.

1.1 Quarks and Leptons in Standard Model

Massless chiral fermions fields are the fundamental objects of matter: left-handed, with helicity \( \lambda = -1/2 \), and right-handed, with helicity \( \lambda = 1/2 \). The former carry different weak charges from the latter. So the left-handed and right-handed fermions behave very differently under the influence of the charged-current weak interactions. Chiral fermion fields are two-component (Weyl) spinors:
1.1. QUARKS AND LEPTONS IN STANDARD MODEL

Left-handed weak-isospin doublets

\[
\begin{pmatrix}
  u \\
  d
\end{pmatrix}_L \quad \begin{pmatrix}
  c \\
  s
\end{pmatrix}_L \quad \begin{pmatrix}
  t \\
  b
\end{pmatrix}_L \quad Quarks
\]

\[
\begin{pmatrix}
  \nu_e \\
  e
\end{pmatrix}_L \quad \begin{pmatrix}
  \nu_\mu \\
  \mu
\end{pmatrix}_L \quad \begin{pmatrix}
  \nu_\tau \\
  \tau
\end{pmatrix}_L \quad Leptons
\]

These are left-handed chiral fields, each describing two massless degrees of freedom: a particle with the helicity \(\lambda = -1/2\) and its antiparticle with \(\lambda = 1/2\).

Right-handed fields in the same representations of \(SU(2)_L \otimes U(1)_Y\) as the left-handed fields (1.1) and (1.2) do not exist in Nature \(^1\). Instead, we have:

Right-handed weak isospin singlets

\[
\begin{aligned}
  u_R & \quad c_R & \quad t_R \\
  d_R & \quad s_R & \quad b_R \\
  e_R & \quad \mu_R & \quad \tau_R \\
\end{aligned} \quad Quarks
\]

\[
\begin{aligned}
  e_R & \quad \mu_R & \quad \tau_R \\
\end{aligned} \quad Leptons
\]

These are right-handed chiral fields.

The quarks are influenced by the strong interaction, and so carry color, the strong-interaction charge(red, blue, green), whereas the leptons do not feel the strong interaction, and are colorless.

All fermions come in three copies called families, as shown in table 1.1. The quantum numbers of quarks and leptons are also listed in the same table. We note that this is the picture of the matter before the discovery of neutrino oscillations, that require neutrino mass and almost surely imply the existence of right-handed neutrinos.

Looking a little more closely at the constituents of matter, we find that our world is not as neat as we have represented here. The striking fact is the asymmetry between left-handed fermion doublets and right-handed fermion singlets, which is manifested in parity violation in the charged-current weak interactions. This nature’s broken mirror, the distinction between left-handed and right-handed fermions, qualifies as one of the great mysteries.

\(^1\)We do not know whether the pairs of quarks and leptons carry a right-handed weak isospin, in other words, whether they make up \(SU(2)_R\) doublets. We do know that we have-as yet-no experimental evidence for right-handed charged-current weak interactions.
CHAPTER 1. THE STANDARD MODEL AND BEYOND

Table 1.1: Weak Isospin and Hypercharge assignments

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>T</th>
<th>T^3</th>
<th>Q</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>νeL</td>
<td>νμL</td>
<td>ντL</td>
<td>1/2</td>
<td>1/2</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>eL</td>
<td>μL</td>
<td>τL</td>
<td>1/2</td>
<td>-1/2</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>eR</td>
<td>μR</td>
<td>τR</td>
<td>0</td>
<td>0</td>
<td>-2</td>
<td>-1</td>
</tr>
<tr>
<td>uL</td>
<td>cL</td>
<td>tL</td>
<td>1/2</td>
<td>1/2</td>
<td>2/3</td>
<td>1/3</td>
</tr>
<tr>
<td>uR</td>
<td>cR</td>
<td>tR</td>
<td>0</td>
<td>0</td>
<td>2/3</td>
<td>4/3</td>
</tr>
<tr>
<td>dL</td>
<td>sL</td>
<td>bL</td>
<td>1/2</td>
<td>-1/2</td>
<td>-1/3</td>
<td>1/3</td>
</tr>
<tr>
<td>dR</td>
<td>sR</td>
<td>bR</td>
<td>0</td>
<td>0</td>
<td>-1/3</td>
<td>-2/3</td>
</tr>
</tbody>
</table>

1.2 Abelian Gauge Symmetries and QED

Invariance under translations, time displacements, rotations and Lorentz transformations leads to the conservation of momentum, energy and angular momentum. Rather than studying these conservation laws, we are interested here in ‘internal’ symmetry transformations that do not mix fields with different space-time properties (that is, transformations that commute with the space-time components of the wavefunction).

For example, an electron is a complex field and Dirac’s Lagrangian:

\[ \mathcal{L} = i \bar{\psi} \gamma_\mu \partial^\mu \psi - m \bar{\psi} \psi \]  

shows that it is invariant under the phase transformation

\[ \psi(x) \rightarrow e^{i \alpha} \psi(x), \]  

where \( \alpha \) is a real constant. The family of phase transformations \( U(\alpha) \equiv e^{i \alpha} \), where a single parameter \( \alpha \) may run continuously over real numbers, forms a unitary Abelian group known as the \( U(1) \) group. Abelian just records the property that the group multiplication is commutative:

\[ U(\alpha_1)U(\alpha_2) = U(\alpha_2)U(\alpha_1). \]  

From a physicist point of view, the existence of a symmetry implies that some quantity is unmeasurable. For example, translation invariance means that we cannot determine an absolute position in space. Similarly, (1.4) implies that the phase \( \alpha \) is unmeasurable, it has no physical meaning and can
be chosen arbitrarily. $\alpha$ is a constant; therefore, once we fix it, the value is specified for all space and time. We speak of *global gauge* invariance. This is surely not the most general invariance, for it would be more satisfactory if $\alpha$ could differ from space-time point to point, that is, $\alpha = \alpha(x)$.

- \textbf{$U(1)_{\text{em}}$ Local Gauge Invariance}

Last paragraph suggests that we should generalize (1.4) to the transformation

$$\psi(x) \rightarrow e^{i\alpha(x)} \psi(x), \quad (1.6)$$

where $\alpha(x)$ now depends on space and time in a completely arbitrary way. This is known as *local gauge* invariance.

The Lagrangian (1.3) is not invariant under the space-time-dependent phase transformation (1.6). If we insist on imposing invariance of the Lagrangian under such transformations, we must seek a modified derivative, $D_\mu$, that transforms covariantly under phase transformations, that is, like $\psi$ itself:

$$D_\mu \psi \rightarrow e^{ia(x)} D_\mu \psi. \quad (1.7)$$

To form the *covariant derivative* $D_\mu$, we must introduce a vector field $A_\mu$ with transformation properties such that the unwanted term is canceled. This can be achieved by the construction

$$D_\mu \equiv \partial_\mu - ieA_\mu, \quad (1.8)$$

where $e$ is the charge in natural units of the particle described by $\psi(x)$ and the field $A_\mu$ transforms as

$$A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha. \quad (1.9)$$

Therefore, local gauge invariance is not possible for a free theory. By demanding it we are forced to introduce a ($4-v$ector) field $A_\mu$, called *gauge field* whose interactions with the charged particle are precisely determined and which undergoes the transformation (1.9). The demand of this type of phase invariance will have then dictated the form of the interaction-this is the basis of the *gauge principle*.

If we are to regard this new field as the physical photon field, we must add to the Lagrangian a term corresponding to its kinetic energy. Since the
CHAPTER 1. THE STANDARD MODEL AND BEYOND

kinetic term must be invariant under (1.9), it can only involve the gauge invariant field strength tensor

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]  

(1.10)

We are thus led to the Lagrangian of QED:

\[ \mathcal{L}_{QED} = \bar{\psi} (i \gamma_\mu \partial_\mu - m) \psi + e \bar{\psi} \gamma_\mu A_\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \]  

(1.11)

Note that the addition of a mass term \( \frac{1}{2} m^2 A_\mu A^\mu \) is prohibited by gauge invariance. The gauge particle, the photon, must be massless (that is, the gauge field have infinite range). This was expected since there is no limit to the distance over which the phases of the electron field might have to be reconciled.

In summary, we see that by imposing the ‘natural’ requirement of local phase invariance on the free fermion Lagrangian, we are led to the interacting field theory of QED. We must emphasize that there is ultimately no compelling logic for the vital leap to a local phase invariance from a global one. The latter is, by itself, both necessary and sufficient in quantum field theory to guarantee local charge conservation. Nevertheless, the gauge principle—deriving interactions from the requirement of local phase invariance—provides a satisfying conceptual unification of the interactions present in the Standard model.

- **U(1)\(Y\) Local Gauge Invariance**

The hypercharge operator \( Y \), which is defined by

\[ Q = T^3 + \frac{Y}{2}, \]

(1.12)
generates the symmetry group \( U(1)_Y \). One representation of the generators of the group \( U(1)_Y \) is: \( \frac{Y}{2} \) and the respective elements representation is: \( e^{i\theta \frac{Y}{2}} \).

We recall the way left (L) and right (R) states behave under the action of the hypercharge operator:

\[ YL = -1L, \quad YR = -2R \]  

(1.13)
Therefore, the $U(1)_Y$ transformation of the left-handed fermion doublets $L$ and the right-handed singlets $R$ are respectively:

$$
\begin{align*}
L & \longrightarrow e^{i\theta(x)}\frac{Y}{2} L = e^{-i\theta(x)/2} L \\
R & \longrightarrow e^{i\theta(x)} \frac{Y}{2} R = e^{-i\theta(x)} R
\end{align*}
$$

(1.14)

In order to have an invariance Lagrangian under the $U(1)_Y$ gauge transformation, we must introduced a vector field $B^\mu$ which transforms as

$$
B^\mu \longrightarrow B^\mu - g' \frac{Y}{2} \partial^\mu \theta(x) \quad \text{where } g' \text{ is the dimensionless coupling constant for } U(1)_Y
$$

(1.15)

end modify the derivative

$$
\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu + ig \frac{Y}{2} B_\mu
$$

(1.16)

We also have to add a kinetic term for the gauge field $B^\mu$, to do so we first define the field strength tensor

$$
B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu.
$$

(1.17)

We therefore obtain the Lagrangian:

$$
\mathcal{L}_1 = \bar{\psi} \gamma^\mu (i\partial_\mu - g \frac{Y}{2} B_\mu) \psi - \frac{1}{4} B_{\mu\nu} B^{\mu\nu},
$$

(1.18)

where $\psi$ can be any right-handed or left-handed fermion state.

### 1.3 Non-Abelian Gauge Invariance

In the preceding section the successful dynamical theory-QED-has been introduced, based on the remarkably simple gauge principle: namely that the theory should be invariant under local phase transformations on the wavefunctions or field operators of charged particles. Such transformations were characterized as Abelian, since the phase factors commuted. This section will be concerned with the formulation and elementary application of the remaining two dynamical theories within the Standard Model-that is, QCD and the electroweak theory. They are built on a generalization of the gauge principle, in which the transformations involve more than one state, or field,
at a time. In that case, the ‘phase factors’ become matrices, which generally do not commute with each other, and the associated symmetry is called a *non-Abelian* one. When the phase factors are independent of the spacetime coordinate $x$, the symmetry is a ‘global non-Abelian’ one; when they are allowed to depend on $x$, one is led to a non-Abelian gauge theory. Both QCD and the electroweak theory which are of the latter type, providing fact that all three dynamical theories in the Standard Model are based on a gauge principle of local phase invariance.

Fermions obtain one extra index, except the Dirac’s one.

$$\psi \rightarrow \psi^a \quad a = 1, 2, 3 \quad \text{strong interactions, color index}$$

$$\psi \rightarrow \psi_\alpha \quad \alpha = 1, 2 \quad \text{weak interactions, isospin index, } \left( ^1_2 \right)$$

We demand the transformation:

$$\psi^a(x) \rightarrow \psi'^a(x) \equiv U^{ab}(x) \psi^b(x)$$

(1.20)

where $U^{ab}(x) \in SU(N)$, one local, non-Abelian operator.

To approach our goal of generating an invariance theory under the local gauge transformation (1.20) we must introduce a vector field $A_\mu(x)$, which is now a matrix $N \times N$ and obeys the transformation rule:

$$A_\mu(x) \rightarrow A'^\mu(x) = U(x) A_\mu(x) U^{-1}(x) + \frac{i}{g} U(x) \partial_\mu U^{-1}(x)$$

(1.21)

where $A_\mu$ hermitian$^2$ with $Tr(A_\mu) = 0$ and $[A_\mu, A_\nu] \neq 0$

We then form the covariant derivative:

$$D_\mu(x) \equiv \partial_\mu + igA_\mu(x)$$

(1.22)

Using the $T_\alpha$ generators of $SU(N)$ group, $A_\mu$ may be written as:

$$A_\mu(x) = A^\alpha_\mu(x) T_\alpha \quad \alpha = 1, 2, \ldots N^2 - 1$$

(1.23)

$N = 2 \quad \Rightarrow \quad SU(2), \text{weak interactions, three gauge fields} \quad H^\alpha_\nu \quad \alpha = 1, 2, 3$

$N = 3 \quad \Rightarrow \quad SU(3), \text{strong interactions, eight gauge fields} \quad G^\alpha_\nu \quad \alpha = 1, \ldots, 8$

All these gauge fields are *vector bosons* with spin = 1 and local gauge invariance requires to be *massless*.

$^2$ hermitian means: $A = A^\dagger$
1.3. NON-ABELIAN GAUGE INVARIANCE

Since not all the generators $T_\alpha$ commute with each other, the group is non-Abelian. The commutation relations of the matrices $T_\alpha$ are:

$$[T_\alpha, T_b] = i f_{abc} T_c$$

(1.24)

where $f_{abc}$ is a set of real numbers called structure constants of the group and is completely antisymmetric.

We can now rewrite the covariant derivative as:

$$D_\mu(x) = \partial_\mu + igA_\mu^\alpha T_\alpha$$

(1.25)

and define the $SU(N)$ field strength tensor by evaluating the commutator of two $D$’s of the previous form:

$$[D_\mu, D_\nu] = igF_{\mu\nu}^\alpha T_\alpha$$

(1.26)

or more explicitly,

$$F_{\mu\nu}^\alpha \equiv (\partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha) - g f_{abc} A_\mu^b A_\nu^c$$

(1.27)

This allows us to write down a gauge invariant kinetic energy term for each of the $A_\mu^\alpha$ fields

$$-\frac{1}{4} F_{\mu\nu}^\alpha F_{\alpha}^{\mu\nu}$$

(1.28)

In the case of QCD-$SU(3)$ group-the final gauge invariant Lagrangian is

$$\mathcal{L}_{QCD} = \bar{q}(i\gamma^\mu \partial_\mu - m)q - g_s(\bar{q}\gamma^\mu T_\alpha q)G_\mu^\alpha - \frac{1}{4} G_\mu^\alpha G_{\mu\nu}^\alpha$$

(1.29)

This is the Lagrangian for interacting colored quarks $q$ and vector gluons $G_\mu^\alpha$ with coupling specified by $g_s$, which follows simply from demanding that the Lagrangian be invariant under local color phase transformations to the quark fields. Since we can arbitrarily vary the phase of three quark color fields, it is not surprising that eight vector gluon fields ($G_\mu^1, G_\mu^2, ..., G_\mu^8$) are needed to compensate all possible phase changes. Just as for the photon, local gauge invariance requires the gluons to be massless.
• SU(2) Local Gauge Invariance

One representation of the generators of the group SU(2) is:

\[ T^a = \frac{\tau^a}{2}, \quad \text{where } \tau^a : \text{Pauli matrices}. \quad (1.30) \]

The respective group-elements representation is the transformation matrix:

\[ U(\alpha_a(x)) = e^{i\alpha_a(x)\frac{\tau^a}{2}}, \quad \text{is one local non-Abelian operator} \quad (1.31) \]

For example the transformation of the left-handed weak-isospin doublet \( L = (\nu_e e_L) \) is:

\[
\begin{pmatrix}
\nu_e \\
e_L
\end{pmatrix}_L \rightarrow e^{i\alpha_a(x)\frac{\tau^a}{2}} \begin{pmatrix}
\nu_e \\
e_L
\end{pmatrix}_L
\]

\[ (1.32) \]

The 1 – d representation of the generators of the group SU(2) is:

\[ T^a = (0) \]

\[ (1.33) \]

now the transformation matrix is

\[ e^{i\alpha(x)0} = 1 \]

\[ (1.34) \]

therefore the right-handed isosinglet \( R = e_R \) is invariance under the SU(2) gauge-transformation

\[ e_R \rightarrow e_R \]

\[ (1.35) \]

To achieve local SU(2) invariance of the Lagrangian we follow our familiar procedure and we replace \( \partial_\mu \) by the covariant derivative

\[ D_\mu = \partial_\mu + ig \frac{\tau_a}{2} H^a_\mu \quad \text{where } g \text{ is the dimensionless coupling constant for } SU(2) \]

\[ (1.36) \]

the tree gauge fields, \( H^a_\mu(x) \) with \( a = 1, 2, 3 \), are massless vector bosons with \( \text{spin} = 1 \) and they transform as the transformation rule (1.21)

\[ H^a_\mu(x) \rightarrow H^a_\mu(x) = U(x)H^a_\mu(x)U^{-1}(x) + \frac{i}{g}U(x)\partial_\mu(U^{-1}(x)) \]

\[ (1.37) \]

\[ 3\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]
The gauge invariant Lagrangian is then

\[ L_2 = \bar{L} \gamma^\mu (i \partial_\mu - g \frac{\tau^a}{2} H^a_\mu) L + \bar{R} i \gamma^\mu \partial_\mu R - \frac{1}{4} H^a_\mu H^a_\mu, \]  

where we have added the kinetic energy term of the gauge fields, with the field strength tensor defined as

\[ H^a_\mu = \partial_\mu H^a_\nu - \partial_\nu H^a_\mu - g \varepsilon^{abc} H^b_\mu H^c_\nu. \]  

The last term in (1.39) and in (1.37) arises from the non-Abelian character of the group.

- **SU(2) × U(1)\text{Y}** Local Gauge Invariance

The electroweak gauge group, SU(2) × U(1)\text{Y}, implies two sets of gauge fields: the weak isovector \( H^a_\mu \), with coupling constant \( g \), and the weak isoscalar \( B_\mu \), with coupling constant \( g' \). Corresponding to these gauge fields are the field-strength tensors: \( H^a_\mu \), Eq.(1.39), for the weak-isospin symmetry and \( B_\mu \), Eq.(1.17), for the weak-hypercharge symmetry. Just as the previous Lagrangians resulted from imposing local gauge invariance, we are led to the electroweak Lagrangian by requiring SU(2) × U(1)\text{Y} invariant form

\[ \mathcal{L} = \bar{L} \gamma^\mu (i \partial_\mu - g \frac{\tau^a}{2} H^a_\mu - g' \frac{Y}{2} B_\mu) L \\
+ \bar{R} \gamma^\mu (i \partial_\mu - g' \frac{Y}{2} B_\mu) R - \frac{1}{4} H^a_\mu H^a_\mu - \frac{1}{4} B_\mu B^\mu, \]  

where the final two terms are the kinetic energy and self-coupling of the \( H^a_\mu \) and \( B_\mu \) fields. We also remind that the operators \( \tau^a \) and \( Y \) are the generators of the SU(2) and U(1)\text{Y} groups of gauge transformations, respectively.

So far, so good. However, note that \( \mathcal{L} \) describes massless gauge bosons and massless fermions. Mass terms such as \( \frac{1}{2} M^2 B_\mu B^\mu \) and \( -m \bar{\psi} \psi \) are not gauge invariant and so cannot be added. The requirement of a massless gauge boson is familiar (for example, the photon of the previous section). If we wanted to add a mass term, for example, for the electron this would be

\[ -m_e \bar{e} e = -m_e \bar{e} \left[ \frac{1}{2} (1 - \gamma^5) + \frac{1}{2} (1 + \gamma^5) \right] e \\
= -m_e (\bar{e}_R e_L + \bar{e}_L e_R). \]  

(1.41)
Since $e_L$ is a member of an isospin doublet and $e_R$ is a singlet, this term manifestly breaks gauge invariance.

To generate the particle masses in a gauge invariant way, we must use the Higgs mechanism. That is, we spontaneously break the gauge symmetry, which has the paramount virtue that the theory remains renormalizable.

1.4 The Higgs Field in the Standard Model

We want to formulate the Higgs mechanism so that the intermediate bosons of the weak interaction, $W^\pm$ and $Z^0$, become massive and the photon remains massless. To do this, we introduce \emph{four real scalar fields} $\phi_i$. We have to add to $\mathcal{L}$ an $SU(2) \times U(1)_Y$ gauge invariant Lagrangian for the scalar fields

$$\mathcal{L}_{\text{scalar}} = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi),$$

where the gauge-covariant derivative is

$$D_\mu = \partial_\mu + ig^Y \frac{Y}{2} B_\mu + ig^a \frac{\tau_a}{2} H^a_\mu,$$

and the potential interaction has the form

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + |\lambda|(\phi^\dagger \phi)^2.$$

To keep $\mathcal{L}$ gauge invariant, the $\phi_i$ must belong to $SU(2) \times U(1)_Y$ multiplets. The most economical choice is to arrange four fields in an isospin doublet with weak hypercharge $Y = 1$:

$$\phi = \left( \begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right) \quad \text{with} \quad \phi^+ \equiv (\phi_1 + i \phi_2)/\sqrt{2}, \quad \phi^0 \equiv (\phi_3 + i \phi_4)/\sqrt{2}.$$  \hspace{1cm} (1.45)

We are also free to add a Yukawa interaction between the scalar fields and the leptons, in order to give masses to the leptons and quarks.

$$\mathcal{L}_{\text{Yukawa}} = -G_e [\bar{R}_i (\phi^\dagger)^a L_i^a + \bar{L}_i^a (\phi)^a R_i] \quad a = 1, \ldots, 4 \ (\text{Dirac index}) \quad a = 1, 2 \ (\text{isospin index})$$

We then arrange the scalar self-interactions so that the vacuum state corresponds to a broken-symmetry solution. The electroweak symmetry is spontaneously broken if the parameters $\mu^2 < 0$ and $\lambda > 0$. The minimum
energy, or vacuum state, may then be chosen to correspond to the vacuum expectation value
\[ \langle \phi \rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \]
where \( v = \sqrt{-\mu^2/|\lambda|} \) and sets the scale of electroweak symmetry breaking. Now the choice \( \langle \phi \rangle_0 \) with \( T = \frac{1}{2}, T^3 = -\frac{1}{2}, \) and \( Y = 1 \) breaks both \( SU(2) \) and \( U(1)_Y \) gauge symmetries. But since \( \langle \phi \rangle_0 \) is neutral, the \( U(1)_{em} \) symmetry with generator
\[ Q = T^3 + \frac{Y}{2} \]
remains unbroken. That is,
\[ Q \langle \phi \rangle_0 = 0, \]
so that: \( \langle \phi \rangle_0 \rightarrow \langle \phi \rangle'_0 = e^{ia(x)}Q \langle \phi \rangle_0 = \langle \phi \rangle_0, \) for any value of \( a(x). \) The vacuum is thus invariant under \( U(1)_{em} \) transformations and the photon remains massless. Out of the four \( SU(2) \times U(1)_Y \) generators \( T^a, Y, \) only the combination \( Q \) obeys relation (1.49). The other three break the symmetry and generate massive gauge bosons. It appears that we have accomplished our goal of breaking \( SU(2) \times U(1)_Y \rightarrow U(1)_{em}. \) Note however that, the parameters \( \mu \) and \( \lambda \) are free parameters of the Standard Model. Equivalently, the scale of the electroweak symmetry breaking is not predicted by the theory and must be taken from experiment.

As a result of spontaneous symmetry breaking, the weak bosons acquire masses. Specifically, the mediator of the charged-current weak interaction:
\[ W^\pm_\mu \equiv (H^1_\mu + iH^2_\mu)/\sqrt{2}, \quad \text{acquires a mass:} \quad m_W = \frac{1}{2}gv. \]

The mediator of the neutral-current weak interaction:
\[ Z^0_\mu \equiv \frac{gH^3_\mu - gB_\mu}{\sqrt{g^2 + g^{'2}}}, \quad \text{acquires a mass:} \quad m_Z = \frac{1}{2}v\sqrt{g^2 + g^{'2}}. \]

After spontaneous symmetry breaking, there remains an unbroken \( U(1)_{em} \) phase symmetry, so that electromagnetism is mediated by a massless photon
\[ A_\mu \equiv \frac{gB_\mu + gH^3_\mu}{\sqrt{g^2 + g^{'2}}} \quad \text{with} \quad m_A = 0, \]
coupled to the electric charge: \( e = gg'/\sqrt{g^2 + g'^2}. \)
We define the weak mixing angle, $\theta_w$, to be the angle that appears in the change of basis from $(H^3, B)$ to $(Z^0, H)$:

\[
\begin{pmatrix}
Z^0 \\
H
\end{pmatrix} = \begin{pmatrix}
\cos \theta_w & -\sin \theta_w \\
\sin \theta_w & \cos \theta_w
\end{pmatrix} \begin{pmatrix}
H^3 \\
B
\end{pmatrix},
\]

then we have the relations,

\[
\frac{q}{g} = \tan \theta_w, \quad g = \frac{e}{\sin \theta_w}.
\]

We see here that the couplings of all of the weak bosons are described by two parameters: the well-measured electron charge $e$, and the new parameter $\theta_w$. The couplings induced by $W$ and $Z$ exchange will also involve the masses of these particles. However, these masses are not independent, since it follows from Eqs. (1.50) and (1.51) that

\[
\frac{m_W}{m_Z} = \cos \theta_w.
\]

All effects of $W$ and $Z$ exchange processes can be written in terms of the three basic parameters $e$, $\theta_w$ and $m_W$. 

As a vestige of the spontaneous breaking of the symmetry, there remains a massive, spin-zero particle, the Higgs boson. The mass of the Higgs scalar is given symbolically as $m_{h_{SM}}^2 = -2\mu^2 > 0$, but we have no prediction for its value. Though what we take to be the work of the Higgs boson is all around us, the Higgs particle itself has not yet been observed. This is an important aim for LHC.

An attractive feature of the Standard Model is that the same Higgs doublet which generates $W^\pm$ and $Z$ masses is also sufficient to give masses to the leptons and quarks. These are determined not only by the scale of electroweak symmetry breaking, $v$, but also by their Yukawa interactions with the scalars. For further analysis about electroweak symmetry breaking see Appendix A.

---

4 The Higgs doublet contains 4 real degrees of freedom and 3 generators are broken when $SU(2)_L \times U(1)_Y \to U(1)_{EM}$. The fourth scalar degree of freedom that remains in the physical spectrum is the CP-even neutral Higgs boson ($h_{SM}$) of the Standard Model.
1.5 The Final Lagrangian

To summarize the Standard Model, we gather together all the ingredients of the Lagrangian. The complete Lagrangian is:

$$\mathcal{L}_{SM} = -\frac{1}{4} H^a_{\mu\nu} H^a_{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$+ \bar{L}\gamma^\mu (i\partial_\mu - g\tau^a H^a_{\mu}) L$$

$$+ \bar{R}\gamma^\mu (i\partial_\mu - g\tau^a H^a_{\mu}) R$$

$$+ \left| (i\partial_\mu - g\tau^a H^a_{\mu}) \phi \right|^2 - V(\phi^\dagger \phi)$$

$$- G_e [\bar{L}_i (\phi^\dagger)^a L_i^a + \bar{L}_i (\phi)^a R_i]$$

$$\begin{align*}
W^\pm, Z^0, A & \text{ Kinetic energies and self-interactions} \\
& \text{lepton and quark kinetic energies} \\
& \text{and their interactions with} \\
W^\pm, Z^0, A & \text{masses and couplings} \\
& \text{lepton and quark masses and} \\
& \text{coupling to Higgs}
\end{align*}$$

$L$ denotes a left-handed fermion (lepton or quark) doublet, and $R$ denotes a right-handed fermion singlet.

It is widely believed that gauge principles may generate the structure of all particle interactions. The Standard Model for weak and electromagnetic interactions is constructed from a gauge theory with four gauge fields, the photon and the massive bosons, $W^\pm$ and $Z^0$. We generate the masses of the gauge fields (as well as the fermions) by spontaneous symmetry breaking, ensuring that one of them (the photon) remains massless. We also require that the theory must reproduce the low-energy phenomenology. Such a theory will be renormalizable and will contain one (or possible more) Higgs scalars but no Goldstone bosons.

The electroweak gauge symmetry is hidden, $SU(2) \times U(1)_Y \longrightarrow U(1)_{em}$. If it were not, the world would be very different: • All the quarks and leptons would be massless and move at the speed of light. • Electromagnetism as we know it would not exist, but there would be a long-range hypercharge force. • The strong interaction, QCD, would confine quarks and generate baryon
masses roughly as we know them. • The Bohr radius of atoms consisting of an electron or neutrino attracted by the hypercharge interaction to the nucleons would be infinite. • Beta decay, inhibited in our world by the great mass of the $W$ boson, would not be weak. • The unbroken $SU(2)$ interaction would confine objects that carry weak isospin.

It is fair to say that electroweak symmetry breaking shapes our world! In fact, when we take into account every aspect of the influence of the strong interactions, the analysis of how the world would be is very subtle and fascinating. Nevertheless, there are several reasons to suspect that the gauge theory of the Standard Model cannot be considered as the final theory. We have already mentioned some of its shortcomings. Further analysis will be conducted in the next section.

## 1.6 Standard Model’s Puzzles

Although the SM provides a correct description of virtually all known microphysical nongravitational phenomena, there are a number of theoretical and phenomenological issues that the SM fails to address adequately:

$\Leftarrow$ First of all, there is three sets of data that are not accounted for by the Standard Model:

* **Dark Matter.** There is a lot (23%) of dark(non-SM) matter in the universe and neutrinos cannot account for it. The SM also does not have a viable candidate for the dark matter of the universe, nor a viable inflaton.

* **Dark Energy.** There is another source of energy in the universe (73%), known as ‘dark energy’ and it’s very much looks like vacuum energy. If we translate the 74% in high energy physics units this corresponds to a vacuum energy which is: $|V_{\text{vac}}| \sim 10^{-12}eV^4$. If we now go to the SM and calculate the vacuum energy, in particular the quantum contribution to the vacuum energy, this is $|V_{\text{vac}}| \gtrsim 10^{44}eV^4$. So there is something more, in fact, that one would have to do to the theory because theory and data are in wild disagreement. This is a version of what is called cosmological constant problem.

* **Massive Neutrinos.** The recent evidence for neutrino oscillations suggest the mass of the third-generation neutrino of about $0.05eV$. This implies that the theory has to be extended, as in the SM the neutrinos are strictly left-handed and massless. Right-handed neutrinos can be added, but achieving ultralight neutrino masses from the seesaw mechanism requires the introduc-
1.6. STANDARD MODEL’S PUZZLES

formation of a new scale much larger than $O(100 \text{ GeV})$.

$\rightarrow$ Gravity Problem. Another important issue is that the SM based on QFT (Quantum field Theory), as we defined it, includes 3 of the 4 known interactions. Gravity which is an important interaction, at least in macroscopic cases, is not part of the SM and eventually we would like to include it into a theory that describes all interactions. In particular we would like to describe a QFT of gravity and this will entail changes also in the theory of the SM. In addition, another important puzzle is that the characteristic scale which is associated with gravity, the Planck scale, defined as $M_{\text{planck}} = \frac{1}{\sqrt{G_N}} \simeq 10^{19} \text{GeV}$ is so much higher than any of the scales that characterize the other 3 interactions. This is in the root of very important problems when we try to put everything together in a unified theory.

$\rightarrow$ Flavor problem. The pattern of SM masses is mysterious at least:

<table>
<thead>
<tr>
<th>family\type</th>
<th>ups</th>
<th>downs</th>
<th>leptons</th>
</tr>
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<tbody>
<tr>
<td>3$^{rd}$</td>
<td>$m_t = 171, 4$</td>
<td>$m_b = 4, 2$</td>
<td>$m_\tau = 1, 7$</td>
</tr>
<tr>
<td>2$^{nd}$</td>
<td>$m_c = 1, 2$</td>
<td>$m_s = 0, 1$</td>
<td>$m_\mu = 0, 1$</td>
</tr>
<tr>
<td>1$^{st}$</td>
<td>$m_u = 3.10^{-3}$</td>
<td>$m_d = 5.10^{-3}$</td>
<td>$m_e = 5.10^{-4}$</td>
</tr>
</tbody>
</table>

Why is there a seemingly unnecessary three-fold repetition of ‘generations’? Even the second generation led the Nobel Laureate I.I. Rabi to ask ”who ordered muon?” Now we face even more puzzling question of having three generations. In fact, we know of no good reason why any, beyond the first one, should exist. And why do the fermions have a mass spectrum which stretches over almost six orders of magnitude between the electron and the top quark? This question becomes even more serious once we include the neutrinos, that today we have upper bounds on the absolute values of their masses coming from $\beta - decay$. Neutrino masses seem to be in the $10^{-12} - 10^{-14} \text{GeV}$ range. So, if we put everything together we see that the SM masses span 16 orders of magnitude! We have no concrete understanding of the mass spectrum nor the mixing patterns. We have to mention that all the masses in SM come from symmetry breaking and they are proportional to the expectation value of the Higgs. So what needs to be explain is the relative size of the associate Yukawa couplings.
Quantum number and hypercharge assignments. The quantum number assignments, table 1.1, of the fermions appear utterly bizarre. Probably the hypercharges are the weirdest of all. These assignments, however, are crucial to guarantee the cancelation of anomalies which could jeopardize the gauge invariance at the quantum level, rendering the theory inconsistent. Another related puzzle is why the hypercharges are quantized in the unit of 1/6. In principle, the hypercharges can be any numbers, even irrational.

CP violation. The origin of CP violation is also a mystery. CP violation was observed in the kaon system in the 1960s and more recently in the B system. CP violation is also a necessary ingredient for baryogenesis. Whether the observed CP violation in the neutral meson systems is related to the CP violation that affects the baryon asymmetry is an open question. However, other CP-violating observables, most notably the fermion electric dipole moments (EDMs), have not been observed experimentally.

The strong CP problem. We have a non-perturbative parameter, the dimensionless coefficient $\theta$ which multiplies a CP-violating term of the SM QCD Lagrangian. The strong CP problem of the SM is that this parameter to be less than $10^{-10}$, when there is no symmetry reason for such a small number. More precisely, the term responsible for the problem is the following CP-odd term:

$$\delta L_{SCPV} = \frac{\theta}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} G_{\mu\nu}^a G_{\rho\sigma}^a,$$

where $G_{\rho\sigma}^a$ is the field strength of the $SU(3)_C$ gluons. We know from experiment that a measure of CP violation, which is the EDM for the neutron, is $D_n \lesssim 10^{-25} e cm$, this leads to the unnaturally small $\theta \lesssim 10^{-10}$.

Electroweak symmetry breaking (EWSB). In the SM, electroweak symmetry breaking is parameterized by the Higgs boson and its potential $V(\phi^\dagger \phi)$, Eq (1.44). However, the Higgs sector is not constrained by any symmetry principles, and it must be put into the theory by hand. This can be rephrased as follows: (i) What decides the scale of Electroweak symmetry breaking $v \simeq 246 \text{ GeV}$? (ii) What decides the mass of the Higgs?

Gauge coupling unification. The gauge group of the Standard Model $SU(3) \times SU(2) \times U(1)_Y$ is not unified, this means that each group factor comes with its own independent coupling strength. The idea that the gauge
couplings undergo renormalization group evolution in such a way that they meet at a point at a high scale lends credence to the picture of grand unified theories (GUTs) and certain string theories. However, precise measurements of the low energy values of the gauge couplings demonstrated that the SM cannot describe gauge coupling unification accurately enough to imply it is more than an accident.

\[ \text{Hierarchy problem.} \] Phenomenologically the mass of the Higgs boson associated with electroweak symmetry breaking must be in the electroweak range. However, radiative corrections to the Higgs mass are quadratically dependent on the UV cutoff \( \Lambda \), since the masses of fundamental scalar fields are not protected by chiral or gauge symmetries.

\[
m_{\text{eff}}^2(E) = m^2 + \frac{\lambda - \lambda_0^2}{4\pi^2}(\Lambda^2 - E^2)
\]

The ‘natural’ value of the Higgs mass is therefore of \( \mathcal{O}(\Lambda) \) rather than \( \mathcal{O}(100 \text{ GeV}) \), leading to a destabilization of the hierarchy of the mass scales in the SM. In other words, to achieve \( m \sim \mathcal{O}(100 \text{ GeV}) \) it is necessary to fine-tune the scalar mass-squared parameter \( m_0^2 \sim \Lambda^2 \) of the fundamental ultraviolet theory to a precision of \( m^2/\Lambda^2 \). If, for example, \( \Lambda = 10^{16} \text{ GeV} \) and \( m = 100 \text{ GeV} \), the precision of tuning must be \( 10^{-28} \).

Therefore, the Standard Model contains several nagging theoretical problems which cannot be solved without the introduction of some new physics. Supersymmetry is, at present, many theorists’ favorite candidate for such new physics. Supersymmetry is the subject of the next chapter.
Chapter 2

Supersymmetry

All the puzzles raised in the previous section (and more) cry out for a more fundamental theory underlying the Standard Model. What history suggests is that the fundamental theory lies always at shorter distances than the distance scale of the problem. For instance, the equation of state of the ideal gas was found to be a simple consequence of the statistical mechanics of free molecules. The van der Waals equation, which describes the deviation from the ideal one, was the consequence of the finite size of molecules and their interactions. Mendeleev’s periodic table of chemical elements was understood in terms of the bound electronic states, Pauli exclusion principle and spin. The existence of varieties of nuclide was due to the composite nature of nuclei made of protons and neutrons. The list would go on and on. Indeed, seeking answers at more and more fundamental level is the heart of the physical science, namely the reductionist approach.

The motivation for supersymmetry is to make the Standard Model applicable to much shorter distances so that we can hope that answers to many of the puzzles in the Standard Model can be given by physics at shorter distance scales. In order to do so, supersymmetry repeats what history did with the positron: doubling the degrees of freedom with an explicitly broken new symmetry. In this chapter, we will give an introduction to supersymmetry from a modern perspective. Emphasis is placed on the technical background requisite for the recent applications of supersymmetric gauge theories to S.M.
2.1 Theoretical Framework

The Standard Model is a theory of spin-$\frac{1}{2}$ matter fermions which interact via the exchange of spin-1 gauge bosons, where the bosons and fermions live in independent representations of the gauge symmetries. Supersymmetry (SUSY) is a symmetry which establishes a one-to-one correspondence between bosonic and fermionic degrees of freedom, and provides a relation between their couplings. It is the largest known symmetry of the $S-matrix^1$. Of importance to us is the fact that SUSY leads to an amelioration of divergences in quantum field theory. This, in turn, protects the electroweak scale from large quantum corrections, and stabilizes the ratio $M_W/M_X$, when the Standard Model is embedded into a larger theory, involving an ultra-high energy scale $M_X$ (e.g., $M_{GUT}$ or $M_{Planck}$). In other words, SUSY models do not require the incredible fine-tuning endemic to the Higgs sector of the SM, provided only that the super-partners exist at or below the TeV energy scale. On the experimental side, while the measurements of the three SM gauge couplings at LEP are incompatible with unification in the minimal $SU(5)$ model, they unify remarkably well in the simplest supersymmetric $SU(5)$ GUT, with SUSY broken at the desired scale $\sim 1$ TeV. The last two arguments are especially important in that they bound the SUSY breaking scale and strongly suggest that supersymmetric partners of ordinary particles should be accessible at colliders designed to probe the TeV energy scale.

The hypothesis that nature is supersymmetric is very compelling to many particle physicists for several reasons.

- It can be shown that the SUSY algebra is the only nontrivial extension of the set of spacetime symmetries which forms one of the foundations of relativistic quantum field theory.

- Locally supersymmetric theories necessarily incorporate gravity. More precisely, if supersymmetry is formulated as a local symmetry, then one is necessarily forced into introducing a massless spin $– 2$ (graviton) field into the theory. The resulting supergravity theory reduces to Einstein’s general relativity theory in the appropriate limit.

---

1 A symmetry of the S-matrix means that the symmetry transformations have the effect of merely reshuffling the asymptotic single and multiparticle states.
2.1. THEORETICAL FRAMEWORK

\[ m_{eff}^2 = m^2 + \lambda + \lambda_t \lambda_t + \ldots + \frac{\lambda}{16\pi^2} \int \frac{d^4 p}{p^2} - \frac{\lambda_t^2}{16\pi^2} \int \frac{d^4 p}{p^2} \]

Figure 2.1: Radiative corrections on Higgs mass

▷ Spacetime supersymmetry appears to be an essential ingredient of superstring theory.

▷ Hierarchy problem. The SM Higgs sector has two ‘naturalness’ problems. One is the technical naturalness problem associated with the absence of a symmetry protecting the Higgs mass at the electroweak scale when the natural cutoff scale is at or above the GUT scale. The second problem is associated with explaining the origin of the electroweak scale, when a more ‘fundamental’ embedding theory such as a GUT or string theory typically is defined at a scale which is at least \(10^{13}\) times larger than the electroweak scale. This is typically referred to as the gauge hierarchy problem.

Incorporation of supersymmetry into the SM leads to a solution of the technical hierarchy problem, as the Higgs mass parameter is not renormalized as long as supersymmetry is unbroken. Namely, since supersymmetry relates the scalar and fermionic sectors, the chiral symmetries which protect the masses of the fermions also protect the masses of the scalars from quadratic divergences. As shown in figure (2.1) if \(\lambda = \lambda_t^2\) quadratic divergences in loop

\[2^\text{In other words, the radiative corrections naturally give the Higgs a mass of order the GUT scale or a similarly large cutoff scale; unlike the fermions, there is no chiral symmetry protecting the scalar sector.}\]
corrections to the Higgs boson mass will cancel between fermionic and bosonic loops. This mechanism works only if the superpartner particle masses are roughly of order or less than the weak scale. Supersymmetry also mitigates the gauge hierarchy problem by breaking the electroweak symmetry radiatively through logarithmic running, which explains the large number $\sim 10^{13}$.

Electroweak symmetry breaking is a derived consequence of supersymmetry breaking in many particle physics models with weak-scale supersymmetry, whereas electroweak symmetry breaking in the SM is put in ‘by hand’. To oversimplify a little, (this will be expanded in Appendix B), the SM effective Higgs potential has the form $V(h) = m^2 h + \lambda h^2$, Eq.(1.44) with $h = \phi^\dagger \phi$. First, supersymmetry requires that the quadric coupling $\lambda$ is a function of the $U(1)_Y$ and $SU(2)$ gauge couplings $\lambda = (g'^2 + g^2)/2$. Second, the $m^2$ parameter runs to negative values at the electroweak scale, driven by the large top quark Yukawa coupling. Thus the ‘Mexican hat’ potential with a minimum away from $h = 0$ is derived rather than assumed. The SUSY radiative electroweak symmetry-breaking mechanism works best if the top quark has mass $m_t \sim 150 - 200 \, GeV$. The recent discovery of the top quark with $m_t = 171 \pm 2.4 \, GeV$ is consistent with this mechanism.

There exists an experimental hint: the three gauge couplings can unify at the Grand Unification scale if there exist weak-scale supersymmetric particles, with a desert between the weak scale and the GUT scale. The extrapolation of the low energy values of the gauge couplings using renormalization group equations and the MSSM particle content shows that the gauge couplings unify at the scale $M_G = 3 \times 10^{16} GeV$. Gauge coupling unification and electroweak symmetry breaking depend on essentially the same physics since each needs the soft masses and $\mu$ to be of order the electroweak scale.

In supersymmetric theories, the lightest superpartner (LSP) can be stable. This stable superpartner provides a nice cold dark matter candidate. Simple estimates of its relic density are of the right order of magnitude to provide the observed amount. LSPs were noticed as good candidates before the need for nonbaryonic cold dark matter was established. (this will be expanded in chapter 4)

Supersymmetry has also made several correct predictions:
• Supersymmetry predicted in the early 1980s that the top quark would be heavy, because this was a necessary condition for the validity of the electroweak symmetry breaking explanation.

• Supersymmetric grand unified theories with a high fundamental scale accurately predicted the present experimental value of $\sin^2 \theta_w$ before it was measured.

• Supersymmetry requires a light Higgs boson to exist, consistent with current precision measurements, which suggest $m_H < 200$ GeV.

Remarkably, supersymmetry was not invented to explain any of the above physics. Supersymmetry was discovered as a beautiful property of string theories and was studied for its own sake in the early 1970s. Only after several years of studying the theory did it become clear that supersymmetry solved the above problems, one by one. Furthermore, all of the above successes can be achieved simultaneously, with one consistent form of the theory and its parameters. Low energy supersymmetry also has no known incorrect predictions; it is not easy to construct a theory that explains and predicts certain phenomena and has no conflict with other experimental observations.

2.2 Construction of Supersymmetry Theories

This section describes how to construct the Lagrangian of a supersymmetric field theory. To that end I first give a formal definition of the supersymmetry (SUSY) algebra. The next two subsections introduce chiral and vector superfields, respectively. The construction of the SUSY Lagrangian will be accomplished in Sec. 2.2.4. Finally, soft SUSY breaking is treated in the last subsection.

Relativistic quantum field theory is formulated to be consistent with the symmetries of the Lorentz/Poincaré group — a non-compact Lie algebra. Mathematically, supersymmetry is formulated as a generalization of the Lorentz/Poincaré group of space-time symmetries to include spinorial generators which obey specific anticommutation relations; such an algebra is known as a graded Lie algebra. Representations of the SUSY algebra include both bosonic and fermionic degrees of freedom. In particular, we need equal numbers of physical (propagating) bosonic and fermionic degrees of freedom.
2.2.1 The SUSY Algebra

It is quite clear from previous considerations that the symmetry we are looking for must connect bosons and fermions. In other words, the generators $Q$ of this symmetry must turn a bosonic state into a fermionic one, and vice versa. This in turn implies that the generators themselves carry half-integer spin, i.e. are fermionic. This is to be contrasted with the generators of the Lorentz group, or with gauge group generators, all of which are bosonic.

The simplest choice of SUSY generators is a 2-component (Weyl) spinor $Q$ and its conjugate $\overline{Q}$. Since these generators are fermionic, their algebra can most easily be written in terms of anti-commutators:

\[
\{Q_\alpha, Q_\beta\} = \{\overline{Q}_\dot{\alpha}, \overline{Q}_\dot{\beta}\} = 0 \quad (2.1)
\]

\[
\{Q_\alpha, \overline{Q}_\dot{\beta}\} = 2\sigma_\mu^{\alpha\dot{\beta}} P_\mu \quad [Q_\alpha, P_\mu] = 0. \quad (2.2)
\]

Here the indices $\alpha, \beta$ of $Q$ and $\dot{\alpha}, \dot{\beta}$ of $\overline{Q}$ take values 1 or 2, $\sigma^\mu = (1, \sigma_i)$ with $\sigma_i$ being the Pauli matrices, and $P_\mu$ is the translation generator (momentum).

For a compact description of SUSY transformations, it will prove convenient to introduce ‘fermionic coordinates’ $\theta, \bar{\theta}$. These are anti-commuting, ‘Grassmann’ variables:

\[
\{\theta, \theta\} = \{\theta, \bar{\theta}\} = \{\bar{\theta}, \bar{\theta}\} = 0. \quad (2.3)
\]

Of course, the objects on which SUSY transformations act must then also depend on $\theta$ and $\bar{\theta}$. This leads to the introduction of superfields, which can be understood to be functions of $\theta$ and $\bar{\theta}$ as well as the spacetime coordinates $x_\mu$. Since $\theta$ and $\bar{\theta}$ are also two-component spinors, one can even argue that supersymmetry doubles the dimension of spacetime, the new dimensions being fermionic.

For most purposes it is sufficient to consider infinitesimal SUSY transformations by using chiral representations where $\theta$ and $\bar{\theta}$ are treated slightly differently. These can be written as (I am suppressing spinor indices from now on):

L-representation:

\[
\delta_s \Phi_L = \left( \alpha \frac{\partial}{\partial \theta} + \bar{\alpha} \frac{\partial}{\partial \bar{\theta}} + 2i\bar{\theta} \sigma_\mu \bar{\alpha} \frac{\partial}{\partial \mu} \right) \Phi_L, \quad D_L = \frac{\partial}{\partial \theta} + 2i\sigma_\mu \bar{\theta} \frac{\partial}{\partial \mu}, \quad \overline{D_L} = -\frac{\partial}{\partial \bar{\theta}}. \quad (2.4)
\]
2.2. CONSTRUCTION OF SUPERSYMMETRY THEORIES

R-representation:

\[ \delta_s \Phi_R = \left( \alpha \frac{\partial}{\partial \theta} + \bar{\alpha} \frac{\partial}{\partial \bar{\theta}} - 2i \alpha \sigma^\mu \bar{\theta} \partial_\mu \right) \Phi_R, \]
\[ \bar{D}_R = -\frac{\partial}{\partial \bar{\theta}} - 2i \theta \sigma^\mu \partial_\mu, \quad D_R = \frac{\partial}{\partial \theta}. \quad (2.5) \]

Where \( D, \bar{D} \) are SUSY-covariant derivatives, \( \alpha, \bar{\alpha} \) are again Grassmann variables and \( \Phi \) is a superfield. Clearly, \( \bar{D} (D) \) has a particularly simple form in the \( L (R) \) representation. The following identity allows to switch between representations:

\[ \Phi(x, \theta, \bar{\theta}) = \Phi_L(x_\mu + i \theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}) = \Phi_R(x_\mu - i \theta \sigma_\mu \bar{\theta}, \theta, \bar{\theta}). \quad (2.6) \]

So far everything has been written for arbitrary superfields \( \Phi \). However, we will only need two kinds of special superfields, which are irreducible representations of the SUSY algebra; they will be discussed in the following two subsections.

2.2.2 Chiral Superfields

The first kind of superfield we will need are chiral superfields. This name is derived from the fact that the SM fermions are chiral, that is, their left- and right-handed components transfer differently under \( SU(2) \times U(1)_Y \). We therefore need superfields with only two physical fermionic degrees of freedom, which can then describe the left- or right-handed component of an SM fermion. Of course, the same superfields will also contain bosonic partners, the sfermions. The simplest way to construct such superfields is to require either

\[ \bar{D} \Phi_L \equiv 0 \quad (\Phi_L \text{ is a left-chiral}) \quad \text{or} \quad (2.7) \]
\[ D \Phi_R \equiv 0 \quad (\Phi_R \text{ is a right-chiral}). \quad (2.8) \]

Clearly these conditions are most easily implemented using the chiral representations of the SUSY generators and SUSY-covariant derivatives. For example, Eq.(2.4) shows that in the \( L \)-representation, Eq.(2.7) simply implies that \( \Phi_L \) only depends on \( x \) and \( \theta \). We can then expand \( \Phi_L \) as:

\[ \Phi_L(x, \theta) = \phi(x) + \sqrt{2} \theta^\alpha \psi_\alpha(x) + \theta^\alpha \theta^\beta \epsilon_{\alpha\beta} F(x), \quad (2.9) \]
where $\epsilon_{\alpha\beta}$ is the anti-symmetric tensor in two dimensions. $\theta$ has mass dimension $-1/2$, assigning the usual mass dimension $+1$ to the scalar field $\phi$ then gives the usual mass dimension $+3/2$ for the fermionic field $\psi$, and the unusual mass dimension $+2$ for the scalar field $F$; the superfield $\Phi$ itself has mass dimension $+1$. The fields $\phi$ and $F$ are complex scalars, while $\psi$ is a Weyl spinor. At first glance, $\Phi_L$ seems to contain four bosonic degrees of freedom and only two fermionic ones; however, we will see later on that not all bosonic fields represent physical (propagating) degrees of freedom. The expression for $\Phi_R$ in the R-representation is very similar; one merely has to replace $\theta$ by $\bar{\theta}$.

Applying the explicit form (2.4) of the SUSY transformation to the left-chiral superfield (2.9) gives: $\delta_s \Phi_L = \delta_s \phi + \sqrt{2} \theta \delta_s \psi + \theta \theta \delta_s F$. Explicitly, we have:

$$
\begin{align*}
\delta_s \phi &= \sqrt{2} \alpha \psi \quad (\text{boson} \to \text{fermion}) \quad (2.10) \\
\delta_s \psi &= \sqrt{2} \alpha F + i \sqrt{2} \sigma^\mu \bar{\alpha} \partial_\mu \phi \quad (\text{fermion} \to \text{boson}) \quad (2.11) \\
\delta_s F &= -i \sqrt{2} \bar{\partial}_\mu \psi \sigma^\mu \bar{\alpha} \quad (F \to \text{total derivative}) \quad (2.12)
\end{align*}
$$

Notice in particular the result (2.12); it implies that $\int d^4x F(x)$ is invariant under SUSY transformations, assuming as usual that boundary terms vanish. We will come back to this point in Sec. 2.2.4.

### 2.2.3 Vector Superfields

The chiral superfields introduced in the previous subsection can describe spin-0 bosons and spin-1/2 fermions, e.g. the Higgs boson and the quarks and leptons of the SM. However, we also have to describe the spin-1 gauge bosons of the SM. To this end one introduces *vector superfields* $V$. They are constrained to be self-conjugate:

$$
V(x, \theta, \bar{\theta}) \equiv V^\dagger(x, \theta, \bar{\theta}). \quad (2.13)
$$
This leads to the following representation of $V$ in component form:

$$V(x, \theta, \bar{\theta}) = \left(1 + \frac{1}{4} \theta \overline{\theta} \sigma^\mu \partial_\mu \right) C(x) + \left(i \theta + \frac{1}{2} \theta \sigma^\mu \bar{\theta} \partial_\mu \right) \chi(x) + \frac{i}{2} \theta \theta [M(x) + iN(x)] \right)$$

$$+ \left( -i \bar{\theta} + \frac{1}{2} \bar{\theta} \sigma^\mu \theta \partial_\mu \right) \lambda(x) - \frac{i}{2} \theta \theta [M(x) - iN(x)]$$

$$- \theta \sigma^\mu \bar{\theta} A^\mu(x) + i \theta \theta \lambda(x) - i \theta \theta \lambda(x) + \frac{1}{2} \theta \theta \theta D(x).$$

Here, $C, M, N$ and $D$ are real scalars, $\chi$ and $\lambda$ are Weyl spinors, and $A^\mu$ is a vector field. If $A^\mu$ is to describe a gauge boson, $V$ must transform as an adjoint representation of the gauge group.

The general form (2.14) is rather unwieldy. Fortunately, we now have many more gauge degrees of freedom than in nonsupersymmetric theories, since now the gauge parameters are themselves superfields. A general non-abelian supersymmetric gauge transformation acting on $V$ can be described by

$$e^{gV} \longrightarrow e^{-ig\Lambda^\dagger} e^{gV} e^{ig\Lambda}$$

where $\Lambda(x, \theta, \bar{\theta})$ is a chiral superfield and $g$ is the gauge coupling. In the case of an abelian gauge symmetry, this transformation rule can be written more simply as

$$V \longrightarrow V + i(\Lambda - \Lambda^\dagger) \quad \text{(abelian case)}.$$

Remembering that a chiral superfield contains four scalar (bosonic) degrees of freedom as well as one Weyl spinor, it is quite easy to see that one can use the transformation (2.15) or (2.16) to chose

$$\chi(x) = C(x) = M(x) = N(x) \equiv 0.$$

This is called the ‘WessZumino’ (WZ) gauge; it is in some sense the SUSY analog of the unitary gauge in ‘ordinary’ field theory, since it removes many unphysical degrees of freedom. Also the WZ gauge can be used in combination with any of the usual gauges. However, the choice (2.17) is sufficient by itself to remove the first two lines of eq.(2.14), leading to a much more compact expression for $V$. Assigning the usual mass dimension +1 to $A^\mu$ gives the canonical mass dimension +3/2 for the fermionic field $\lambda$, while the field $D$ has the unusual mass dimension +2, just like the $F$-component of the
chiral superfield (2.9). Notice also that the superfield $V$ itself has no mass dimension.

Applying a SUSY transformation to eq. (2.14) obviously gives a lengthier expression than in case of chiral superfields. Here I only quote the important result

$$\delta S_D = -\alpha \sigma^\mu \partial_{\mu} \bar{\lambda} + \bar{\alpha} \sigma^\mu \partial_{\mu} \lambda,$$

(2.18)

which shows that the $D$ component of a vector superfield transforms into a total derivative. Together with the analogous result (2.12) for chiral superfields, this provides the crucial clue for the construction of the Lagrangian, to which we turn next.

### 2.2.4 The Supersymmetric Lagrangian

$$\mathcal{L} = \sum_i \left( F_i F_i^* + | \partial_{\mu} \phi |^2 - i \bar{\psi}_i \sigma_{\mu} \partial^{\mu} \psi_i \right)$$

$$+ \left[ \sum_j \frac{\partial f(\phi_i)}{\partial \phi_j} F_j - \frac{1}{2} \sum_{j,k} \frac{\partial^2 f(\phi_i)}{\partial \phi_j \partial \phi_k} \psi_j \psi_k + h.c. \right]$$

$$+ | D_{\mu} \phi |^2 - i \bar{\psi} \sigma_{\mu} D^{\mu} \psi + g \phi^* D \phi + ig \sqrt{2} \left( \phi^* \lambda \psi - \bar{\lambda} \bar{\psi} \phi \right) + | F |^2$$

$$- \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} D_a D^a + \left( - \frac{i}{2} \lambda^a \sigma_{\mu} \partial^{\mu} \bar{\lambda} + \frac{1}{2} g f^{abc} \lambda_a \sigma_{\mu} \Lambda^a_{\mu b c} + h.c. \right)$$

I. The first line contains kinetic energy terms for the scalar component $\phi$ as well as the fermionic component $\psi$ of chiral superfields! Equally importantly, this line does not contain kinetic energy terms for $F$. This field does therefore not propagate (it does not represent a physical degree of freedom); it is a mere auxiliary field, which can be integrated out exactly using its purely algebraic equation of motion. A chiral superfield therefore only has two physical bosonic degrees of freedom, described by the complex scalar $\phi$, i.e. it contains equal numbers of propagating bosonic and fermionic degrees of freedom.
Figure 2.2: (a) Yukawa and four-scalar couplings arising from the supersymmetric Lagrangian with superpotential \((2.20)\); (b) Diagrams which give the leading radiative corrections to the scalar field mass term.

II. In the second line we have introduced the superpotential \(f\):

\[
f(\Phi_i) = \sum_i k_i \Phi_i^* + \frac{1}{2} \sum_{i,j} m_{ij} \Phi_i \Phi_j + \frac{1}{3} \sum_{i,j,k} g_{ijk} \Phi_i \Phi_j \Phi_k, \tag{2.20}
\]

where the \(\Phi_i\) are all left-chiral superfields, and the \(k_i, m_{ij}\) and \(g_{ijk}\) are constants with mass dimension 2, 1 and 0, respectively. The second term in this line describes fermion masses and Yukawa interactions, while the last term describes scalar mass terms and scalar interactions. Since these interactions do not involve any spacetime derivatives, choosing the superpotential to be a globally gauge invariant function of superfields is sufficient to guarantee the gauge invariance of the Lagrangian. For a renormalizable theory, the superpotential must be a polynomial of degree \(\leq 3\).

III. The gauged interactions are introduced in the third line. The coupling of the gauge (super)fields to the (chiral) matter (super)fields is accomplished by a SUSY version of the familiar ‘minimal coupling’:

\[
\int d^2 \theta d^2 \bar{\theta} \Phi^\dagger \Phi \rightarrow \int d^2 \theta d^2 \bar{\theta} \Phi^\dagger e^{2g_V} \Phi. \tag{2.21}
\]
In order to obtain the final result we have used the W-Z gauge (2.17), and introduced the usual gauge-covariant derivative $D_\mu = \partial_\mu + igA_\mu^\alpha T_\alpha$, where the $T_\alpha$ are group generators. Note that this piece of the Lagrangian not only describes the interactions of the matter fields (both fermions and scalars) with the gauge fields and the auxiliary fields, but also contains gauge-strength Yukawa-interactions between fermions (or higgsinos) $\psi$, sfermions (or Higgs bosons) $\phi$, and gauginos $\lambda$.

IV. The last line in addition to the familiar kinetic energy term for the gauge fields, also contains a kinetic energy terms for the gauginos $\lambda_\alpha$, as well as the canonical coupling of the gauginos to the gauge fields, which is determined by the group structure constants $f^{abc}$. Note that we do not have a kinetic energy term for the $D_\alpha$ fields. They are therefore also auxiliary fields, and can again easily be integrated out.

2.2.5 Supersymmetry Breaking

The supersymmetric Lagrangian constructed in the previous subsection satisfies the equation: $m_{\tilde{f}} = m_f$, that is, the masses of the ‘ordinary’ SM particles and their superpartners are identical. This is clearly not realistic; there is no selectron with mass 511 keV, nor is there a smuon with mass 106 MeV, etc. Indeed, no superpartners have been discovered yet. Searches at the $e^+e^-$ collider LEP imply that all charged sparticles must be heavier than 60 to 80 GeV. Similarly, searches at the Tevatron $p\bar{p}$ collider imply bounds on squark and gluino masses between 150 and 220 GeV. Hence supersymmetry must be broken spontaneously. In other words, the underlying model should have a Lagrangian density that is invariant under supersymmetry, but a vacuum state that is not. In this way, supersymmetry is hidden at low energies in a manner analogous to the fate of the electroweak symmetry in the ordinary Standard Model.

Therefore, at low energies, supersymmetry must be a broken symmetry. Since this implies the appearance of supersymmetry-breaking terms in the Lagrangian, an immediate question is whether such terms spoil supersymmetry’s elegant solution to the hierarchy problem. As generic quantum field the-

---

3Spontaneous breaking of supersymmetry is achieved in the same way as the electroweak symmetry breaking. One introduces the field whose vacuum expectation value is nonzero and breaks the symmetry. However, due to a special character of SUSY, this should be a superfield whose auxiliary $F$ and $D$ components acquire nonzero v.e.v.s.
ories with scalars generally have a hierarchy problem, if all supersymmetry-
breaking terms consistent with other symmetries of the theory are allowed
the dangerous UV divergences may indeed be reintroduced.

Fortunately, such dangerous divergences are not generated to any or-
der in perturbation theory if only a certain subset of terms, called ‘soft
SUSY-breaking terms’ (defined to be those that do not destabilize the ra-
tio $M_W/M_X$) consistent with the SM symmetries, are present in the theory.
‘Soft’ here means that we want to maintain the cancellation of quadratic di-
vergencies. Such operators, are said to break supersymmetry softly. The part
of the Lagrangian which contains these terms is generically called the soft
supersymmetry-breaking Lagrangian $L_{\text{soft}}$, or simply the soft Lagrangian.

The soft supersymmetry-breaking Lagrangian is defined to include all
allowed terms that do not introduce quadratic divergences in the theory:
all gauge invariant and Lorentz invariant terms of dimension two and three
(i.e., the relevant operators from an effective field theory viewpoint). It can
be shown that, at least to one-loop order, quadratic divergencies still cancel
even if we introduce

- scalar mass terms $-m^2_{\phi_i}|\phi_i|^2$, for various left- and right- spin-0 (squark,
slepton, Higgs) fields.

- trilinear (A terms) scalar interactions $-A_{ijk}\phi_i\phi_j\phi_k + \text{h.c.}$

into the Lagrangian. Girardello and Grisaru have shown that this result
survives in all orders in perturbation theory. They also identified three ad-
ditional types of soft breaking terms:

- gaugino mass terms $-\frac{1}{2}m_{\lambda_l}\bar{\lambda}_l\lambda_l$, where $l$ labels the group factor, $(U(1),
SU(2), SU(3))$;

- bilinear (B term) scalar interactions $-B_{ij}\phi_i\phi_j + \text{h.c.}$; and

- linear terms $-C_i\phi_i$.

Of course, linear terms are gauge invariant only for gauge singlet fields. Note
that we are not allowed to introduce additional masses for chiral fermions,
beyond those contained in the superpotential. Also, the relations between
dimensionless couplings imposed by supersymmetry must not be broken.

This completes our discussion of the construction of ‘realistic’ supersym-
metric field theories. Let us now apply these results to the simplest such
model.
2.3 Minimal Supersymmetric Standard Model

The simplest supersymmetric model of particle physics which is consistent with the SM is called the Minimal Supersymmetric Standard Model (MSSM). As implied by the name, MSSM is essentially a straightforward supersymmetrization of the SM. In particular, ‘minimal’ means that we want to keep the number of superfields and interactions as small as possible. The recipe for this model is to start with the SM of particle physics, but in addition add an extra Higgs doublet of opposite hypercharge. (This ensures cancelation of triangle anomalies due to Higgsino partner contributions.) Next, proceed with supersymmetrization, following well-known rules to construct supersymmetric gauge theories. At this stage one has a globally supersymmetric SM theory.

Since, the MSSM is defined to be the minimal supersymmetric extension of the SM, is an \( SU(3) \times SU(2)_L \times U(1)_Y \) supersymmetric gauge theory with a general set of soft supersymmetry-breaking terms. The known matter and gauge fields of the SM are promoted to superfields in the MSSM: each known particle has a (presently unobserved) superpartner. The superpartners of the SM chiral fermions are the spin-zero sfermions, the squarks and sleptons. The superpartners of the gauge bosons are the spin-1/2 gauginos. Since the SM matter fermions reside in different representations of the gauge group than the gauge bosons, we have to place them in different superfields; no SM fermion can be identified as a gaugino. The fundamental field content of the MSSM is listed in Table 2.1, for one generation of quark and lepton (squark and slepton) fields.

As we have already mentioned in section 2.2.2, the chiral fermion field with the corresponding scalar field and the auxiliary field together constitute a chiral matter superfield, in much the same way that the proton and the neutron together constitute the isospin doublet. The left-handed and right-handed pieces of the quarks and leptons are separate two-component Weyl fermions with different gauge transformation properties in the SM, so each must have its own complex scalar partner. Note that this means that the number of bosonic and fermionic degrees of freedom in a chiral superfield are the same. The names for the spin \(-0\) partners of the quarks and leptons are constructed by prepending an ‘s’, for scalar. So, generically they are called squarks and sleptons (short for ‘scalar quark’ and ‘scalar lepton’), or sometimes sfermions. The symbols for the squarks and sleptons are the same as for the corresponding fermion, but with a tilde (\(\tilde{}\)) used to denote the
### Table 2.1: The MSSM Particle Spectrum for one generation of Quarks and Leptons

<table>
<thead>
<tr>
<th>Gauge multiplets</th>
<th>Boson fields</th>
<th>Fermion fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SU(3)$</td>
<td>$g^a$</td>
<td>$\tilde{g}^a$ gluinos</td>
</tr>
<tr>
<td>$SU(2)$</td>
<td>$W^i$</td>
<td>$\tilde{W}^i$ winos</td>
</tr>
<tr>
<td>$U(1)$</td>
<td>$B$</td>
<td>$\tilde{B}$ bino</td>
</tr>
<tr>
<td><strong>Matter multiplets</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>leptons</td>
<td>$\tilde{L}^j = (\tilde{\nu}, \tilde{e}_L^j)$</td>
<td>$(\nu, e^-)_L$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{E}^c = \tilde{e}_R^c$</td>
<td>$e^c_L$</td>
</tr>
<tr>
<td>quarks</td>
<td>$\tilde{Q}^j = (\tilde{u}_L^j, \tilde{d}_L^j)$</td>
<td>$(u, d)_L$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{U}^c = \tilde{u}_R^c$</td>
<td>$u^c_L$</td>
</tr>
<tr>
<td></td>
<td>$\tilde{D}^c = \tilde{d}_R^c$</td>
<td>$d^c_L$</td>
</tr>
<tr>
<td>Higgs bosons</td>
<td>$H_d^j$</td>
<td>$(\tilde{H}_d^0, \tilde{H}_d^-)_L$</td>
</tr>
<tr>
<td></td>
<td>$H_u^j$</td>
<td>$(\tilde{H}_u^0, \tilde{H}_u^+)_L$</td>
</tr>
</tbody>
</table>

superpartner of a Standard Model particle. For example, the superpartners of the left-handed and right-handed parts of the electron Dirac field are called left- and right-handed *selectrons*, and are denoted $\tilde{e}_L$ and $\tilde{e}_R$. It is important to keep in mind that the ‘handedness’ here does not refer to the helicity of the selectrons (they are spin $-0$ particles) but to that of their superpartners. Finally, a complete list of the squarks is $\tilde{q}_L, \tilde{q}_R$ with $q = u, d, s, c, b, t$. The gauge interactions of each of these squark and slepton fields are the same as for the corresponding Standard Model fermions; for instance, the left-handed squarks $\tilde{u}_L$ and $\tilde{d}_L$ couple to the $W$ boson, while $\tilde{u}_R$ and $\tilde{d}_R$ do not. Note that, for the quarks and leptons, we normally have left-handed and right-handed fields in the Standard Model. In order to promote them to chiral superfields, however, we need to make all fields left-handed Weyl spinors. This can be done by charge-conjugating all right-handed fields\(^4\). So that a right-handed field (i.e., $u_R$) will be denoted as a left-handed antiparticle field (i.e., $\bar{u}_L$). One generation of the SM is therefore described by five left-chiral superfields: $Q$ contains the quark and squark $SU(2)$ doublets, $U^c$ and $D^c$ contains the

\(^4\)anti-particles are assigned to the complex conjugate of the representation to which the corresponding particles belong i.e., $\bar{u} = u^*$
(s)quark singlets, $L$ contains the (s)lepton doublets, and $E^c$ contains the (s)lepton singlets. We remind that the $SU(2)$ singlet superfields contain left-handed anti-fermions; their scalar members therefore have charge $+1$ for $\tilde{e}_R^+$, $-2/3$ for $\tilde{u}_R$, and $+1/3$ for $\tilde{d}_R$. Of course, we need three generations to describe the matter content of the SM.

As discussed in Sec. 2.2, we have to introduce vector superfields to describe the gauge sector. In particular, we need eight gluinos $\tilde{g}$ as partners of the eight gluons of QCD, three winos $\tilde{W}$ as partners of the $SU(2)$ gauge bosons, and a bino $\tilde{B}$ as $U(1)_Y$ gaugino. Since $SU(2) \times U(1)_Y$ is broken, the winos and the bino are in general not mass eigenstates; rather, they mix with fields with the same charge but different $SU(2) \times U(1)_Y$ quantum numbers.

Finally we have to introduce dedicated Higgs superfields to break $SU(2) \times U(1)_Y$. Indeed, we need at least two such superfields: $H_d$ has hypercharge $Y = -1/2$, while $H_u$ has $Y = +1/2$. There are at least three reasons for this. First, is that a model with a single Higgs doublet superfield suffers from quadratic divergencies, since the trace of the hypercharge generator does not vanish. This already hints at the second problem: A model with a single Higgs doublet superfield has nonvanishing gauge anomalies associated with fermion triangle diagrams. The contribution from a complete generation of SM fermions does vanish, of course, since the SM is anomalyfree. However, if we only add a single higgsino doublet, anomalies will be introduced; we need a second higgsino doublet with opposite hypercharge to cancel the contribution from the first doublet. Finally, as discussed in Sec. 2.2.5, the masses of chiral fermions must be supersymmetric, i.e. they must originate from terms in the superpotential. On the other hand, the superpotential must not contain products of left-chiral and right-chiral superfields. This means that we are not allowed to introduce the hermitean conjugate of a Higgs superfield (or of any other chiral superfield) in $f$. It would then be impossible to introduce $U(1)_Y$ invariant terms that give masses to both up-type and down-type quarks if there is only one Higgs superfield; we again need (at least) two doublets.

The conditions for cancellation of gauge anomalies include $\text{Tr}[T_3^2 Y] = \text{Tr}[Y_3] = 0$, the traces run over all of the left-handed Weyl fermionic degrees of freedom in the theory.
2.3. MINIMAL SUPERSYMMETRIC STANDARD MODEL

2.3.1 Supersymmetric Interactions

Having specified the field content of the MSSM, we have to define the interactions. Of course, the gauge interactions are determined uniquely by the choice of gauge group, which we take to be $SU(3) \times SU(2) \times U(1)_Y$ as in the SM. However, while the gauge symmetries constrain the superpotential $f$, they do not fix it completely. We can therefore appeal to a principle of minimality and only introduce those terms in $f$ that are necessary to build a realistic model\(^6\).

In the SM, gauge invariance implies that all operators of dimension less than 4 automatically (but accidentally) preserve both baryon number and lepton number. However, supersymmetric extensions of the SM have the additional complication that in general there are additional renormalizable terms that one could write in the superpotential that are analytic, gauge invariant, and Lorentz invariant, but violate $B$ and/or $L$. Such couplings would lead to rapid proton decay. This would, of course, be a phenomenological disaster. Hence at least certain combinations of these terms must be forbidden by imposing additional symmetries on the theory. A common, though not absolutely necessary, choice is to impose a discrete symmetry known as $R$–parity, which forbids all baryon and lepton number violation in the renormalizable superpotential. R-parity is defined as follows:

$$R = (-1)^{3(B-L)+2s} \quad (2.22)$$

It is a discrete symmetry in which all SM fields (matter fermions, Higgs and gauge bosons) are even while all sparticles (sfermions, higgsinos and gauginos) are odd.

The above approach leads to the following MSSM superpotential:

$$f_{MSSM} = \sum_{i,j=1}^{3} \left[ (\lambda_E)_{ij} H L_i E^c_j + (\lambda_D)_{ij} H Q_i D^c_j + (\lambda_U)_{ij} \bar{H} Q_i U^c_j \right] + \mu \bar{H} H \quad (2.23)$$

Here $i$ and $j$ are generation indices, and contractions over $SU(2)$ and $SU(3)$ indices are understood. For example,

$$H \bar{H} \equiv H_1 \bar{H}_2 - H_2 \bar{H}_1$$

$$Q D^c_R \equiv \sum_{n=1}^{3} Q_n (D^c_R)_n \quad (2.24)$$

\(^6\)The model is minimal in that it not only contains the fewest new particles, but also the fewest number of interactions necessary to be phenomenologically viable.
The matrices $\lambda_D$ and $\lambda_U$ give rise to quark masses and to the mixing between quark current eigenstates. Since the superpotential (2.23) leaves neutrinos exactly massless, as in the SM, the matrix $\lambda_E$ can be taken to be diagonal.

The fact that the interactions produced by the $f_{MSSM}$ respect R-parity means that in the MSSM one has to produce sparticles in pairs, if one starts with beams of ordinary particles. For example, one can produce a pair of sleptons from the decay of a (virtual) $Z$ boson using the first term in the third line of eq.(2.19). Since we saw in Sec.2.2.5 that sparticles have to be quite heavy, this constraint reduces the ‘mass reach’ of a given collider for sparticle searches. For example, at $e^+ e^-$ colliders one can generally only produce sparticles with mass below the beam energy, which is only half the total center-of-mass energy.

Furthermore, a sparticle can only decay into an odd number of other sparticles and any number of SM particles. For example, a squark might decay into a quark and a higgsino via a Yukawa interaction described by the second term in the second line of eq.(2.19), if this decay is kinematically allowed. In the MSSM the lightest supersymmetric particle (LSP) therefore cannot decay at all; it is absolutely stable. This gives rise to characteristic signatures for sparticle production events at colliders, which allow to distinguish such events from ‘ordinary’ SM events. The argument goes as follows. Since LSPs are stable, some of them must have survived from the Big Bang era. If LSPs had strong or electromagnetic interactions, many or most of these cosmological relics would have bound to nuclei. Since the LSPs would have to be quite massive in such scenarios, this would give rise to ‘exotic isotopes’, nuclei with very strange mass to charge ratios. Searches for such exotics have led to very stringent bounds on their abundance, which exclude all models with stable charged or strongly interacting particles unless their mass exceeds several TeV. In the context of the MSSM this means that the LSP must be neutral. As far as collider experiments are concerned, an LSP will then look like a heavy neutrino, that is, it will not be detected at all, and will carry away some energy and momentum. Since all sparticles will rapidly decay into (at least) one LSP and any number of SM particles, the MSSM predicts that each SUSY event has some ‘missing (transverse) energy/momentum’.
2.3.2 Breaking of SUSY in the MSSM

The basic question to be addressed is how to understand the explicit soft supersymmetry breaking encoded in the $L_{\text{soft}}$ parameters as the result of spontaneous supersymmetry breaking in a more fundamental theory. To predict the values of the $L_{\text{soft}}$ parameters unambiguously within a more fundamental theory requires a knowledge of the origin and dynamics of supersymmetry breaking. Despite significant effort and many model-building attempts, the mechanism of spontaneous supersymmetry breaking and how it might be implemented consistently within the underlying theory is still largely unknown.

Since none of the fields of the MSSM can develop non-zero v.e.v. to break SUSY without spoiling the gauge invariance, it is supposed that spontaneous supersymmetry breaking takes place via some other fields. The most common scenario for producing low-energy supersymmetry breaking is called the hidden sector one \footnote{1}. According to this scenario, the theory can be split into at least two sectors with no direct renormalizable couplings between them: the usual matter belongs to the ‘visible’ one, while the second, ‘hidden’ sector, contains fields which lead to breaking of supersymmetry. These two sectors interact with each other by exchange of some fields called messengers, which mediate SUSY breaking from the hidden sector, where it originates, to the visible sector (see Fig. 2.3). The result is the effective soft supersymmetry breaking Lagrangian, $L_{\text{soft}}$, in the observable sector. Since the mediator interactions which generate $L_{\text{soft}}$ are suppressed, the hidden sector framework implies that the fundamental scale of supersymmetry breaking $M_S$, as exemplified by the F and/or D term VEVs, is hierarchically larger than the TeV scale. Indeed $M_S$ may be related to other postulated heavy mass scales, such as the Majorana neutrino mass scale, the GUT scale, or scales in extra-dimensional braneworlds. Because both $M_S$ and the scales associated with the mediator interactions are much larger than the TeV scale, renormalization group analysis is necessary in order to obtain the low energy values of the $L_{\text{soft}}$ parameters. Specific mechanisms for how supersymmetry breaking is mediated between the hidden and observable sectors imply specific energy scales at which the soft terms are generated. These generated values are then used to compute the values at observable energy scales, using the scale dependence of the $L_{\text{soft}}$ parameters as dictated by their RGEs.

Surprisingly, the pattern of the soft terms usually turns out to be relatively insensitive to the exact mechanism of the supersymmetry breaking
initiated in the hidden sector. While this is good news in that our ignorance of the origin of supersymmetry breaking does not prevent us from doing phenomenological analysis of theories such as the MSSM with softly broken supersymmetry, it is unfortunate that it becomes more difficult to infer the mechanism of supersymmetry breaking from data.

So far there are known four main mechanisms to mediate SUSY breaking from a hidden to a visible sector:

- Gravity mediation (SUGRA)
- Gauge mediation
- Anomaly mediation
- Gaugino mediation

All four mechanisms of soft SUSY breaking are different in details but are common in results. For comparison and a detail analysis of the four above-mentioned mechanisms see ref[1, 2]. Here we will only present the gravity mediation.

2.3.3 Gravity mediated supersymmetry breaking

This mechanism is based on effective nonrenormalizable interactions arising as a low-energy limit of supergravity theories. In this case, two sectors interact with each other via gravity. There are two types of scalar fields that develop nonzero v.e.v.s, namely moduli fields $T$, which appear as a result of compactification from higher dimensions, and the dilaton field $S$, part of
2.3. MINIMAL SUPERSYMMETRIC STANDARD MODEL

SUGRA supermultiplet. These fields obtain nonzero v.e.v.s for their $F$ components: $\langle F_T \rangle \neq 0$, $\langle F_S \rangle \neq 0$, which leads to spontaneous SUSY breaking. Since the supergravity interactions are Planck-suppressed, on dimensional grounds the soft parameters generated in this way are of order

\[ m \sim \frac{F}{M_{Pl}}. \]  

(2.25)

For $m \sim \mathcal{O}(\text{TeV})$, the scale of spontaneous supersymmetry breaking $M_S \sim \sqrt{F}$ is $10^{11-13} \text{GeV}$.

Since in SUGRA theory supersymmetry is local, spontaneous breaking leads to Goldstone particle which is a Goldstone fermion in this case, called the Goldstino $\tilde{G}$. With the help of a super-Higgs effect this particle will be eaten by the gravitino (the spin 3/2 partner of the spin 2 graviton), which becomes massive, with

\[ m_{\tilde{G}} \sim \frac{M_S^2}{M_{Pl}}. \]  

(2.26)

In gravity mediated supersymmetry breaking, the gravitino mass $m_{\tilde{G}}$ generically sets the overall scale for all of the soft supersymmetry breaking mass parameters.

In spite of attractiveness of these mechanism in general, since we know that gravity exists anyway, it is not truly substantiated due to the lack of a consistent theory of quantum (super)gravity. Among the problems of a supergravity mechanism also are the large freedom of parameters and the absence of automatic suppression of flavour violation.

2.3.4 The parameters of the MSSM

Having presented in, sec. 2.2, the soft supersymmetry-breaking Lagrangian of the MSSM, we now count its physical parameters.

With the exception of $m_{H_u}^2$, $m_{H_d}^2$ and the diagonal entries of the soft mass-squared parameters of the squarks and sleptons, every parameter can in principle be complex. The Yukawa couplings of the SM and the soft supersymmetry-breaking trilinear couplings are each general complex $3 \times 3$ matrices which involve a total of 54 real parameters and 54 phases. The soft mass-squared parameters for the squarks and sleptons are each Hermitian $3 \times 3$ matrices which have in total 30 real and 15 imaginary parameters. Taking into account the real soft Higgs mass-squared parameters, complex
gaugino masses, the MSSM would appear to have 91 real parameters (masses and mixing angles) and 74 phases\textsuperscript{7}. However, a subset of parameters can be eliminated by global rephasings of the fields and thus are not physical. Including the rest of the SM parameters: the gauge couplings, the QCD $\theta$ angle, etc., there are 79 real parameters and 45 phases in the MSSM.

Therefore the final count gives 124 independent parameters for the MSSM of which 110 (69 real parameters and 41 phases) are associated with the flavor sector. Of these 124 parameters, 18 correspond to Standard Model parameters, one corresponds to a Higgs sector parameter (the analogue of the Standard Model Higgs mass), and 105 are genuinely new parameters of the model. Thus, an appropriate name for the minimal supersymmetric extension of the Standard Model is MSSM-124.

The masses, mixings, and couplings of the superpartners and Higgs bosons depend in complicated ways on the soft-SUSY Lagrangian parameters as well as on the SM parameters. There are 32 mass eigenstates in the MSSM: 2 charginos, 4 neutralinos, 4 Higgs bosons, 6 charged sleptons, 3 sneutrinos, 6 up-squarks, 6 down-squarks, and the gluino. If it were possible to measure all the mass eigenstates it would in principle be possible to determine 32 of the 105 soft parameters. However, inverting the equations to go from observed mass eigenstates to soft parameters requires a knowledge of soft phases and flavor-dependent parameters, or additional experimental information, and hence in practice it may be difficult or impossible.

Constraints on the 105-dimensional soft-SUSY Lagrangian parameter space arise from many phenomenological and theoretical considerations, as well as direct and indirect experimental bounds. The restrictions on the soft parameters can be loosely classified into two categories:

- **Constraints from flavor physics.**
  Many of the parameters of the MSSM-124 are present only in flavor-changing couplings. Even flavor-conserving MSSM couplings can lead to flavor-violating effective couplings at higher-loop level. Such couplings potentially disrupt the delicate cancellation of flavor-changing neutral currents (FCNCs) of the SM. The constraints are particularly stringent for the parameters associated with the first and second generations of squarks and sleptons. This issue, is known as the supersymmetric flavor problem.

\textsuperscript{7}One can also include the complex gravitino mass in the parameter count.
• Constraints from CP violation.

The parameters of the MSSM include a number of CP-violating phases, which can be classified into two general categories:

I. Certain phases are present in flavor-conserving as well as flavor-changing interactions. These phases include the phases of the gaugino mass parameters $\phi_{Ma}$, the phases of higgsino mass parameter $\mu$ and soft SUSY-breaking bilinear couplings $B_{i,j}$, $\phi_{\mu}, \phi_{B}$ and the overall phases of soft SUSY-breaking trilinear couplings $A_{u,d,e}$: physical observables depend on the reparameterization invariant phase combinations spanned by the basis Eq.

$$\phi_{1f} = \phi_{\mu} + \phi_{A_{f}} - \phi_{B}. \quad (2.27)$$

A subset of these phases play a role in electroweak baryogenesis. However, these phases are also constrained by electric dipole moments (EDMs).

In summary, the phases, if nonnegligible, not only can have significant phenomenological implications for CP-violating observables, but also can have nontrivial consequences for the extraction of the MSSM parameters from experimental measurements of CP-conserving quantities, since almost none of the Lagrangian parameters are directly measured. The phases will be addressed in the context of neutralino dark matter in next chapter.

II. The remaining phases are present in the off-diagonal entries of the $A$ and $m^2$ parameters, and hence occur in flavor-changing couplings. In this sense they are analogous to the $CKM$ phase of the SM. These phases are generically constrained by experimental bounds on CP violation in flavor-changing processes.

• Constraints from EWSB, cosmology, and collider physics.

The gaugino masses, $\mu$ parameter, and the third family soft mass parameters play dominant roles in MSSM phenomenology, from electroweak symmetry breaking to dark matter to collider signatures for the superpartners and Higgs sector. Cosmological questions such as dark matter will be addressed in chapter 4. Issues related to electroweak symmetry breaking will be discussed in Appendix A and Appendix B.
2.4 Minimal Supergravity Model

As mentioned at the end of the previous section, a general parametrization of supersymmetry breaking in the MSSM introduces 105 free parameters. This is not very satisfactory. The predictive power of a theory with such a large number of parameters is clearly quite limited. It is therefore desirable to try and construct models that make do with fewer free parameters. The currently most popular such model is the minimal Supergravity (mSUGRA).

This model is based on the local version of supersymmetry. Eqs. (2.2) show that invariance under local SUSY transformations implies invariance under local coordinate change; this invariance is the principle underlying Einsteins construction of the theory of General Relativity. Local supersymmetry therefore naturally includes gravity, and is usually called supergravity.

Here one assumes that SUSY is broken spontaneously in a ‘hidden sector,’ so that some auxiliary field(s) get vev(s) of order $M_Z \cdot M_{Pl} \simeq (10^{10} \text{GeV})^2$. Gravitational-strength interactions then automatically transmit SUSY breaking to the ‘visible sector,’ which contains all the SM fields and their superpartners; the effective mass splitting in the visible sector is by construction of order of the weak scale, as needed to stabilize the gauge hierarchy. In minimal supergravity one further assumes that the kinetic terms for the gauge and matter fields take the canonical form: as a result, all scalar fields (sfermions and Higgs bosons) get the same contribution $m_0^2$ to their squared scalar masses, and that all trilinear $A$ parameters have the same value $A_0$, by virtue of an approximate global $U(n)$ symmetry of the SUGRA Lagrangian. Finally, motivated by the apparent unification of the measured gauge couplings within the MSSM at scale $M_{GUT} \simeq 2 \cdot 10^{16}$ GeV, one assumes that SUSY-breaking gaugino masses have a common value $m_{1/2}$ at scale $M_{GUT}$. In practice, since little is known about physics between the scales $M_{GUT}$ and $M_{Planck}$, one often uses $M_{GUT}$ as the scale at which the scalar masses and $A$ parameters unify. We note that R parity is assumed to be conserved within the mSUGRA framework.

This ansatz has several advantages. First, it is very economical; the entire spectrum can be described with a small number of free parameters. Second, degeneracy of scalar masses at scale $M_{GUT}$ leads to small flavor-changing neutral currents. Finally, this model predicts radiative breaking of the electroweak gauge symmetry because of the large top-quark mass.

Radiative symmetry breaking together with the precisely known value of $M_Z$ allows one to trade two free parameters, usually taken to be the
absolute value of the *supersymmetric Higgsino mass parameter* $|\mu|$ and the $B$ parameter appearing in the scalar Higgs potential, for the *ratio of vevs*, $\tan\beta$. The model then has four continuous and one discrete free parameter not present in the SM:

$$m_0, m_{1/2}, A_0, \tan\beta, \text{sign}(\mu)$$

This model is now incorporated in several publicly available MC codes, in particular ISAJET\(^8\). An approximate version is incorporated into Spythia, which reproduces ISAJET results to 10%. Most SUSY spectra studied at various workshops have been generated within mSUGRA; one generically finds the following features:

- $|\mu|$ is large, well above the masses of the $SU(2)$ and $U(1)$ gauginos. The lightest neutralino is therefore mostly a Bino (and an excellent candidate for cosmological CDM) and the second neutralino and lighter chargino are dominantly $SU(2)$ gauginos. The heavier neutralinos and charginos are only rarely produced in the decays of gluinos and sfermions (except possibly for stop decays). Small regions of parameter space with $|\mu| \simeq M_W$ are possible.

- If $m_0^2 \gg m_{1/2}^2$, all sfermions of the first two generations are close in mass. Otherwise, squarks are significantly heavier than sleptons, and $SU(2)$ doublet sleptons are heavier than singlet sleptons. Either way, the lighter stop and sbottom eigenstates are well below the first generation squarks; gluinos therefore have large branching ratios into $b$ or $t$ quarks.

- The heavier Higgs bosons (pseudoscalar $A$, scalar $H^0$, and charged $H^\pm$) are usually heavier than $|\mu|$ unless $\tan\beta \gg 1$. This also implies that the light scalar $h^0$ behaves like the SM Higgs.

These features have already become something like folklore. We want to emphasize here that even within this restrictive framework, quite different spectra are also possible.

\(^8\)ISAJET is a Monte Carlo program which simulates $pp$, $\bar{p}p$, and $e^+e^-$ interactions at high energies. It is based on perturbative QCD plus phenomenological models for parton and beam jet fragmentation.
Chapter 3

Cosmology

In the last chapter we laid out what we believe that the world is maid of, or in other words what may be the theory of the world. That is the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ gauge theory of the strong and electroweak interactions, which provides an understanding of physics up to the weak scale (about 250 GeV), in addition with particle physics at very short distances, e.g., grand Unification, Supersymmetry, Superstring theory, etc. Particle physicists and cosmologists have applied these theoretical constructs to the study of the earliest moments of the universe. Their speculations have led to very interesting scenarios about the events that may have taken place at these early times: baryogenesis, inflation, the production of exotic relics—monopoles, strings, axions, neutralinos and so on. This chapter will outline and apply a brief mathematical framework for the description of the universe, so we will be able in the next chapter to calculate the relic abundance of the universe.

The preceding summary describes what may be called the ‘standard model’ of the universe, based on the Cosmological Principle and Einstein’s field equations. I will begin our discussion with these equations and the energy-momentum tensor. I shall next show that the Cosmological Principle allows the specification of the cosmic metric entirely in terms of ‘radius’ $R(t)$ and a constant $K$ and we shall then see how astronomical observations can be interpreted as measurements of $R(t)$ and $K$. In the forth section we are going to talk about dynamical Cosmology, as it is described by the Friedmann equation. A brief summary of the thermal history of the universe is given at the end of the chapter.

Of course, the standard cosmological model may be partly or wholly
However, its importance lies not in its certain truth, but in the common meeting ground that it provides for an enormous variety of observational data. Nevertheless some other possible models are discussed in the last section.

### 3.1 Einstein’s Field Equations

We start from the Einstein equations which follow directly from an action principle: *The action is stationary under small variations of the metric tensor*. The action that leads to the Einstein field equations is $S = S_{E-H} + S_M$

\[
\text{The Einstein-Hilbert action : } S_{E-H} = -\frac{1}{16\pi G} \int d^4x \sqrt{-g}(R + 2\Lambda)
\]

\[
\text{and the ‘matter’ action : } S_M = \sum_{\text{fields}} \int d^4x \sqrt{-g}L_{\text{fields}}
\]

(3.1)

Where the sum runs over the lagrangian densities for all the fundamental fields. The variation of the actions are:

\[
\delta S_{E-H} = \frac{1}{16\pi G} \int d^4x \sqrt{-g}(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu} R) \delta g_{\mu\nu}
\]

\[
\delta S_M = -\frac{1}{2} \sum_{\text{fields}} \int d^4x \sqrt{-g} T^{\mu\nu}_{\text{fields}} \delta g_{\mu\nu}
\]

(3.2)

and so the principle that $\frac{\delta S}{\delta g_{\mu\nu}} = 0$ yields the Einstein equations:

\[
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} R \equiv G_{\mu\nu} = -8\pi GT_{\mu\nu} + \Lambda g_{\mu\nu}
\]

(3.3)

where $R_{\mu\nu}$\(^1\) is the Ricci tensor, $\mathcal{R}$\(^2\) is the Ricci scalar, $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ is the energy momentum tensor for all the fields present - matter, radiation etc. The term $\Lambda g_{\mu\nu}$ was originally introduced by Einstein for cosmological reasons, which is why $\Lambda$ is called the cosmological constant.

---

\(^1\)The fully covariant form of the curvature tensor $R_{\lambda\mu\nu\kappa}$ may be contracted to give the Ricci tensor: $R_{\mu\nu} = g^{\lambda\nu} R_{\lambda\mu\nu}$

\(^2\)And the Ricci scalar $\mathcal{R} \equiv g^{\lambda\nu} g^{\mu\kappa} R_{\lambda\mu\nu\kappa}$
3.2 SPACES WITH MAXIMALLY SYMMETRIC SUBSPACES

To be consistent with the symmetries of the metric\(^3\), the total energy-momentum tensor \(T_{\mu\nu}\) must be diagonal, and by isotropy the spatial components must be equal so that the energy-momentum tensor of the universe takes the same form as for a perfect fluid characterized by a time-dependent energy density \(\rho(t)\) and pressure \(\rho(t)\):

\[
T^\mu_{\nu} = \text{diag}(\rho, -\rho, -\rho, -\rho)
\]  

(3.4)

The energy-momentum tensor obeys the conservation equation: \(T^\mu_{\nu;\nu} = 0\) while for the \(\mu = 0\) component it leads to the first law of thermodynamics:

\[
d(\rho R^3) = -\rho d(R^3)
\]

(3.5)

given an equation of state \(\rho = \omega \rho^4\) we can use Eq. (3.5) to determine \(\rho\) as a function of \(R\). For instance, if the energy density of the universe is dominated by non relativistic matter with negligible pressure then Eq. (3.5) gives:

\[
(MD) \text{ Matter Dominated } (\rho \ll \rho) \implies \rho \sim R^{-3}
\]

(3.6)

Whereas if the energy density is dominated by relativistic particles, such as photons, then we have:

\[
(RD) \text{ Radiation dominated } (\rho = \frac{1}{3} \rho) \implies \rho \sim R^{-4}
\]

(3.7)

If the universe underwent inflation there was a ‘very early’ period when the energy density was dominated by vacuum energy:

\[
\text{Vacum Energy } (\rho = -\rho) \implies \rho \sim \text{const.}
\]

(3.8)

3.2 Spaces with Maximally Symmetric Subspaces

Euklid implicitly assumed that metric relations are unaffected by translations or rotations. Real gravitational fields do not usually have such a high degree of form invariance. A tensor as \(T_{\mu\nu}\) is required to be form invariant with respect to those coordinates transformation which leave the metric \(dt^2 = dt^2 - R^2(t)[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2]\) form invariant. This ‘isometry’ is purely spatial.

\(^3\)where the constant of proportionality \(\omega\) is independent of time
of symmetry, but they often admit some group of approximate symmetry transformation to help solve the Einstein equations. I shall not give the elaborate mathematical theory of symmetric spaces, I will only give attention to the maximally symmetric spaces and this because in next section we shall deal with such a space, for more detailed analysis see ref[3].

The maximally symmetric spaces are uniquely specified by a ‘curvature constant’ K, and by the numbers of eigenvalues of the metric that are positive or negative. That is given two maximally symmetric metrics with the same K and the same numbers of eigenvalues of each sign, it will always be possible to find a coordinate transformation that carries one metric into the other.

In many cases of physical importance, whole space (or space-time) is not maximally symmetric, but it can be decomposed into maximally symmetric subspaces. For instance, a spherically symmetric 3-dimensional space can be decomposed into a family of spherical surfaces centered on the origin, each of which is described by a metric of the form: \( ds^2 = k^{-1} [d\theta^2 + \sin^2 \theta d\phi^2] \). The maximal symmetry of a family of subspaces imposes very strong constrains on the metric of the whole space.

Armed with these theorems, we shall be able, in next section, to study space-times which the metric is spherically symmetric and homogenous on each ‘plane’ of constant time.

### 3.3 The Cosmological Principle

The Cosmological Principle is the hypothesis that the universe is spatially homogenous and isotropic.

We can formulate our intuitive ideas of homogeneity and isotropy in precise mathematical terms and find one particular space-time coordinate system: the coordinates \( x^\mu \), \( t \) and will be called the cosmic standard coordinate system.

The Cosmological Principle can be formulated as a statement about the existence of equivalent coordinate systems. Suppose that we use the cosmic standard coordinate system to carry out astronomical observations, determining (never mind how!) the metric \( g^{\mu\nu} \), the energy momentum tensor \( T_{\mu\nu} \) and all other cosmic fields, as functions of the cosmic standard coordinates \( x^\mu \). A different set of space-time coordinates \( x'^\mu \) may be considered equivalent to the cosmic standard coordinates, if the whole history of the universe appears the same in the \( x'^\mu \) coordinates system as in the \( x^\mu \). This requires
3.3. THE COSMOLOGICAL PRINCIPLE

that every cosmic field \( g_{\mu\nu}(x') \), \( T_{\mu\nu}(x') \) and so on must be the same function of the \( x'^\mu \) as the corresponding quantities \( g_{\mu\nu}(x) \), \( T_{\mu\nu}(x) \) and so on are of the standard coordinates \( x^\mu \). That is, at any coordinate point \( y^\mu \), we must have:

\[
\begin{align*}
g_{\mu\nu}(y) &= g'_{\mu\nu}(y) \\
T_{\mu\nu}(y) &= T'_{\mu\nu}(y) \quad \text{etc.}
\end{align*}
\] (3.9)

The first equation says that the coordinate transformation \( x \rightarrow x' \) must be an isometry and the second says that \( T^{\mu\nu} \) and so on must be form invariant under this transformation.

Also Eq. (1.10) will have to hold for the scalar \( s \) used to define our cosmic standard time \( t \).\(^5\) since \( s \) is by definition a function only of \( t \), and a scalar, Eq. (1.10) for \( s \) reads at \( y = x' \),

\[
s(t') = s'(x') \equiv s(x) = s(t)
\]

and so

\[
t = t
\] (3.11)

All coordinate systems that are equivalent to the cosmic standard system necessarily use cosmic standard time.

I summary, the Cosmological Principle can formulated, in the language of the previous section, as follows:

I. The hypersurfaces with constant cosmic standard time are maximally symmetric subspaces of the whole of space-time

II. Not only the metric \( g_{\mu\nu} \), but all cosmic tensors such as \( T_{\mu\nu} \) are form-invariant with respect to the isometries of these subspaces.

The universe therefor satisfies the requirements for a four-dimensional space with three-dimensional maximally symmetric subspaces \( t = \text{const} \). The metric for such a space is the \( \text{Robertson-Walker metric} \), which can be written in the form:

\[
d\tau^2 = dt^2 - R^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 \right]
\] (3.12)

\(^5\)It is believed that several cosmic scalar fields, such as the the proper energy density \( \varrho \) or the black-body radiation temperature \( T_\gamma \) are everywhere decreasing monotonically. choose anyone of the these, say a scalar \( s \), and let the time of any event be any definite decreasing function \( t(s) \) of the chosen scalar, when and where the event occurs.
CHAPTER 3. COSMOLOGY

where \( R(t) \) is an unknown function of time named \textit{cosmic scale factor}. \( k \) is a constant which by a suitable choice of units for \( r \) can be chosen to have the value: +1, 0, or −1. (These are not necessarily the same as the cosmic standard coordinates introduced previously, although \( t \) in Eq. (3.12) is the cosmic standard time, or a function of it.)

The beautiful thing is that this metric can be derived solely from the assumption homogeneity and isotropy, with no use of the Einstein field equations.

3.4 The Friedmann Equation

The discussion in the previous section concerned the ‘kinematics’ of a universe described by a Robertson-Walker metric. Now we shall talk about dynamical cosmology. We know that the dynamics of the expanding Universe only appeared implicitly in the time dependence of the scale factor \( R(t) \). So the goal is to find the dynamical equations that describe the evolution of the scale factor \( R(t) \).

The non-zero components of the Ricci tensor for the Robertson-Walker metric are:

\[
R_{00} = \frac{3\dddot{R}}{R} \\
R_{ij} = -\left[ \frac{\dddot{R}}{R} + \frac{2\dddot{R}^2}{R^2} + \frac{2k}{R^2} \right] g_{ij}
\]

(3.13)

and the Ricci scalar \( \mathcal{R} \) is:

\[
\mathcal{R} = -6 \left[ \frac{\dddot{R}}{R} + \frac{\dddot{R}^2}{R^2} + \frac{k}{R^2} \right]
\]

(3.14)

The time-time component of the Einstein equation (3.3) gives:

\[
3\dddot{R} = -4\pi G (\varrho + 3\rho) R
\]

(3.15)

the space-space components give the single equation:

\[
R\dddot{R} + 2\dddot{R}^2 + 2k = 4\pi G (\varrho - \rho) R^2
\]

(3.16)
3.4. THE FRIEDMANN EQUATION

and the space-time components give: \( 0 = 0 \)

By eliminating \( \ddot{R} \) from Eq. (1.15) and (1.16) we find a first order differential equation for \( \dot{R}(t) \)

\[
\dot{R}^2 + k = \frac{8}{3}\pi G\varrho R^2
\]

(3.17)

the so-called Friedmann equation.

The fundamental equations of dynamic cosmology are thus the Friedmann equation (3.17), the energy conservation equation (3.5), and the equation of state. Knowing \( \varrho \) as a function of \( R \), we can determine \( R(t) \) for all time by solving Eq. (3.17)

By differentiating the Friedmann equation with respect to time and using the equation of energy conservation (3.5) we find:

\[
2\ddot{R}\dot{R} = -\frac{8\pi}{3}G\frac{\dot{R}}{R}(\varrho R^2 + 3\rho R^2)
\]

(3.18)

or, equivalently,

\[
\frac{\dot{R}}{R} = -\frac{4\pi G}{3}(\varrho + 3\rho)
\]

(3.19)

As long as the quantity \( \varrho + 3\rho \) remains positive the ‘acceleration’ \( \frac{\ddot{R}}{R} \) is negative.

Since at present \( R > 0 \) (by definition) and \( \frac{\dot{R}}{R} > 0 \) (because we see red shifts, not blue shifts) it follows that the curve of \( R(t) \) versus \( t \) must be concave downward and must have reached \( R(t) = 0 \), at some finite time in the past. Let us call the time of this singularity \( t = 0 \), so that:

\[
R(0) = 0
\]

(3.20)

The present time \( t_0 \) is then the time elapsed since this singularity and may justly be called the age of the universe. The expansion rate of the universe is determined by the Hubble parameter:

\[
H \equiv \frac{\dot{R}}{R}
\]

(3.21)

The Hubble parameter is not constant and in general varies as \( t^{-1} \). So the Hubble time is \( H^{-1} \) and sets the time scale for the expansion: \( R \) roughly doubles in a hubble time.
The Friedmann equation can be written in the form:

\[ \frac{k}{H^2 R^2} = \frac{\varrho}{3 H^2 / 8 \pi G} - 1 \equiv \Omega - 1 \quad (3.22) \]

where: \( \Omega \equiv \frac{\varrho}{\varrho_c} \) and \( \varrho_c \equiv \frac{3 H^2}{8 \pi G} \) is the critical density

since \( H^2 R^2 \geq 0 \) then if:

\[ k = +1 \implies \Omega > 1 \quad \text{CLOSED} \quad (3.23) \]

the expansion will eventually cease and be followed by a contraction back to a singular state with \( R(t) = 0 \)

end if:

\[ k = 0 \implies \Omega = 1 \quad \text{FLAT} \]
\[ k = -1 \implies \Omega < 1 \quad \text{OPEN} \quad (3.24) \]

the universe will go on expanding forever.

### 3.5 The Expansion Age of the Universe

Before going on, we want to find a formula for the expansion age of the universe. If we do so we will be able to comprehend better the early history of the universe, the subject of the next section.

The integration of the Friedmann equation gives us the age of the universe in terms of present cosmological parameters. So by integrating Eq.(3.17) we find:

\[ \left( \frac{\dot{R}}{R_0} \right)^2 + \frac{k}{R_0^2} = \frac{8 \pi G}{3} \varrho_0 \frac{R_0}{R} \quad (MD) \]

\[ \left( \frac{\dot{R}}{R_0} \right)^2 + \frac{k}{R_0^2} = \frac{8 \pi G}{3} \varrho_0 \left( \frac{R_0}{R} \right)^2 \quad (RD) \quad (3.25) \]

Using the fact that \( \frac{k}{R_0^2} \equiv H_0^2(\Omega_0 - 1) \) the age of the universe as a function of
\( \frac{R_0}{R} = 1 + z \) is given by:

\[
t = \int_0^{R(t)} \frac{dR'}{R'} = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{[1 - \Omega_0 + \Omega_0 x^{-1}]^{1/2}} \quad (MD)
\]

\[
t = H_0^{-1} \int_0^{(1+z)^{-1}} \frac{dx}{[1 - \Omega_0 + \Omega_0 x^{-2}]^{1/2}} \quad (RD) \quad (3.26)
\]

where sub-0 denotes the present value for a quantity.

As we can see the time scale for the age of the universe is set by the Hubble time \( H_0^{-1} \). We define the zero of time to be that time when the scale factor \( R \) extrapolates to zero.

In calculating the age of the universe arises a problem: earlier than some time, say \( \tilde{t} \), (more accurately, for \( R \) less than some \( \tilde{R} \)) our knowledge of the universe is uncertain and so the time elapsed from \( R = 0 \) to \( R = \tilde{R} \) cannot be reliably calculated. However, this contribution to the age of the universe is very small.

### 3.6 Thermal History of the Early Universe

The key to understanding the thermal history of the early universe is: the comparison of the particle interaction rates and the expansion rate. Ignoring the temperature variation of \( g_* \), we will assume for this discussion that \( T \sim R^{-1} \) and the rate of change of the temperature is set by the expansion rate: \( \dot{T}/T = -H \). So long as the interactions necessary for particle distribution functions to adjust to the changing temperature are rapid compared to the expansion rate, the universe will, to a good approximation, evolve through a succession of nearly thermal states with temperature decreasing as \( R^{-1} \). A useful rule of thumb is that a reaction is occurring rapidly enough to maintain thermal distributions when \( \Gamma \gtrsim H \), where the \( \Gamma \) is the interaction rate per particle\(^7\).

\(^6\)g_* (t) = \( \sum_{i=bosons} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{i=fermions} g_i \left( \frac{T_i}{T} \right)^4 \)

where \( g_* \) counts the total number of effectively massless degrees of freedom and \( g \) is the internal degrees of freedom.

\(^7\)\( \Gamma \equiv n \sigma |v| \), \( n \) is the number density of target particles and \( \sigma |v| \) is the cross section for interaction times relative velocity.
The correct way to evolve particle distributions is to integrate the Boltzmann equation. This approach will be used in next section to precisely calculate the relic density of a stable particle species which decouples. For the moment we will use $\Gamma > H$ ($\Gamma < H$) as the criterion for whether or not a species is coupled to (decoupled from) the thermal plasma in the universe.

To get a rough understanding of the decoupling of a particle species in the expanding universe, consider two types of interactions:

I. interactions mediated by a massless gauge bosons, e.g., the photon

II. interactions mediated by a massive gauge bosons, e.g., a $W^\pm$ or $Z^0$ boson below the scale of electroweak symmetry breaking ($T \lesssim 300 \text{GeV}$)

In the first case the cross section for $2 \leftrightarrow 2$ scattering of relativistic particles with significant momentum transfer is $\sigma \sim a^2/T^2$ ($g = \sqrt{4\pi a} = \text{gauge coupling strength}$). So $\Gamma \sim n\sigma|v| \sim a^2T$ and during the RD epoch: $H \sim T^2/m_{pl}$

so that:

$$\frac{\Gamma}{H} \sim \frac{a^2m_{pl}}{T} \begin{cases} \text{for } T \lesssim a^2m_{pl} \sim 10^{16}\text{GeV} \text{ or so such reactions are occurring rapidly} \\
\text{for } T \gtrsim a^2m_{pl} \sim 10^{16}\text{GeV such reactions are effectively ‘frozen out’} \end{cases}$$

In the second case, for $T \gg m_x$ the cross section is the same as that for the massless gauge boson exchange. But for $T \lesssim m_x$ the corresponding cross section is $\sigma \sim G_x^2T^2$ where $G_x \sim a/m_x$ and $m_x$ is the mass of the gauge boson. Therefore $\Gamma \sim n\sigma|v| \sim G_x^2T^3$

$$\frac{\Gamma}{H} \sim G_x^2m_{pl}T^3 \begin{cases} \text{for } m_x \gtrsim T \gtrsim G^{-2/3}m_{pl}^{-1/3} \sim (m_x/100\text{GeV})^{4/3}\text{MeV such reactions are occurring rapidly} \\
\text{for } T \gtrsim (m_x/100\text{GeV})^{4/3}\text{MeV such reactions are effectively ‘frozen out’} \end{cases}$$

We should emphasize that for $T \gtrsim a^2m_{pl} \sim 10^{16}\text{GeV}$ corresponding to times earlier than about $10^{-38}\text{sec}$, the known interactions plus any new interactions

---

8the $m_x^{-2}$ factor results from the propagator of the massive gauge boson.
3.6. THERMAL HISTORY OF THE EARLY UNIVERSE

arising from grand unification, are not capable of thermalizing the universe. So we should keep in mind the fact that the universe may not have been in thermal equilibrium during its earliest epoch.

Fig. 3.1 provides a brief summary of the thermal history of the universe, based on our present knowledge of particle physics. At the earliest times the universe was a plasma of relativistic particles including the quarks, leptons, gauge bosons and Higgs bosons. If the current ideas are correct a number of spontaneous symmetry breaking (SSB) phase transitions should take place during the early history of the universe. They include:

- the grand unification (GUT) phase transition at a temperature of $10^{14}$ to $10^{16}$ GeV
- the electroweak SSB phase transition at a temperature of about 300 GeV

During these SSB phase transitions some of the gauge bosons and other particles acquire mass via the Higgs mechanism and the full symmetry of the theory is broken to a lower symmetry. Subsequent to the phase transition the interactions mediated by the $X$ bosons which acquire mass will be characterized by a coupling strength $G_X$ and particles which only interact via such interactions will decouple from the thermal plasma at $T \sim G_X^{-2/3} m_{pl}^{-1/3}$.

At a temperature of about 100 to 300 MeV ($t \sim 10^{-5} \text{sec}$) the universe should undergo a transition associated with chiral symmetry breaking and color confinement, after which the strongly-interacting particles are color-singled-quark-triplet states (baryons) and color-singled-quark-antiquark states (mesons).

The epoch of primordial nucleosynthesis follows when $t \sim 10^{-2}$ to $10^{2} \text{sec}$ and $T \sim 10$ to 0.1 MeV. This epoch can be studied in three steps:

→ step 1 ($t = 10^{-2}\text{sec}, T = 10\text{MeV}$) : As we have seen, at this epoch the energy density of the universe is dominated by radiation. The relativistic degrees of freedom are: $e^{\pm}$, photons and 3 neutrino species. All the weak rates are much larger than the expansion rate $H$, so the light elements are in Nuclear Statistical Equilibrium (NSE)\footnote{If the nuclear reactions that produce nucleus A out of Z protons and A-Z neutrons occur rapidly compared to the expansion rate, Nuclear Statistical Equilibrium obtains.}
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$CHAPTER 3. COSMOLOGY$

$\sim 13,710^9$ yrs. $T \sim 2,725K$ Internet, chinese food, cybersex, etc.

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$\sim 300K$ Universe becomes fully transparent (no matter/photon interactions)

$t = 350,000$ yrs. $T \sim 300K$ Matter and Radiation decouple ending the long epoch of near thermal equilibrium

$t = 10^{13}$ sec $T = 10^4K \sim 1eV$ Ions and electrons combine to form atoms and Matter and Radiation decouple ending the long epoch of near thermal equilibrium

$t = 10^{11}$ sec $T = 10^5K \sim 10eV$ Matter density $=$ Radiation density (marks the beginning of current MD epoch, and start of structure formation)

$t = 10^{12}$ sec $T = 10^{15}K \sim 300GeV$ Electroweak Symmetry breaks (end of Electroweak Unification)

$t = 10^{-6}$ sec $T = 10^{13}K \sim 1GeV$

we can distinguish quarks from leptons because the previous ones interact with gluons.

Production of Magnetic Monopoles, as a result of GUT braking

$t = 10^{-5}$ sec $T \sim 3.10^{12}K \sim 100 - 300MeV$ Quarks confined complete

Color-singlet-quark-triplet states (baryons)

Color-singlet-quark-antiquark states (mesons)

$t = 10^{-2}$ sec $T = 10^{11}K \sim 10MeV$

$t = 10^2$ sec $T = 10^9K \sim 0,1MeV$

$t = 10^{-12}$ sec $T = 10^{15}K \sim 300GeV$ Quarks start to confine in hadrons, ending era of quark-gluon plasma

$t = 10^{-38}$ sec $T = 10^{14}GeV$ GUT breaks

$GUT$ epoch

SU(5) Symmetry

Quark-Gluon plasma

$t = 10^{-32}$ sec $T = 10^{14}GeV$ GUT breaks

$PLANK$ EPOCH

All Forces Unified

Quantum Gravity

Supergravity?

Extra Dimensions?

Supersymmetry?

Superstrings?

$t = 10^{-43}$ sec $T = 10^{32}K \sim 10^{19}GeV$ Gravity separates

Big Bang Nucleosynthesis

$t = 10^{-2}$ sec $T = 10^{11}K \sim 10MeV$

$t = 10^2$ sec $T = 10^9K \sim 0,1MeV$

$t = 10^{-6}$ sec $T = 10^{13}K \sim 1GeV$

$t = 10^{-5}$ sec $T \sim 3.10^{12}K \sim 100 - 300MeV$

$t = 10^{-12}$ sec $T = 10^{15}K \sim 300GeV$

$t = 10^{-38}$ sec $T = 10^{14}GeV$

$t = 10^{-32}$ sec $T = 10^{14}GeV$

$t = 10^{-43}$ sec $T = 10^{32}K \sim 10^{19}GeV$ Gravity separates

$t = 10^{-2}$ sec $T = 10^{11}K \sim 10MeV$

$t = 10^2$ sec $T = 10^9K \sim 0,1MeV$

$t = 10^{-6}$ sec $T = 10^{13}K \sim 1GeV$

$t = 10^{-5}$ sec $T \sim 3.10^{12}K \sim 100 - 300MeV$ Quarks confined complete

Color-singlet-quark-triplet states (baryons)

Color-singlet-quark-antiquark states (mesons)

$t = 10^{-12}$ sec $T = 10^{15}K \sim 300GeV$ Quarks start to confine in hadrons, ending era of quark-gluon plasma

$t = 10^{-38}$ sec $T = 10^{14}GeV$ GUT breaks

$GUT$ epoch

SU(5) Symmetry

Quark-Gluon plasma

$t = 10^{-32}$ sec $T = 10^{14}GeV$ GUT breaks

$PLANK$ EPOCH

All Forces Unified

Quantum Gravity

Supergravity?

Extra Dimensions?

Supersymmetry?

Superstrings?
→ step 2 \( (t \approx 1 \text{sec}, T = T_F \approx 1 \text{MeV}) \) : shortly before this epoch, the 3 neutrinos species decouple from the plasma and a little later (at temperatures about \( T \approx m_e/3 \)) the \( e^\pm \) pairs annihilate, transferring their entropy to the photons alone and thereby raising the photon temperature relative to that of the neutrinos by a factor of \( (11/4)^{1/3} \). As a consequence of this the dominant constituents of the universe are only photons, neutrinos and antineutrinos in essentially free expansion. At about this time \( \Gamma \) becomes smaller than \( H \) so the weak interactions that interconvert neutrons and protons freeze out. When this occurs the neutron-to-proton ratio is given approximately by its equilibrium value,

\[
\left( \frac{n}{\rho} \right)_{\text{freeze-out}} = \exp(-Q/T_F) \approx \frac{1}{6}.
\]  

(3.27)

→ step 3 \( (t = 1 \text{ to } 3 \text{ minutes}, T = 0.3 \text{ to } 0.1 \text{ MeV}) \): At about this time the \( e^\pm \) pairs have disappeared and transferred their entropy to the photons, so the total number of effectively massless degrees of freedom has decreased to its value today \( (g_* = 3.36) \). Also the neutron-to-proton ratio has decreased from \( \sim 1/6 \) to \( \sim 1/7 \) due to occasional weak interactions. At \( T = 0.3 \text{ MeV} \) the NSE value of the mass fraction of \( ^4 \text{He} \) rapidly approaches unit. Shortly before this \( (T \sim 0.5 \text{ MeV}) \), the actual amount of \( ^4 \text{He} \) present first falls below its NSE value. This occurs because the rates for the processes that synthesize \( ^4 \text{He} \) are not fast enough to keep up with the rapidly increasing ‘NSE demand’ for \( ^4 \text{He} \). The reaction rates, \( \Gamma \), are not fast enough for two reasons: (i) while the abundances of D, \( ^3 \text{He} \) and \( ^3 \text{H} \) are beginning to exceed their NSE values, the NSE abundances are still very small. (ii) Coulomb-barrier suppression is beginning to become significant. At a temperature of about 0.1 MeV \( (t \approx 180 \text{ sec}) \) the neutrons rapidly began to fuse with protons into heavier nuclei, leaving an ionized gas of hydrogen and \( ^4 \text{He} \), with about 27% helium by weight and a trace of \( D, ^3 \text{He} \) and other elements.

After this epoch the free expansion of the photons, neutrinos and antineutrinos confined. The ionized gas temperature remained locked to the photon temperature until the hydrogen recombined at \( T \approx 4000^0 K \). At some temperatures between \( 10^5 K \) and \( 10^3 K \) the energy density of the photons, neutrinos and antineutrinos dropped below the rest-mass density of hydrogen and Helium and we entered upon the MD era.
In the early universe the matter and radiation were in good thermal contact, because of rabid interactions between photons and electrons. However while the universe was expanding and the temperatures were dropping eventually the density of free electrons became too low to maintain thermal contact and matter and radiation decoupled. This approximately took place when \( \Gamma_\gamma \simeq H \), where \( \Gamma_\gamma \): the interaction rate of the photons.\(^\text{10}\) Therefore the decoupled from the matter photons move freely in space creating our known microwave radiation. As universe expands the photons wavelength expands with the same rate, as well as the galaxies distances. This has as a result the decreasing of their energy and frequency along with their velocity and temperature (all with the same rate). The current photon distribution of microwave radiation is \( \sim 3K \) and \( \lambda \) is in the order of a few millimeters. Regarding electrons, they were combining with ions and forming atoms.

### 3.7 New models

The recent release of the WMAP three year data \(^4\) illustrates the extent to which cosmological model-building is now constrained and guided by precision data. It also emphasizes the extent to which a Standard Model of cosmology has emerged. This concordance model is denoted as \( \Lambda CDM \) model. In this model the universe on large scales is \textit{homogeneous}, \textit{isotropic}, and \textit{spatially flat}. The universe contains matter (dominated by \textit{dark matter}), radiation, and \textit{dark energy}. The dark energy consists either of a \textit{cosmological constant} or of a \textit{quintessence field} whose equation of state parameter \( w(t) \) is close to -1 now. Primordial density perturbations have a nearly scale invariant spectrum. At scales comparable to the current Hubble radius, the scalar perturbations have a slightly negative spectral tilt \(^4\).

A weakness of the cosmological data set is that direct observations only cover the era in which the scale factor \( a(t) \) was about 0.001 or larger. The success of Big Bang Nucleosynthesis (BBN) suggests that the the universe was strongly radiation-dominated at \( a \simeq 10^{-10} \), and that not much entropy was added to the radiation bath after the time of BBN. Beyond this, and especially in order to describe earlier times, we have to add additional theoretical baggage. A primordial epoch of inflaton-driven accelerated expansion provides an attractive (though not unique) explanation for many features of

\(^{10} \Gamma_\gamma = n_e \sigma_T \)

\( n_e \): number density of free electron, \( \sigma_T = 6.65 \times 10^{-25} \text{cm}^2 \) Thomson cross section.
cosmological data. Thus the assumption of an inflaton seems like a minimal theoretical input allowing us to make models of early time cosmology that can be confronted with data.

It is possible to deviate from the standard cosmological paradigm of inflation-assisted $\Lambda CDM$, keeping within current observational constraints, and \textit{without adding to or modifying any theoretical assumptions}. Within a minimal framework there are many new possibilities, some of them wildly different from the standard picture \cite{3}. We present only one illustrative example of a new model, described phenomenologically by a noncanonical scalar field coupled to radiation and matter. This model has interesting implications for inflation, quintessence, reheating, electroweak baryogenesis, and the relic densities of WIMPs and other exotics. For more possible models see \cite{5}

To be completely concrete, let us restate the theoretical assumptions that underlie standard cosmology:

I. The large scale evolution of the universe is well-described by solutions of the four dimensional Einstein equations (3.3) which are spatially homogeneous and isotropic (FRW). This assumption holds for any prior epoch when the temperature is (very roughly) less than $M_{\text{Planck}}$. During this time the universe has been expanding monotonically

II. The primordial distribution of light nuclei was produced by the standard BBN processes.

III. The solution to the horizon and flatness problems is provided by inflationary expansion in earlier epochs. This process is parametrized by a single scalar inflaton, which also produces primordial scalar and curvature perturbations which are nearly scale invariant, consistent e.g., with observations of the Cosmic Microwave Background (CMB).

IV. The dominant form of matter now is cold dark matter, consistent with a total indistinguishable from unity, and thus with the above assumption of primordial inflation.

V. Large scale evolution has lately undergone a transformation from a matter-dominated expansion to an accelerating expansion. The source of this accelerating expansion is parametrized either by a small positive cosmological constant, or by a quintessence scalar whose equation of state parameter $w(t)$ is close to -1 now.
We regard the above assumptions as minimal, and will refer to them as the minimal standard assumptions.

In next subsection we describe a new model, named Slinky, which obey the minimal standard assumptions, described phenomenologically in the non-canonical framework. This framework parametrizes FRW cosmological histories in terms of a single scalar field $\theta$ with a potential $V(\theta)$ and (in general) a noncanonical kinetic function $F(\theta)$. Slinky model solves the horizon, flatness, and ‘why now’ problems. Without any additional tuning of parameters and satisfies all constraints from CMB, large scale structure, and nucleosynthesis. In this model a somewhat abbreviated epoch of primordial inflation is followed by a second period of inflation which begins just before the electroweak phase transition (EWPT), and ends before BBN and we are currently beginning the third cosmic epoch of accelerated expansion. The radiation temperature decreases much more slowly than in the standard picture, and there is no period of large reheating. The Hubble expansion rate is much smaller during the EWPT; for Higgs sectors such that the phase transition is first order, this will enhance electroweak baryogenesis.

Slinky model is a new approach to quintessential inflation, in which both dark energy and inflation are explained by the evolution of a single scalar field whose potential and/or kinetic function have both oscillatory and exponential behavior. In such models the equation of state parameter $w(z)$ has a periodic component, leading to occasional periods of accelerated expansion during epochs where $w(z) \simeq -1$, as it is today and and $\dot{w}$ is nonzero over most or all of FRW cosmological history. The model uses the conventional reheating mechanism of new inflation, in which the scalar decays to light fermions with a decay width that is proportional to the scalar mass. Because our scalar mass is proportional to the Hubble rate, this gives adequate reheating at early times while shutting off at late times to preserve quintessence and satisfy nucleosynthesis constraints.

It is a successful model with only three adjustable parameters: $b$, $k$, $f_m$. One parameter controls the period between inflationary epochs, a second controls the overall decay width, and the third parametrizes our ignorance about the relative fraction of matter versus radiation produced by reheating. These three parameters are adjusted to produce sufficient inflation along with the correct fractions $\Omega_r/\Omega_\Lambda$, $\Omega_m/\Omega_\Lambda$ of radiation, matter, and dark energy, as measured today. We do not distinguish between the excess of baryonic matter density and the much larger excess of dark matter density, thus $f_m$ refers to the production of dark matter. Here we are assuming that the dark matter
3.7. NEW MODELS

results from decay of the noncanonical scalar. If instead the dark matter is a thermal relic, then one can set $f_m = 0$ and obtain the dark matter relic density by standard methods. Note however that such calculations must take into account the nonstandard expansion histories in our model. Having thus fixed the model we find that we automatically satisfy all constraints of primordial nucleosynthesis, CMB, and large-scale structure.

An FRW cosmological history is completely specified by a scale factor $a_i = a(t_i)$, with $t_i$ the initial time at which we begin the FRW evolution. Without loss of generality we can take the scale factor now to be unity: $a_0 = 1$. Thus the cosmological history is equivalently specified by the Hubble parameter $H(t) = \dot{a}/a$. Since we assumed that the FRW expansion is monotonic, we can trade the co-moving time variable $t$ for the scale factor $a$. Thus an FRW cosmological history is fully specified by some $H(a)$.

The parameters $b$ and $a_i$ are constrained by several phenomenological requirements:

- Since we assume standard BBN to explain the abundances of light nuclei, the universe should be radiation dominated when the temperature is in the MeV range. In addition, we should not produce very much entropy, in the form of radiation from reheating, at any time after BBN.

- To solve the horizon problem, the ratio of the comoving horizon to the comoving Hubble radius, as measured today, should be greater than one:

$$aH \int_0^a \frac{da'}{a'} \frac{1}{a'H(a')} > 1.$$  (3.28)

- The scalar spectral index, for perturbations which are now on scales comparable to the Hubble radius, should be close to the WMAP-preferred value [4].

- The temperature $T = (30g_r/\pi^2g_\ast)^{1/4}$ should not exceed $M_{\text{Planck}}$ at any time after $t_i$.

- Late time variations of $w(a)$ should not interfere with structure formation, or cause too much distortion of the imprint from baryon acoustic oscillations (BAO).

- The coupling of the scalar to radiation and ordinary matter must be very suppressed at late times, to satisfy bounds on Equivalence Princi-
ple violations, Faraday rotation of light from distant sources, and time variation of Standard Model parameters.

While \( b \) and \( a_i \) are nontrivially constrained by the above considerations, many solutions remain. The tunings required are not very strong; our example was obtained from a few minutes of hand-tuning, not from a systematic parameter scan.

### 3.7.1 Slinky Inflation

It is not widely appreciated that any FRW cosmological history can be parametrized by a single real scalar quintessence field \( \theta(x^\mu) \) coupled to Einstein gravity \([6]\). In general the scalar field action will be noncanonical, meaning that it has the form

\[
\int d^4x\sqrt{-g}\left[\frac{1}{2}f(\theta)P(X) - V(\theta)\right],
\]

where \( X = \frac{1}{2}g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \). When \( V = 0 \) and \( P(0) = 0 \), this is the class of noncanonical theories which generate k-essence models of inflation \([7]\). When \( P(X) = X \) reduces to

\[
\int d^4x\sqrt{-g}\left[\frac{1}{2}f(\theta)g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta - V(\theta)\right].
\]

where the kinetic function \( f(\theta) \) and potential \( V(\theta) \) are given by:

\[
f(\theta) = \frac{3M_{Pl}^2}{\pi b^2}\sin^2 \theta,
\]

\[
V(\theta) = \varrho_0 \cos^2 \theta \exp\left[\frac{3}{b}(2\theta - \sin 2\theta)\right],
\]

where \( M_{Pl} \) is the Planck mass: \( 1.22 \times 10^{19} \text{ GeV} \); \( \varrho_0 \) is the dark energy density observed today: \( \simeq (10^{-4} eV)^4 \); \( b \) is a dimensionless parameter which controls the periodic behavior. A canonical kinetic term can be restored via a field redefinition \( \theta(x) \rightarrow \phi(x) \), where

\[
\phi(x) = \varrho_0 \cos \theta,
\]

with \( \varrho_0 \equiv \sqrt{3M_{Pl}^2/\pi b^2} \).
In the approximation where we ignore the energy density of radiation and matter, and where the only friction is from the metric expansion, the evolution of the model can be solved analytically. Energy conservation requires:

$$\dot{\varrho}_\theta = -3H(1 + w)\varrho_\theta,$$

(3.34)

where $H$ is the Hubble rate, $w$ is the equation of state parameter, and $\varrho_\theta$ is the dark energy density. The solution to equation 3.34 as a function of the scale factor $a(t)$ is:

$$\varrho_\theta(a) = \varrho_0 \exp \left[-3 \int_1^a \frac{da}{a}(1 + w(a))\right],$$

(3.35)

where $\varrho_0$ is the dark energy density at $a = 1$ (today).

We also know that

$$V(\theta) = \frac{1}{2}(1 - w)\varrho_\theta.$$

(3.36)

Asking for a simple periodic behavior in the equation of state parameter:

$$w(a) = -\cos 2\theta(a),$$

(3.37)

one immediately gets a solution to the (flat) Friedmann equation combined with the relations (3.35, 3.36):

$$\theta(a) = -\frac{b}{2}\ln a;$$

(3.38)

$$w(a) = -\cos[b \ln a].$$

(3.39)

The expectation value of the quintessence field $\theta$ evolves logarithmically with scale factor from a positive initial value to zero today. The potential $V(\theta)$ has the ‘Slinky’ form shown in Figure 3.2. Accelerated expansion corresponds to epochs (such as today) where $\theta$ is evolving through one of the flat ‘steps’ of the potential. From Eq.(3.31) we see that the kinetic function is simultaneously suppressed in this epochs, slowing the roll of the scalar field evolution. Note also that our potential $V(\theta) \to 0$ as $\theta \to -\infty$, which corresponds to $t \to \infty$; this is as desired for a quintessence model.

We can understand the same behavior by looking at the ‘canonical’ scalar $\phi$ in Eq.(3.33). The potential $V(\phi)$ resembles a series of descending ramps, with the value of $\phi$ bounded between $-\phi_0$ and $\phi_0$. Instead of a suppression of the kinetic function when $\theta$ is a multiple of $\pi$, the scalar $\phi$ has an effective
mass that diverges when $\phi = \pm \phi_0$, i.e., at each junction of the descending ramps.

To complete the model, we will assume that the quintessence field $\phi$ has a weak perturbative coupling to light fermions. This is the standard reheating mechanism of new inflation [8]. To avoid the strong constraints on long-range forces mediated by quintessence scalars [9], it is simplest to imagine that our scalar only has a direct coupling to a sterile neutrino. This is sufficient to hide the quintessence force from Standard Model nonsinglet particles [10], while still allowing the generation of a radiative thermal bath of Standard Model particles from quintessence decay.

With this assumption for reheating the evolution equations for quintessence, radiation, and matter become:

\begin{align*}
\dot{\varrho}_\theta &= -3H(1 + w)\varrho_\theta - k_0 m_\phi (1 + w)\varrho_\theta, \quad (3.40) \\
\dot{\varrho}_r &= -4H\varrho_r + (1 - f_m)k_0 m_\phi (1 + w)\varrho_\theta, \quad (3.41) \\
\dot{\varrho}_m &= -3H\varrho_m - f_m k_0 m_\phi (1 + w)\varrho_\theta \quad (3.42)
\end{align*}

where $k_0$ and $f_m$ are small dimensionless constants. As long as $\theta$ is not near
a multiple of $\pi/2$, it is a reasonable approximation to make the replacement

$$k_0 m_\phi \to k H,$$  \hspace{1cm} (3.43)

where $k$ is another small dimensionless parameter. This replacement decouples the $\theta$ evolution equation from the Friedmann equation, giving an immediate analytic solution:

$$\rho_\theta(a) = \rho_0 \exp \left[ \frac{1}{b} (3 + k) (2\theta - \sin 2\theta) \right],$$  \hspace{1cm} (3.44)

where $\theta(a)$ and $w(a)$ are still given by (3.38), (3.39).

We have used this convenient approximation in generating the figures shown below. Clearly this approximation overestimates the reheating effect for epochs where $\theta$ is close to an odd multiple of $\pi/2$, however this is a small effect since these are the epochs of maximum radiation or matter domination. The approximation also breaks down in the epochs of maximum inflation, i.e. when $\sin \theta \to 0$. However when $\sin \theta \to 0$ we also expect strong-coupling physics to enter in a full theory, providing a cutoff for the naive vanishing of the kinetic function of $\theta$. Since we cannot compute this effect without a full Planck-scale theory, we may as well stay with the approximation (3.43).

The temperature history of Slinky model near the EWPT is shown in Figure 3.3. Also shown is the Hubble parameter $H$ of Slinky model normalized to the expansion rate $H_{rad}$ for pure radiation. $H_{rad}$ corresponds to what is assumed in the standard paradigm. Notice that for temperatures of a few GeV the expansion rate is actually somewhat larger than normal, but at higher temperatures it is much less than normal.

Such a nonstandard thermal history will impact on electroweak baryogenesis. For a Higgs sector such that the EWPT is first order, the change in the net baryon asymmetry is proportional to $-\log(H/H_{rad})$, where $H$ is the expansion rate during the phase transition and $H_{rad}$ is the corresponding expansion rate for pure radiation [11]. If the Higgs sector is such that the EWPT is second order, the baryon asymmetry is proportional to the expansion rate [11]. Clearly one should reevaluate the popular scenarios for electroweak baryogenesis [12] in this light.

Figure 3.4 show the results obtained from Slinky model with $b = 1/7$, $k = 0.06$, and $f_m = 10^{-11}$. Shown are the relative energy density fractions in dark energy, radiation, and matter, as a function of $\log a$. We have chosen to integrate the evolution equations starting from $a = 10^{-42}$, which in our
CHAPTER 3. COSMOLOGY

Figure 3.3: The temperature history of Slinky Model (red/solid) near the electroweak phase transition. Shown in green/dashed is the Hubble parameter H of Slinky Model 2 normalized to the expansion rate $H_{rad}$ for pure radiation.

model corresponds to a temperature of slightly less than $10^{16}$ GeV, and an initial comoving Hubble radius of about 100 Planck lengths. For simplicity we have also chosen the value of $w(a)$ now to be exactly -1. Neither of these choices corresponds to a necessary tuning.

Our three adjustable parameters have been chosen such that the values of $\Omega_r/\Omega_\Lambda$, $\Omega_m/\Omega_\Lambda$ come out to their measured values at $a = 1$, and such that we have sufficient inflation. The latter is checked by computing the ratio of the fully inflated size of the initial comoving Hubble radius to the current comoving Hubble radius. This ratio is about 3 in Slinky model, indicating that the total amount of inflation is indeed enough to solve the horizon problem. The flatness problem is solved because the total $\Omega_r + \Omega_m + \Omega_\Lambda = 1$ within errors.

From Figure 3.4 we see that we are currently beginning the third epoch of accelerated expansion, as mentioned before. Quantum fluctuations, during the first epoch of inflation, produced the spatial inhomogeneities responsible for large scale structure and CMB anisotropies observed today. Constraints on the physics responsible for the primordial power spectrum of these density fluctuations can be set with WMAP and 2dF data, under the assumption that the Hubble rate is dominated by the contribution from $\varphi$ during the observable part of inflation [13].

Slinky Model will have major implications for predictions of the relic abundance of dark matter particles with Terascale masses. The dominant production mechanism for such particles may be scalar decays, as suggested
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Figure 3.4: The cosmological history of Slinky Model. Shown are the relative energy density fractions in radiation (green/dashed), matter (blue/dotdashed), and the noncanonical scalar (red/solid), as a function of the logarithm of the scale factor $a(t)$.

by Figure 3.4. Even if the dark matter particles are thermal relics, their abundance now will be affected by the nonstandard expansion rates at earlier times [14, 15].
Chapter 4

Dark Matter

By measuring the motions of stars and gas, astronomers can ‘weigh’ galaxies. In our own solar system, we can use the velocity of the Earth around the Sun to measure the Sun’s mass. The Earth moves around the Sun at 30 kilometers per second. If the Sun were four times more massive, then the Earth would need to move around the Sun at 60 kilometers per second in order for it to stay on its orbit. The Sun moves around the Milky Way\(^1\) at 225 kilometers per second. We can use this velocity (and the velocity of other stars) to measure the mass of our Galaxy. Similarly, radio and optical observations of gas and stars in distant galaxies enable astronomers to determine the distribution of mass in these systems.

We believe that most of the matter in the universe is dark, i.e. cannot be detected from the light which it emits (or fails to emit). This is ‘stuff’ which cannot be seen directly - so what makes us think that it exists at all? Its presence is inferred indirectly from the motions of astronomical objects, specifically stellar, galactic, and galaxy cluster/supercluster observations. It is also required in order to enable gravity to amplify the small fluctuations in the Cosmic Microwave Background (CMB) enough to form the large-scale structures that we see in the universe today. For each of the stellar, galactic, and galaxy cluster/supercluster observations the basic principle is that if we measure velocities in some region, then there has to be enough mass there for gravity to stop all the objects flying apart. When such velocity measurements are done on large scales, it turns out that the amount of inferred mass is much

\(^1\)The Milky Way is a gravitationally bound collection of roughly a hundred billion stars. Our Sun is one of these stars and is located roughly 24,000 light years (or 8000 parsecs) from the center of our the Milky Way.
To be more specific, the mass that astronomers infer for galaxies including our own is roughly ten times larger than the mass that can be associated with stars, gas and dust in a Galaxy. This mass discrepancy has been confirmed by observations of gravitational lensing, the bending of light predicted by Einstein’s theory of general relativity. By measuring how the background galaxies are distorted by the foreground cluster, astronomers can measure the mass in the cluster. The mass in the cluster is more than five times larger than the inferred mass in visible stars, gas and dust. Recent analysis combining high-redshift supernova luminosity distances, microwave background fluctuations and the dynamics and baryon fraction of galaxy clusters indicate that the present mass density of matter in the Universe \( \Omega_M = \rho_M/\rho_{\text{crit}} \) normalized to the critical density \( \rho_{\text{crit}} = 3H_0^2/(8\pi G_N) = h^2 \times 1.9 \times 10^{-29} \text{g cm}^{-3} \) is \( 0.1 \lesssim \Omega_M h^2 \lesssim 0.2 \), which is considerably higher than the value \( \Omega_B h^2 \lesssim 0.023 \) allowed by big bang nucleosynthesis [16]. Hence we infer that there is dark matter in the Universe.

### 4.1 Nature of Dark Matter

The most favored cosmological model today inferred from WMAP\(^2\) and other cosmological data maintains a cosmological expansion driven by an energy density comprised of the following approximate fractions, as shown in figure 4.1:

- \( 0.73 \pm 0.04 \) negative pressure dark energy
- \( 0.23 \pm 0.03 \) dark matter
- \( 0.04 \) atoms, the building blocks of stars and planets.

Let us consider each of these components in turn.

\( \leftarrow \text{Negative pressure dark energy} \) [17] is defined to be an energy density component whose pressure \( \rho \) to energy density \( \rho \) ratio (i.e., its equation of state) is \( \rho/\rho < -1/3 \). A cosmological constant can qualify as such an energy component, because its equation of state is \(-1\). The most sensitive probe of this

\(^2\)The Wilkinson Microwave Anisotropy Probe is a NASA satellite mission measuring the temperature of the cosmic background radiation over the full sky with unprecedented accuracy.
4.1. NATURE OF DARK MATTER

Figure 4.1: Estimated distribution of dark matter and dark energy in the universe

Energy is the combination of CMB and supernovae data [18]. Scalar fields whose potential energy dominates the kinetic energy can also be responsible for this energy component. If such fields are time varying as well as weakly spatially varying, it is fashionable to refer to these fields as *quintessence* [19]: for a sense of the evolution of this idea, see [20, 21] and the review [22]. As the required energy scale is far removed from the electroweak scale, the MSSM fields are not likely candidates for quintessence fields. The only connection of quintessence with $L_{\text{soft}}$ is that supersymmetry breaking terms will induce radiative masses for such fields which are large compared to the Hubble expansion rate today and generically give a cosmological constant contribution which is at least of order $\tilde{m}^4$, where $\tilde{m}$ denotes a typical scale of the $L_{\text{soft}}$ parameters. Generically one might also expect a cosmological constant contribution of order $M_S^4$, where $M_S$ is the scale of supersymmetry breaking in the hidden sector [23, 24].

$\rightarrow$ *Dark matter.* What is the nature of the dark matter, this mysterious material that exerts a gravitational pull, but does not emit nor absorb light? There is no definite answer. There are a number of plausible speculations on the nature of the dark matter:

- **Baryonic dark matter:** consists of white dwarfs, brown dwarfs, neutron stars, and black holes. In almost every case, a serious theoretical or observational problem is encountered. The current problem with baryonic dark matter is that not only is it very difficult to hide baryons,
but given the amount of dark matter required on large scales, there is a direct conflict with primordial nucleosynthesis if all of the dark matter is baryonic. We will not discuss baryonic dark matter further because it has little direct relation to the $L_{soft}$ parameters. The main progress with respect to baryonic dark matter which is relevant for $L_{soft}$ is indirect, mainly pointing to the necessity of non-baryonic matter.

- New forms of matter. Since the universe was very dense and hot in the early moments following the Big Bang, the universe itself was a wonderful particle accelerator. Cosmologists speculate that the dark matter may be made of particles produced shortly after the Big Bang. These particles would be very different from ordinary baryonic matter. That is why we refer to this new form of matter as non-baryonic. Among the non-baryonic particles, many share a set of common properties. They are heavy, neutral, weakly-interacting particles with interaction cross sections nevertheless large enough that they were in thermal equilibrium for some period in the early universe. It is these particles that we refer to collectively as WIMPs, or Weakly Interacting Massive Particles. These particles interact through the weak nuclear force and gravity, and possibly through other interactions no stronger than the weak force. Because they do not interact electromagnetically they cannot be seen directly, and because they do not interact with the strong nuclear force they do not react strongly with atomic nuclei. Therefore WIMPs serving as one possible solution to the dark matter problem. Also from the point of view of structure formation, non-baryonic dark matter seems to be necessary, and the main part of it should consist, as we will discuss later in this section, of particles that were non-relativistic at the time when structure formed (cold dark matter, CDM), thus excluding light neutrinos.

In conclusion, we know two things: Dark matter exists, since we don’t see $\Omega = 1$ in luminous objects, and most (about 90%) of the dark matter is not baryonic. The latter conclusion is a result of Big Bang Nucleosynthesis (BBN) which constrains the baryon-to-photon ratio and hence $\Omega_B$. Thus $1 - \Omega_B$ is not only dark but also non-baryonic.

---

3 One of the primary motivations for building supercolliders is to try to produce this matter in the laboratory.

4 Under reasonable assumptions, the WMAP collaboration, using also galaxy survey and Ly-α forest data, limit the contribution of neutrinos to $\Omega_\nu h^2 < 0.0076$ (95% c.l.).
4.2 Cold Dark Matter candidates

Cold dark matter (CDM) is defined as matter which is nonrelativistic at the time of matter-radiation equality: when the relativistic energy density, characterized by its positive nonvanishing pressure, is equal to the nonrelativistic energy density, which has vanishing pressure. Similarly, hot dark matter is defined as matter which is relativistic at the time of matter-radiation equality. In between lies warm dark matter, which is similar to hot dark matter except that it becomes nonrelativistic at a much earlier epoch, and hence has a much smaller free-streaming scale of about 1 Mpc (3 million light years). Dark matter is categorized in this manner because the time of matter-radiation equality marks the beginning of the matter-dominated universe, which is the beginning of the time during which the universe is expanding slowly enough for matter to gravitationally cluster appreciably. Whether the dark matter is relativistic or nonrelativistic changes the clustering property during this matter domination period. A comparison of cosmological observations, such as CMB and galaxy observations, with various theoretical calculations (including numerical simulations) favors the nonnegative pressure component of the dark matter to be CDM. As we will see in detail, there are natural candidates for CDM in the MSSM.

Among the various dark matter candidates, $L_{\text{soft}}$ has its closest connection with cold dark matter because if R-parity is conserved, the lightest supersymmetric particle (LSP)—which has a mass controlled by the $L_{\text{soft}}$ parameters—naturally provides just the right abundance today to account for the CDM if the LSPs were once in chemical thermal equilibrium with the background radiation. The beauty of LSP cold dark matter is that it was motivated mostly independently of any cosmological considerations. In the MSSM, the R-parity which guarantees LSP stability is needed to eliminate rapid proton decay, while the electroweak scale interactions and mass scales that determine the relic abundance are motivated from naturalness considerations of the SM. As there are strong bounds on charged dark matter [25, 26], the viable MSSM parameter region is usually that within which the LSP is neutral. Among the neutral LSP candidates, neutralinos and sneutrinos each have electroweak scale interactions that can naturally lead to dark matter densities consistent with observations. However, the possibility of sneutrinos as significant CDM is ruled out for most models from LEP constraints and direct detection [27]. In the mass range allowed by these constraints, the sneutrinos annihilate too rapidly via s-channel $Z$ exchange, and hence not
enough remain today to make up the dark matter. However, sneutrinos can of course be the LSPs without violating experimental bounds if LSPs are not required to compose the CDM.

One particular LSP does not have electroweak scale interactions, but only gravitational interactions. This is the gravitino, which usually is the LSP in gauge mediation. Even when the gravitino is not the LSP and can decay, as in most gravity-mediated scenarios, its lifetime can be very long due to its weak gravitational interactions, leading to nontrivial consequences for late time cosmology. As we will explain below, the typical upper limit on the temperature of the universe due to the gravitino decay constraint is about $10^9$ GeV.

Another well-motivated dark matter candidate, although not strictly related to supersymmetry and the $L_{soft}$ parameters, is the axion. Remarkably, axions can still naturally live long enough to be the CDM even though they decay to photons. In many instances the axino, the supersymmetric partner of the axion, can also serve as the LSP dark matter. We discuss these candidates below because (i) axions are arguably the most appealing solution to the strong CP problem, and (ii) the interpretation of MSSM cosmology can be misleading without taking axions and axinos into consideration.

There are rare instances when the NLSP (the next-to-lightest supersymmetric particle) can be an absolutely stable dark matter candidate. This can occur if the LSP is strongly interacting, such that its bound state to other strongly interacting fields has a mass large enough that kinematics allow a decay to the weakly interacting NLSP [28, 29]. We will not discuss this and other rare situations.

### 4.3 Calculation of relic density

Here we summarize the standard technique for calculating the relic abundance of a particle species $\chi$ in the standard cosmological scenario. For more details, see ref [30, 31].

Consider the evolution in the early Universe of $N$ supersymmetric particles $\chi_i$ ($i = 1, \ldots, N$) with masses $m_i$, internal degrees of freedom (statistical weights) $g_i$ and a total number density $n$. Also, assume that the particles are labeled such that $m_i < m_j$, when $i < j$; that is, $\chi_1$ has mass $m_1$ and is the lightest, while $\chi_2$ is the second lightest, etc.

Note that since we are interested in the case where the lightest of these
particles is stable and perhaps even a dark-matter candidate, the assumption of the existence of a conserved quantum number is not particularly ad hoc. Reactions of the following types change the $\chi_i$ number densities and determine their abundances in the early Universe:

$$\chi_i \chi_j \leftrightarrow XX' \quad (4.1)$$

$$\chi_i X \leftrightarrow \chi_j X' \quad (4.2)$$

$$\chi_j \leftrightarrow \chi_i XX' \quad (4.3)$$

where $X, X'$ denote any standard-model particles. There may be many choices for the pair $X$ and $X'$, but generally their are not independent; a choice of $\chi_i, \chi_j,$ and $X$ will determine $X'$. Reactions such as $\chi_i \chi_j \leftrightarrow \chi_k X$ and $\chi_i X \leftrightarrow XX$ are forbidden by the assumed symmetry. The first equation denotes annihilation of the supersymmetric particles and the second denotes scattering between supersymmetric particle and particle in the thermal background. Reactions of type Eq. (4.3) are called coannihilations. This case occurs when the relic particle is the lightest of a set of similar particles whose masses are nearly degenerate. In this case the relic abundance of the lightest particle is determined not only by its annihilation cross section, but also by the annihilation of the heavier particles, which will later decay into the lightest. In supersymmetric theory the squarks or selectrons are only slightly more massive than the LSP, usually taken to be a neutralino. Previous calculation of the relic abundance which consider only the LSP annihilation can be in error by more than two orders of magnitude. Note that as long as reactions of type (4.3) take place at a reasonable rate, we expect all the $\chi_j$ ($j > 1$) particles to have decayed into $\chi_1$ particles by today.

The abundances of the $\chi_i$ are determined by a set of $N$ Boltzmann equations:

$$\frac{dn_i}{dt} = -3Hn_i - \sum_{j,X} \left[ \langle \sigma_{ij} v \rangle (n_in_j - n_i^{eq} n_j^{eq}) ight. $$

$$\left. - \langle \sigma'_{ij} v \rangle (n_in_X - (\sigma_{ji}v)n_jn_X) \right] - \Gamma_{ij} (n_i - n_i^{eq}), \quad (4.4)$$

where the $n_X$ are number densities of the standard-model particles involved in the interactions. Their nature will not be important, apart from the assumption that they are light enough that they are relativistic at freeze-out. The sum over $X$ implies that all relevant reactions are to be included.
Also \( \langle \sigma v \rangle \) denotes the thermal averaging of the cross section \( \sigma \) multiplied by the Moeller speed \( v \). In Eq. (4.4) we have defined the cross sections and decay rates

\[
\begin{align*}
\sigma_{ij} &= \sigma(\chi_i \chi_j \rightarrow XX'), \\
\sigma'_{ij} &= \sigma(\chi_i \rightarrow \chi_j X'), \\
\Gamma_{ij} &= \Gamma(\chi_i \rightarrow \chi_j XX').
\end{align*}
\]

The first term on the right of Eq. (4.4) is the dilution of number density due to the expansion of the Universe. The second term is due to both forward and backward reactions of type Eq. (4.1). The third term is due to forward and backward reactions of type Eq. (4.2), which only change \( n_i \) if \( i \neq j \). The fourth term refers to decays and inverse decays of type Eq. (4.3)

Normally, the decay rate of supersymmetric particles \( \chi_i \), other than the lightest which is stable, is much faster than the age of the Universe. Since we assume that R-parity holds, all supersymmetric particles, which survive annihilation, will eventually decay to the LSP. So its final abundance is simple described by the sum of the density of all supersymmetric particles: \( n = \sum_{i=1}^{N} n_i \). Using Eq. (4.4), we get the evolution equation

\[
\frac{dn}{dt} = -3Hn - \sum_{i,j=1}^{N} \langle \sigma_{ij}v \rangle (n_i n_j - n_i^{eq} n_j^{eq}),
\]

where the last two sums in Eq. (4.4) cancel in the sum.

Furthermore, the scattering rate of supersymmetric particles off particles in the thermal background is much faster than their annihilation rate, because the scattering cross sections \( \sigma'_{Xij} \) are of the same order of magnitude as the annihilation cross sections \( \sigma_{ij} \) but the background particle density \( n_X \) is much larger than each of the supersymmetric particle densities \( n_i \) when the former are relativistic and the latter are nonrelativistic, and so suppressed by a Boltzmann factor. On the other hand it is reactions of type Eq. (4.1) which determine the freeze-out [see Eq. (4.6)]. Hence, the \( \chi_i \) distributions remain in kinetic thermal equilibrium during and after their freeze-out and in particular their ratios are equal to the equilibrium values:

\[
\frac{n_i}{n} \sim \frac{n_i^{eq}}{n^{eq}}.
\]
Combining these effects, we arrive at the following Boltzmann equation for the summed number density of supersymmetric particles

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n_{\text{eq}}^2),
\]

(4.8)

where

\[
\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{\text{eq}}^i n_{\text{eq}}^j}{n_{\text{eq}}^i n_{\text{eq}}^j}.
\]

(4.9)

This very convenient form of the Boltzmann equation that we have just derived has all the ‘features’ that we would have expected: The destruction rate of \( \chi_i \)'s per comoving volume is just proportional to the annihilation rate of \( \chi_i \)'s and the net rate of destruction is just balanced by inverse (creation) processes when \( n = n_{\text{eq}} \). Equation (4.8) can be solved numerically, but a convenient and accurate approximation exists. At early times \( \chi \) was in thermal equilibrium. In other words, \( \chi_i \chi_j \) rapidly annihilated into lighter states and vice versa. So for \( T \geq m_{\chi} \), \( n \) is approximated by \( n_{\text{eq}} \). Moreover, creation processes are Boltzmann suppressed for \( T \ll m_{\chi} \), because only a small fraction of \( \chi \chi' \) pairs have sufficient KE to create a \( \chi_i \chi_j \). So as the temperature drops below the mass of the relic, \( n_{\text{eq}} \) drops exponentially\(^5\) and eventually the point, denoted ‘freeze-out’, is reached where the reaction rate is not fast enough to maintain equilibrium. The temperature at which the particle decouples from the thermal bath is called freeze-out temperature \( T_F \). From this point on, the \( n_{\text{eq}} \) term in Eq. (4.8) can be ignored and the resulting equation is easily integrated.

For ease of presentation we will follow the method detailed in Ref.[3]. The freeze-out point is given in terms of the scaled inverse temperature \( x = m/T \):

\[
x_f = \ln \left( \frac{0.038 g_{\text{eff}} m_{\text{pl}} m_1 \langle \sigma_{\text{eff}} v \rangle}{g^{1/2} s^{1/2}} \right)
\]

(4.10)

where \( m_{\text{pl}} = 1.22 \times 10^{19} \) GeV and \( g_s \) is the total number of effectively relativistic degrees of freedom at the time of freeze-out. Equation (4.10) is usually solved iteratively. In Ref.[3] this is done analytically by substituting for \( x_f \) on the right side of Eq. (4.10). Here we consider that the thermal averaged cross section changes rapidly with temperature and we will take a numerical approach to solving Eq. (4.10).

\(^5\)In the non-relativistic regime, \( n_{\text{eq}} \sim (mT)^{3/2} \exp(-m/T) \)
At freeze-out the abundance of relic particles is usually taken to be the equilibrium abundance; however, after freeze-out there is further significant annihilation of relic particles which reduces the abundance to its final and present value. The efficiency of this post-freeze-out annihilation is expressed through an integral $J$:

$$J(x_f) = \int_{x_f}^{\infty} \frac{\langle \sigma_{\text{eff}} v \rangle}{x^2} dx. \quad (4.11)$$

We can Taylor expand the cross sections $\sigma_{ij} = a_{ij} + b_{ij}v^2$, $\sigma_{\text{eff}} = a_{\text{eff}} + b_{\text{eff}}v^2$ and write the annihilation integral in the form

$$J = \frac{(a_{11}I_a + 3b_{11}I_b/x_f)}{x_f}, \quad (4.12)$$

where:

$$I_a = x_f a_{11}\int_{x_f}^{\infty} x^{-2}a_{\text{eff}}dx,$$

$$I_b = \frac{2x_f^2}{b_{11}}\int_{x_f}^{\infty} x^{-3}b_{\text{eff}}dx. \quad (4.13)$$

The relic density today in units of the critical density, is then given by

$$\Omega_{\chi}h^2 = \frac{1.07 \times 10^9 x_f}{g_*^{1/2}m_{Pl}(GeV)(a_{11}I_a + 3b_{11}I_b/x_f)} \quad (4.14)$$

We have performed a technique for calculating the relic density of the LSP including all coannihilations processes, now we have to list the known situations in which the previous method fails.

$\rightarrow$ The first case concerns annihilation into particles which are more massive than the relic particle. Previous treatments regarded this as kinematically forbidden, but if the heavier particles are only $5 - 15\%$ more massive, these channels can dominate the annihilation cross section and determine the relic abundance. We call this the ‘forbidden’ channel annihilation case. For example, suppose that $2m_1 \gtrsim m_{H_2} + m_{H_3}$, where $m_1$ is the mass of the LSP. It is known that when the channel $\chi\chi \rightarrow H_2H_3$ is open, it dominates the annihilation cross section by up to a factor of 500 and therefore determines the relic abundance. In the standard treatment this channel would not be
4.4 Neutralino parameter dependence

For annihilation reactions of neutralinos significant for the final dark matter abundance, one must have either neutralino+neutralino, neutralino+sfermion, or neutralino+chargino in the initial state. As we have already mentioned in section 4.3, the annihilation reactions are broadly classified into two categories:

- The LSPs are self-annihilating: i.e., LSP+LSP in the initial state.
- The LSPs are coannihilating: i.e., LSP + other superpartner in the initial state.

\(^6\)That poles should not be as sharp and deep as the simple approximations suggest has been realized by E. W. Kolb and by G. Gelmini and E. Roulet.

considered when it was forbidden at zero relative velocity. However, since freeze-out occurs at a temperature \(T_f \sim m_1/20\) and since the \(\chi\) particles are Boltzmann distributed, the annihilation into heavier particles does take place at some rate. If the masses of the annihilation products are not too much greater, this kind of annihilation can dominate the cross section.

\(\rightarrow\) The second case occurs when the annihilation takes place near a pole in the cross section. This happens, for example, in \(Z^0\)-exchange annihilation when the mass of the relic particle is near \(m_Z/2\). Previous treatments have incorrectly handled the thermal averages and the integration of the Boltzmann equation in these situations. The dip in relic abundance caused by a pole is broader and not nearly as deep as previous treatments imply\(^6\).

There has been a great deal of activity in computing the relic density for various regions of MSSM parameter space [32, 33, 34, 35, 36, 37, 38]. The state of the art numerical programs take into account nearly 8000 Feynman diagrams. Typically, the parameter exploration is done in the context of mSUGRA/CMSSM models, in which the independent parameters at \(M_{\text{GUT}}\) are the universal scalar mass \(m_0\), gaugino mass \(m_{1/2}\), trilinear scalar coupling \(A_0\), \(\tan\beta\), and \(\text{sign}(\mu)\). These parameters are then run from \(M_{\text{GUT}}\) to low energies using the MSSM RGEs. In CP-violating extensions of mSUGRA models, there are \(L\) soft phases present in the neutralino and sfermion mass matrices, which consequently affect the annihilation rate (see e.g. [39]).
Due to the strong thermal suppression for initial states heavier than the LSP, the self-annihilation channels usually dominate in the determination of the relic abundance. However, if there are other superpartners with masses close to $m_{\text{LSP}}$ (within an $\mathcal{O}(m_{\text{LSP}}/20)$ fraction of $m_{\text{LSP}}$), then the coannihilation channels become significant.

Since we have so many different diagrams contributing, in Appendix B we classify diagrams according to their topology (s, t, or u channel) and to the spin of the particles involved. In typical nonresonant situations, the t-channel slepton exchange self-annihilation diagrams dominate. However, many s-channel contributions exist, and if the neutralino masses are light enough such that they sum approximately to the mass of one of the s-channel intermediate particles such as the Higgs or the Z, the resonance contribution dominates the annihilation process. When the resonance dominates, unless the resonance is wide as is possible e.g. for the Higgs, some fine tuning is required to obtain a nonnegligible final abundance of LSPs because the final relic density is inversely correlated with the strength of the annihilations. The relative strengths of the nonresonant reactions are determined mostly by the mass of the intermediate particle (e.g. suppressed if it is heavy) and the kinematic phase space available for the final states (i.e., their masses relative to $m_{\text{LSP}}$).

### 4.5 Neutralino direct detection

A great deal of work has been done on both theoretical and experimental aspects of direct detection (see e.g. the reviews [40, 41, 42]). Direct detection of WIMPs can be accomplished through elastic scattering off a nucleus in a crystal [43, 44]. The recoil energy is then measured by a variety of means: scintillation detection, cryogenic detection of phonons (usually relying on superconductor transitions [45]), ionization detection, or some combination thereof. Inelastic nuclear scattering methods have also been considered [46], but most of the proposed experiments use the elastic scattering method due to event rate considerations.

Many direct detection experiments have already produced quite strong limits on the elastic scattering cross section with protons or neutrons of potential dark matter candidates. Furthermore, experiments in the coming years will improve on current limits by several orders of magnitude making the prospects for discovery very great.
Presently, the best direct detection limits come from the CDMS, DAMA, EDELWEISS and ZEPLIN-I experiments. The CDMS experiment \cite{47, 48}, located at the Soudan mine in Minnesota, uses 100 g of Silicon and 495 g of germanium at 20 mK. The EDELWEISS experiment \cite{49}, located under the French-Italian Alps, uses three 320 g Ge detectors operating at 17 mK. The ZEPLIN I experiment uses liquid Xenon (a high mass nucleus) corresponding to 4 kg fiducial mass \cite{50} located in Boulby Mine (England). DAMA, located in the Gran Sasso underground laboratory, uses 58 kg of NaI \cite{51}; it has already claimed positive detection of dark matter. Particularly, DAMA has reported an annual modulation of their event rate consistent with the detection of a WIMP with a mass of approximately 60 GeV and a scattering cross section on of the order of $10^{-41} \text{ cm}^2$ (see fig. 4.2).

However, other experiments, such as EDELWEISS and CDMS have explored the parameter space favored by DAMA without finding any evidence of dark matter. A recent model independent analysis has shown that it is difficult to reconcile the DAMA result with other experiments \cite{52} (see also Ref. \cite{53}) although it may still be possible to find exotic particle candidates and halo models which are able to accommodate and explain the data from all current experiments.

Figure 4.2: CDMS limits in Dark Matter search
The accompanying mass versus cross-section plot above, shows the regions of parameter space that have been ruled out by a given experiment. For example, under standard assumptions for the galactic halo and the interaction of WIMPs with nuclei, WIMPs with mass and cross section above the solid blue curve can be excluded at the 90% confidence level by the CDMS-II data taken at Soudan. The red-colored region corresponds to the claim of a signal by the DAMA collaboration, which is clearly incompatible with CDMS for this set of assumptions. The other colored regions illustrate where different Supersymmetry models provide WIMP candidates, or alternatively, can be ruled out by dark matter null results.

In addition, plot shows the results (black curve) from 2004 with the first six-detector tower [PRL 93, 211301 (2004) and astro-ph/0405033]. Those limits were then the best in the world by a factor of four over EDELWEISS (brown curve) and ZEPLIN (green curve). The combined 2004 and new preliminary 2005 result from CDMS-II (blue curve) represents an additional factor of 2.5 improved sensitivity, with two towers containing twelve detectors. This result is a full factor of ten better than EDELWEISS and ZEPLIN, meaning that, while no signal has been seen, CDMS-II is exploring virgin territory for dark matter in the form of weakly interacting massive particles.

The lightest supersymmetric particle is an excellent candidate in many models, as shown by the colored regions in the plot background, and the new CDMS-II results exclude many SUSY models as well as the signal claimed by DAMA (red closed region). CDMS-II is part of the newly-organized Particle Astrophysics Center at Fermilab.

### 4.6 Neutralino indirect detection

Indirect detection of LSP dark matter is the technique of observing the cosmic ray particles resulting from the annihilation of LSP neutralinos [54]. The flux of such radiation is proportional to the annihilation rate, which in turn depends on the square of the dark matter density, $\Gamma_A \propto \rho_{DM}^2$. Therefore, the ‘natural’ places to look at, when searching for significant fluxes, are the regions where large LSP dark matter densities accumulate. We will also refer to these regions or objects as amplifiers.

- Dense regions of the galactic halo, such as the galactic center, may be excellent amplifiers for the purposes of detecting gamma-rays, lower energy photons such as radio waves [55], and antimatter such as positrons and an-
tiprotons [56, 57]. In fact, the recent positron excess reported by the HEAT balloon borne experiment [58] may be attributable to WIMP annihilations if certain nonstandard astrophysical phenomena are assumed to take place [59]. Specifically, the HEAT collaboration has reported an excess of positrons that are consistent with arising from LSP annihilation if the LSP is heavier than the $W$. While further study is needed to argue that this excess does not arise from conventional sources, there has not been a convincing alternative scenario which leads to an excess with a peak at an energy of order 10 GeV. The excess has been seen in several sets of data with different detectors.

- Other astrophysical objects, such as the Sun or the Earth, could also act as amplifiers for dark matter annihilations by capturing dark matter particles as they lose energy through scattering with nucleons in the interiors of these objects. The accumulated LSPs can annihilate, giving rise to observable final products. Among the SM decay products of the primary annihilation products, the muon neutrino can escape without being absorbed by the core of the sun or the earth and can reach terrestrial detectors (neutrino telescopes). Since $\chi\chi \rightarrow \nu\nu$ is suppressed by the small neutrino masses, the neutrinos primarily arise due to decays of primary products of annihilation with a mean energy of $\sim m_\chi/2$. In the water/terrestrial material immersing the detectors, the muon neutrinos induce muon production, which can easily be measured by its Cerenkov radiation.

There have been several experiments under the category of neutrino telescopes which had put bounds through indirect detection, including Macro [60], Baksan, Super-Kamiokande, and AMANDA [61]. Future experiments have potential to indirectly detect the neutralinos. One is the Antares 0.1 km$^2$ project which covers a volume of around 0.02 km$^3$ (which may be upgraded upgraded to 1 km$^3$ in the future) in the Mediterranean sea at a depth of 2.4 km down south of France. Another project, ICECUBE, will cover 1 km$^3$ volume under about 2.4 km of ice [62]. The reaches of these experiments are compared to the direct detection experiments in Figure 4.3. The typical energy thresholds are between 5 to 10 GeV.
Figure 4.3: Taken from [63], the left figure shows the direct detection scalar elastic scattering cross section for various neutralino masses, and the right figure shows the indirect detection experiments’ muon flux for various neutralino masses. The scatter points represent ‘typical’ class of models. Specifically, the model parameters are $A_0 = 0$, $\tan \beta = 45$, $\mu > 0$, $m_0 \in [40, 3000]$, $m_{1/2} \in [40, 1000]$. The dotted curve, dot dashed curved, and the dashed curve on the right figure represents the upper bound on the muon flux coming from Macro, Baksan, and Super-Kamiokande experiments, respectively. This plot should be taken as an optimistic picture, because the threshold for detection was set at 5 GeV, where the signal-to-noise ratio is very low in practice.
Chapter 5

Cosmology effects on the SUSY parameter space

In this thesis we deal with the important question of whether we can quantitatively evaluate the claim that supersymmetry provides a natural explanation of dark matter. We will show that a physically meaningful approach to this question is to compare the robustness of the connection between SUSY and dark matter under alternative well-motivated cosmological scenarios.

In most SUSY models the dark matter candidate is the neutralino, although there are other interesting possibilities as we have already mentioned in the previous section. The mass and annihilation cross section of the neutralino are determined by the SUSY model parameters. Adapted to a $\Lambda CDM$ cosmological history, the neutralino becomes a cold thermal relic, freezing out during a radiation-dominated era at a temperature equal to approximately $1/20$th of its mass. SUSY model parameters thus determine the current neutralino abundance, which is usually expressed as a fraction, $\Omega_\chi$, of the critical density.

On the other hand, if we assume that neutralinos constitute nearly all of the non-baryonic matter at late times, then their abundance is quite constrained by astrophysical data. For example, combining the WMAP three year data [4] with Sloan Digital Sky Survey data on large scale structure, one obtains the limits:

$$0.095 < \Omega_\chi h^2 < 0.122 .$$

We consider a SUSY model with the neutralino as the LSP. The most popular example is the mSUGRA scenario, where the SUSY parameter set is
written \( m_0, m_{1/2}, A, \tan \beta \) and \( \text{sign}(\mu) \). Imposing very rough considerations of naturalness, we can require \( m_0, m_{1/2} \) and \( |A| \) to be less than 3 TeV. If we assume \( \Lambda CDM \) cosmology we can scan over this parameter space, placing a mark at each point which would predict a relic density satisfying the ‘WMAP’ constraints (5.1). We, also, only plot points which satisfy the existing experimental bounds on the superpartner particles and the Higgs. The results are shown in Figure 5.1, where we have chosen \( A = 0, \mu > 0, \tan \beta = 5 \) and \( m_{top} = 171.4 \) GeV. As we can see, the ‘WMAP-allowed’ regions are a very small fraction of the total parameter space. More significantly, once we impose experimental bounds the WMAP-allowed points are not generic; for a Bino-like neutralino generic points predict a relic density that is much too large, while for a Higgsino-like or Wino-like neutralino generic points predict a relic density that is much too small. Getting a WMAP-allowed relic density thus requires tuning; either enhancing the neutralino annihilation (or co-annihilation) cross section, or balancing the Bino-Higgsino-Wino content of the LSP. Since tuning is the opposite of naturalness, such an analysis casts doubt upon the mSUGRA, and perhaps SUSY in general, as a natural explanation of dark matter.
5.1. A NEW COSMOLOGICAL BENCHMARK

This Neutralino Tuning Problem could be a hint of nonstandard cosmology during and/or after the Terascale era. To quantify this possibility, we introduce an alternative cosmological benchmark based upon a simple model of quintessential inflation. This benchmark has no free parameters, so for a given supersymmetry model it allows an unambiguous prediction of the dark matter relic density. The dark matter predictions of SUSY can then be compared for this cosmological benchmark scenario.

5.1 A new cosmological benchmark

As an alternative cosmological benchmark, we propose the Slinky model of quintessential inflation described in section 3.7.1. This model has no adjustable parameters once we require that:

I. The current radiation and dark energy fractions take their WMAP-preferred values.\(^1\)

II. The universe is overwhelming radiation-dominated during the time of BBN.

Since the same inflaton is responsible for primordial inflation and for dark energy, it is not surprising that the universe is not completely radiation-dominated at the time that neutralinos freeze out. Also at that time, the Hubble expansion rate \(H\) differs from what it would be in standard \(\Lambda CDM\), since the universe is near the end of an epoch of accelerated expansion. Furthermore, between the time that the neutralinos freeze out and BBN, there is a significant entropy increase due to the inflaton coupling to radiation. Therefore, by making the minimal assumption that *inflatons do not decay to neutralinos*, we have a picture in which thermally produced neutralinos are diluted by a predictable amount, over and above the dilution of standard \(\Lambda CDM\). This additional dilution of the neutralino abundance can be expressed (to an accuracy of a few percent) as a polynomial function of the square root of the neutralino freezeout temperature [64]. Thus \(\Omega_\chi(Slinky) = \gamma\Omega_\chi(\text{standard})/\gamma\), where \(\gamma\) is the Slinky dilution factor. From

\(^1\)The coefficient controlling the coupling of the inflaton to radiation \((k_0)\), is always small. This weak coupling is fixed by requiring that the present day ratio of radiation density to quintessence takes the WMAP-preferred value.
a numerical analysis, we have found a heuristic formula for $\gamma$ as a function of $X_F$:

$$\gamma = 0.6 + 0.5\sqrt{X_F} - 0.06X_F + 0.01X_F^2. \quad (5.2)$$

This formula is accurate to within a few percent. The exact solution can be obtained by calculating the entropy production through numerically solving the evolution equations for the inflaton, radiation and matter, i.e.

$$\dot{\varrho}_\theta = -3H(1 + w)\varrho_\theta - k_0m_\phi(1 + w)\varrho_\theta, \quad (5.3)$$

$$\dot{\varrho}_r = -4H\varrho_r + k_0H(1 + w)\varrho_\theta,$$

$$\dot{\varrho}_m = -3H\varrho_m \quad (5.5)$$

where $H$ is the Hubble rate, $w(t)$ is the inflaton equation of state parameter and $\varrho_\theta$, $\varrho_r$, $\varrho_m$ are the energy density of the inflaton, radiation and matter, respectively.

Thus for any SUSY model we can compute the predicted neutralino relic abundance for two contrasting cosmological benchmarks, $\Lambda CDM$ and Slinky. We will see that for the Slinky benchmark the WMAP-allowed points are more generic, without balanced mixings, mass degeneracies, or resonance-inducing mass relations.

### 5.2 Computing Neutralino Abundance

In section 4.3 we have presented the standard technique for calculating the relic density of the lightest supersymmetric particle in the mSUGRA scenario. We now follow Gondolo and Gelmini [65, 66] to put Eq. 4.8 in a more convenient form by considering the ratio of the number density to the entropy density:

$$Y = \frac{n}{s}. \quad (5.6)$$

Therefore, with the assumption that neutralinos are thermal relics, their abundance $Y(X)$ can be computed as a function of $X = T/m_\chi$, the ratio of the temperature of the thermal bath divided by the mass of the LSP. The evolution equation for $Y(X)$ is:

$$\frac{dY}{dX} = \frac{m_\chi}{X^2}\sqrt{\frac{\pi g_*(m_\chi/X)}{45}} M_p <\sigma v> (Y_{eq}(X)^2 - Y(X)^2) \quad (5.7)$$

where $g_*$ is an effective number of degree of freedom, $M_p$ is the Planck mass, $Y_{eq}(X)$ the thermal equilibrium abundance, and $<\sigma v>$ is the relativistic
5.2. COMPUTING NEUTRALINO ABUNDANCE

thermally averaged annihilation cross section. The SUSY model determines the cross section, summing over the relevant annihilation and co-annihilation channels. We have used micrOMEGAs 2.0, which solves Eq. (5.7) numerically [67] and returns the predicted neutralino relic density for the standard cosmological model, $\Lambda CDM$, as $\Omega_\chi h^2$. We have modified the program in order to also return the $\Omega_\chi(Slinky)$. In addition returns the freeze-out value of $X$, called $X_F$, which is roughly equal to 1/20. The program uses SUSPECT 2.3 to compute the SUSY spectrum from mSUGRA input parameters [68, 69]. We repeat the procedure but we now use ISAJET instead of SUSPECT.

We bound the parameter space by requiring $m_0$, $m_{1/2}$ and $|A|$ to be less than, say, 3 TeV. We remove regions in which the lightest neutralino is not the LSP, or in which we do not get proper electroweak symmetry breaking. We also remove regions which are in direct conflict with experiment. For most of the MSSM parameter space, the most restrictive experimental constraint is the direct lower bound on the lightest Higgs mass from LEP. We will conservatively write this bound as $m_h \geq 114\,\text{GeV}$. We fix the mass of the top quark to the new combined Tevatron average [70] of 171.4 GeV.
Figure 5.3: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 10$. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

5.3 Results

The results of the mSUGRA scans by using SUSPECT 2.3, are shown in Figures (5.2 – 5.5), for $A = 0$, $\mu > 0$, and various values of $\tan \beta$. The blue points predict neutralino relic densities which satisfy the WMAP bounds (5.1). The red points predict neutralino relic densities which satisfy the WMAP bounds when combined with the Slinky dilution factor as given by (5.2).

In Figures (5.2), (5.3) and (5.4) the lower slivers of WMAP-allowed points, for the $\Lambda CDM$ as for the Slinky case, are made possible by tuning the lightest stau to be nearly degenerate with the lightest neutralino, producing co-annihilation at the time of freeze-out. The upper sliver in Figures (5.3) and (5.4) is a ‘focus point’ region, where we tune to give the LSP a significant Higgsino component. This allows neutralino pairs to annihilate more efficiently through t-channel chargino exchange into dibosons. The focus point region does not show up in Figure (5.2) because it produces only points with $m_h < 114$ GeV. Apart from this, shifting $\tan \beta$ from 5 to 30 does not make a qualitative difference.

For a window of very large $\tan \beta$ we obtain results as in Figure (5.5).
5.3. RESULTS

Figure 5.4: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 30$. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

Figure 5.5: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 50$. The scan was performed in 10 GeV increments, using SUSPECT 2.3.
Here, assuming $\Lambda CDM$ cosmological history, the focus point region has expanded, the co-annihilation sliver remains a sliver, and a new region, the ‘A-funnel’, has opened up. In this region we are tuning $m_A \simeq 2m_{LSP}$, allowing neutralinos to annihilate efficiently through an s-channel resonance. For the Slinky model the focus point is excluded, the co-annihilation sliver has expanded and a funnel region has, as in the $\Lambda CDM$, opened up.

If we now use ISAJET instead of SUSPECT, apart from $\tan \beta = 50$, does not make a qualitative difference. This can easily be concluded from the figures below (5.6 – 5.8). In Figure (5.9) we see that for the standard model the ‘A-funnel’ is is excluded but for the Slinky the focus point remains and the ‘A-funnel’, has opened up. This is in contrast with the SUSPECT analysis for $\tan \beta = 50$.

From a detailed comparison of the particle spectra, we find that the WMAP-allowed points in the Slinky case are more generic. This is not obvious from the figures, since except for the case of $\tan \beta = 50$ the red regions are fairly close to the blue regions where stau co-annihilation or Bino-Higgsino mixing is occurring. Looking at the spectra, we find that the red regions do not involve the kind of balanced mixings, mass degeneracies, or resonance-inducing mass relations that characterize the blue regions.

In order to see what the parameter space looks like if the mass of the top differs slightly, we repeat the procedure fixing the mass of the top quark at 172, 4 GeV and 174 GeV. As we see from the figures (5.10 – 5.17) there is no important difference except for $\tan \beta = 50$ where new regions are opened up and $\tan \beta = 10$ where the ‘focus’ point is contracted.
5.3. RESULTS

Figure 5.6: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 5$. The scan was performed in 10 GeV increments, using ISAJET.

Figure 5.7: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 10$. The scan was performed in 10 GeV increments, using ISAJET.
Figure 5.8: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 30$. The scan was performed in 10 GeV increments, using ISAJET.

Figure 5.9: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, and $\tan \beta = 50$. The scan was performed in 10 GeV increments, using ISAJET.
Figure 5.10: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 5$ and $m_{\text{top}} = 172.4$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

Figure 5.11: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 10$ $m_{\text{top}} = 172.4$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.
CHAPTER 5. COSMOLOGY EFFECTS ON THE SUSY PARAMETER SPACE

Figure 5.12: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 30$ $m_{top} = 172.4$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

Figure 5.13: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 50$ $m_{top} = 172.4$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.
Figure 5.14: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 5$ $m_{\text{top}} = 174$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

Figure 5.15: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 10$ $m_{\text{top}} = 174$ GeV. The scan was performed in 10 GeV increments, using SUSPECT 2.3.
Figure 5.16: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 30$ $m_{\text{top}} = 174 \text{ GeV}$. The scan was performed in 10 GeV increments, using SUSPECT 2.3.

Figure 5.17: The WMAP-allowed points for the mSUGRA, with $A = 0$, $\mu > 0$, $\tan \beta = 50$ $m_{\text{top}} = 174 \text{ GeV}$. The scan was performed in 10 GeV increments, using SUSPECT 2.3.
5.4 Conclusion

In this thesis we have reviewed the dark matter problem of SM. We have seen that in supersymmetric theories, the lightest superpartner (LSP) can be stable. This stable superpartner provides a nice cold dark matter candidate. If we suppose that by making the previous assumption our problem is somehow solved, then we face a new problem! The Neutralino Tuning Problem of the MSSM, as we have presented it in the beginning of this chapter. We have quantified this new problem by an explicit comparison of the mSUGRA for two different cosmological benchmark scenarios: standard $\Lambda CDM$ and Slinky. The comparison shows that the WMAP-allowed region is an order of magnitude larger if we assume the modified Slinky cosmology rather than the standard cosmology. Areas that have been restricted from the standard cosmological model now open-up. In addition, the Slinky spectra do not exhibit the mass degeneracies, or resonance-inducing mass relations that occur in the standard case. Thus it should be possible to discriminate between these spectra using initial data runs from the LHC.

We expect that in a different cosmological benchmark with a larger dilution factor would look even more natural than Slinky. This opportunity deserves detailed study.

If we now suppose that experiments at the LHC do discover a spectrum of new particles consistent with the CMSSM, but not consistent with the WMAP upper bound as derived assuming a neutralino LSP and standard or slinky cosmology. Then we will face the interesting question: Did we constrain SUSY from Cosmology?
Appendix A

Electroweak Symmetry Breaking

Deciphering the mechanism that breaks the electroweak symmetry and generates the masses of the known fundamental particles is one of the central challenges of particle physics. Two broad classes of electroweak symmetry breaking mechanisms have been pursued theoretically. In one class of theories, electroweak symmetry breaking dynamics is weakly-coupled, while in the second class of theories the dynamics is strongly-coupled.

The electroweak symmetry breaking dynamics that is employed by the Standard Model posits a self-interacting complex doublet of scalar fields, which consists of four real degrees of freedom. Renormalizable interactions are arranged in such a way that the neutral component of the scalar doublet acquires a vacuum expectation value, $v = 246 \text{ GeV}$, which sets the scale of electroweak symmetry breaking. Consequently, three massless Goldstone bosons are generated, while the fourth scalar degree of freedom that remains in the physical spectrum is the CP-even neutral Higgs boson ($h_{SM}$) of the Standard Model. This approach, which we have in detail explained in section 1.4, is an example of weak electroweak symmetry breaking. Assuming that $m_{h_{SM}} \leq 200 \text{ GeV}$, all fields remain weakly interacting at energies up to the Planck scale. In the weakly-coupled approach to electroweak symmetry breaking, the Standard Model is very likely embedded in a supersymmetric theory [71] in order to stabilize the large gap between the electroweak and the Planck scales in a natural way [72, 73]. These theories predict a spectrum of Higgs scalars, with the properties of the lightest Higgs scalar often resembling that of the Standard Model (SM) Higgs boson.
Alternatively, strong breaking of electroweak symmetry is accomplished by new strong interactions near the TeV scale \[74\]. More recently, so-called ‘little Higgs models’ have been proposed in which the scale of the new strong interactions is pushed up above 10 TeV \[75\], and the lightest Higgs scalar resembles the weakly-coupled SM Higgs boson. In a more speculative direction, a new approach to electroweak symmetry breaking has been explored in which extra space dimensions beyond the usual 3 + 1 dimensional spacetime are introduced \[76\] with characteristic sizes of order \((\text{TeV})^{-1}\). In such scenarios, the mechanisms for electroweak symmetry breaking are inherently extra-dimensional, and the resulting phenomenology may be significantly different from the usual approaches mentioned above.

Although there is as yet no direct evidence for the nature of electroweak symmetry breaking dynamics, present data can be used to discriminate among the different approaches. For example, precision electroweak data, accumulated in the past decade at LEP, SLC, the Tevatron and elsewhere, strongly support the Standard Model with a weakly-coupled Higgs boson, for a recent review, see [http://lepewwg.web.cern.ch/LEPEWWG/]. Moreover, the contribution of new physics, which can enter through \(W^\pm\) and \(Z\) boson vacuum polarization corrections, is severely constrained. This fact has already served to rule out several models of strongly-coupled electroweak symmetry breaking dynamics. The Higgs boson contributes to the \(W^\pm\) and \(Z\) boson vacuum polarization through loop effects, and so a global Standard Model fit to the electroweak data yields information about the Higgs mass. The results of the LEP Electroweak Working Group analysis provide a 95% CL upper limit of \(m_{h_{SM}} < 193 \text{ GeV}\). These results reflect the logarithmic sensitivity to the Higgs mass via the virtual Higgs loop contributions to the various electroweak observables. The 95% CL upper limit is consistent with the direct searches at LEP that show no conclusive evidence for the Higgs boson, and imply that \(m_{h_{SM}} > 114.4 \text{ GeV}\) at 95% CL, for additional updates see at [http://lephiggs.web.cern.ch/LEPHIGGS/www/Welcome.html].

Henceforth, we shall assume that the dynamics of electroweak symmetry breaking is a result of a weakly-coupled scalar sector. The Standard Model is an effective field theory and provides a very good description of the physics of elementary particles and their interactions at an energy scale of \(\mathcal{O}(100) \text{ GeV}\) and below. However, there must exist some energy scale, \(\Lambda\), at which the Standard Model breaks down. That is, the Standard Model is no longer adequate for describing the theory above \(\Lambda\), and degrees of freedom associated with new physics become relevant. In particular, we know that \(\Lambda \leq M_{Pl}\),
since at an energy scale above the Planck scale, $M_{Pl} = 10^{19}$ GeV, quantum gravitational effects become significant and the Standard Model must be replaced by a more fundamental theory that incorporates gravity. (Similar conclusions also apply to recently proposed extra-dimensional theories in which quantum gravitational effects can become significant at energies scales as low as $\mathcal{O}(1 \text{ TeV})$.) Of course, it is possible that new physics beyond the Standard Model exists at an energy scale between the electroweak and Planck scale, in which case the value of $\Lambda$ might lie significantly below $M_{Pl}$.

The value of the Higgs mass itself can provide an important constraint on the value of $\Lambda$. If $m_{h_{SM}}$ is too large, then the Higgs self-coupling blows up at some scale $\Lambda$ below the Planck scale [77]. If $m_{h_{SM}}$ is too small, then the Higgs potential develops a second (global) minimum at a large value of the scalar field of order $\Lambda$. Thus new physics must enter at a scale $\Lambda$ or below in order that the global minimum of the theory correspond to the observed $SU(2) \times U(1)$ broken vacuum with $v = 246$ GeV. Thus, given a value of $\Lambda$, one can compute the minimum and maximum Higgs mass allowed. Consequently, a Higgs mass range $130 \text{ GeV} \lesssim m_{h_{SM}} \lesssim 180 \text{ GeV}$ is consistent with an effective Standard Model that survives all the way to the Planck scale.

However, the survival of the Standard Model as an effective theory all the way up to the Planck scale is unlikely based on the following naturalness argument. In an effective field theory, particle masses and dimensionless couplings of the low-energy theory are calculable in terms of parameters of a more fundamental theory that describes physics at the energy scale $\Lambda$. All scalar squared-masses are quadratically sensitive to $\Lambda$. Thus, in this framework, the observed Higgs mass has the following form:

$$m_{h_{SM}}^2(p^2) = m^2(\Lambda^2) + Cg^2 \int_{p^2}^{\Lambda^2} dk^2 + ... ,$$  

(A.1)

where $\Lambda$ defines a reference scale at which the value of $m^2$ is known, $g$ is the coupling constant of the theory, and the coefficient $C$ is calculable in any particular theory. Instead of dealing with the relationship between observables and parameters of the Lagrangian, we choose to describe the variation of an observable with the momentum scale. In order for the mass shifts induced by radiative corrections to remain under control (i.e., not to greatly exceed the value measured on the laboratory scale), either $\Lambda$ must be small, so the range of integration is not enormous, or new physics must intervene to cut off the integral.
If the fundamental interactions are described by an \( SU(3)_c \times SU(2)_L \times U(1)_Y \) gauge symmetry, i.e., by quantum chromodynamics and the electroweak theory, then the natural reference scale is the Planck mass,

\[
\Lambda \sim M_{\text{Planck}} = \left( \frac{\hbar c}{G_{\text{Newton}}} \right)^{1/2} \approx 1.22 \times 10^{19} \text{ GeV} \quad (A.2)
\]

In a unified theory of the strong, weak, and electromagnetic interactions, the natural scale is the unification scale,

\[
\Lambda \sim U \approx 10^{15} - 10^{16} \text{ GeV}. \quad (A.3)
\]

Both estimates are very large compared to the scale of electroweak symmetry breaking: \( v = \sqrt{-\mu^2/|\lambda|} \equiv (G_F\sqrt{2})^{-1/2} \approx 246 \text{ GeV} \). We are therefore assured that new physics must intervene at an energy of approximately 1 TeV, in order that the shifts in \( m_{h_{SM}}^2 \) not be much larger than

\[
\langle \phi \rangle_0 = \sqrt{-\mu^2/2|\lambda|} \equiv (G_F\sqrt{8})^{-1/2} \approx 174 \text{ GeV}. \quad (A.4)
\]

A viable theoretical framework that incorporates weakly-coupled Higgs bosons is that of Supersymmetry. As discussed in Appendix B, supersymmetry breaking effects, whose origins may lie at energy scales much larger than 1 TeV, can induce a radiative breaking of the electroweak symmetry due to the effects of the large Higgs-top quark Yukawa coupling. In this way, the origin of the electroweak symmetry breaking scale is intimately tied to the mechanism of supersymmetry breaking. Thus, supersymmetry provides an explanation for the stability of the hierarchy of scales, provided that supersymmetry-breaking masses in the low-energy effective electroweak theory are of \( O(1 \text{ TeV}) \) or less. Therefore, if we incorporate Supersymmetry in our theory we will have a new physics at an energy of approximately 1 TeV with a symmetry that protects the mass of the Higgs from quadratic divergences.
Appendix B

MSSM

The MSSM superpotential is given by

\[ f_{\text{MSSM}} = \epsilon_{\alpha\beta} \left[ H_\alpha^a L_i^b Y_{ij}^c E_j^c + H_u^a Q_i^b Y_{ij}^c D_j^c - H_u^a Q_i^b Y_{ij}^c U_j^c - \mu H_d^a H_u^a \right] \]  

(B.1)  

In the above expression, i and j are family indices, while \( \alpha \) and \( \beta \) are \( SU(2)_L \) doublet indices (the color indices are suppressed). \( \epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha} \) and \( \epsilon_{12} = +1 \). The Yukawa couplings \( Y \) are general \( 3 \times 3 \) matrices. The superpotential terms include the Yukawa couplings of the quarks and leptons to the Higgs doublets, as well as a mass term which couples \( H_u \) to \( H_d \). The superpotential and gauge couplings thus dictate the couplings of the Higgs potential of the theory. This would appear to reduce the number of independent parameters of the MSSM; for example, the treelevel Higgs quadric couplings are fixed by supersymmetry to be gauge couplings rather than arbitrary couplings as in the SM. However, the phenomenological requirement of supersymmetry breaking terms in the Lagrangian introduces many new parameters, which play crucial roles in the phenomenology of the model.

The soft supersymmetry-breaking Lagrangian \( \mathcal{L}_{\text{soft}} \) takes the form (dropping ‘helicity’ indices):

\[
- \mathcal{L}_{\text{soft}} = \frac{1}{2} \left[ M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} \right] + \epsilon_{\alpha\beta} \left[ -b H_u^a H_u^a - H_u^a \tilde{Q}_i^a \tilde{A}_{aij} \tilde{U}_j^c + H_u^a \tilde{Q}_i^a \tilde{A}_{aij} \tilde{D}_j^c + H_u^a \tilde{L}_i^a \tilde{A}_{aij} \tilde{E}_j^c + \text{h.c.} \right] + m_{H_u}^2 |H_u|^2 + m_{H_u}^2 |H_u|^2 + \tilde{Q}_i^a m_{\tilde{Q}_i} \tilde{Q}_j^a \]  

(B.2)  

\[
+ \tilde{L}_i^a m_{\tilde{L}_i} \tilde{L}_j^a + \tilde{U}_i^a m_{\tilde{U}_i} \tilde{U}_j^a + \tilde{D}_i^a m_{\tilde{D}_i} \tilde{D}_j^a + \tilde{E}_i^a m_{\tilde{E}_i} \tilde{E}_j^a.
\]

The \( SU(2) \) representations of the squark, slepton, and Higgs doublets were defined in Sec. 2.3.

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B.1 The Higgs Sector of the MSSM

The Higgs sector of the MSSM differs from that of the SM (apart from the presence of superpartners, the spin 1/2 higgsinos). The SM Higgs sector consists of a single doublet \( h \) which couples to all of the chiral matter. In the MSSM, two complex Higgs doublets \( H_u \) and \( H_d \), which couple at tree level to up and down type chiral fermions separately, are required. The need for two Higgs doublets can be seen from the holomorphic property of the superpotential: couplings involving \( h^* \), necessary in the SM for the up-type quark Yukawa couplings, are not allowed by supersymmetry. Two Higgs doublets are also required for the model to be anomaly free. Since the chiral fermion content of the theory includes the higgsinos, anomaly constraints require that the Higgs sector be vectorlike, i.e., that the two Higgs doublets have opposite hypercharges. Thus, the MSSM contains the particle spectrum of a two-Higgs-doublet extension of the Standard Model and the corresponding supersymmetric partners. The two-doublet Higgs sector contains eight scalar degrees of freedom:

\[
\begin{align*}
\text{one complex } Y = -1 \text{ doublet:} & \quad H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix}, \\
\text{and one complex } Y = +1 \text{ doublet:} & \quad H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix}.
\end{align*}
\]

(B.3)

(B.4)

The notation reflects the form of the MSSM Higgs sector coupling to fermions.

The tree-level scalar potential for the two Higgs doublets is a sum of \( F \) terms, \( D \) terms, and soft supersymmetry-breaking terms:

\[
V_{\text{higgs}} = (|\mu|^2 + m_{H_u}^2)|H_u^a|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^a|^2 \right. \\
+ \frac{1}{8}(g^2 + g'^2)(|H_u^a|^2 - |H_d^a|^2)^2 + \frac{1}{2}g^2|H_u^aH_d^{a*}|^2 \\
- (\epsilon_{ab}BH_a^aH_u^b + h.c.),
\]

(B.5)

in which \( g \) is the \( SU(2)_L \) gauge coupling and \( g' \) is the hypercharge gauge coupling. Electroweak symmetry breaking requires that the parameters of this potential must take on correlated values, such that the potential is minimized with nonzero VEVs for the neutral components of the Higgs doublets:

\[
\langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix}, \quad \langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix},
\]

(B.6)
in which \( v_d^2 + v_u^2 = v^2 \), \( v = 246 \text{GeV} = 2m_Z/\sqrt{g^2 + g'^2} \), and \( \tan \beta = v_u/v_d \).

It is always possible by \( SU(2)_L \) gauge transformations to set the vacuum expectation values of the charged Higgs components to zero. Furthermore, we can see that in this tree-level potential it is always possible to choose global phases of the Higgs fields to eliminate any complex phase in the B parameter, such that \( v_{u,d} \) can be chosen real and positive. CP symmetry is thus not broken at tree level and the Higgs mass eigenstates have definite CP quantum numbers.

Spontaneous electroweak symmetry breaking results in three Goldstone bosons, which are absorbed and become the longitudinal components of the \( W^\pm \) and \( Z \). As the two Higgs doublets each contain 4 real degrees of freedom and 3 generators are broken when \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \), there are 5 physical Higgs bosons. The physical spectrum of Higgs bosons includes 3 neutral Higgs bosons (the CP-even \( h \), \( H \) and CP-odd \( A \)) and 1 charged Higgs (\( H^\pm \)):

\[
H^\pm = H_d^\pm \sin \beta + H_u^\pm \cos \beta, \tag{B.7}
\]

one CP-odd pseudoscalar

\[
A = \sqrt{2} (\text{Im} H_d^0 \sin \beta + \text{Im} H_u^0 \cos \beta), \tag{B.8}
\]

and two CP-even scalars

\[
\begin{align*}
    h &= -(\sqrt{2} \text{Re} H_d^0 - v_d) \sin \alpha + (\sqrt{2} \text{Re} H_u^0 - v_u) \cos \alpha, \\
    H &= (\sqrt{2} \text{Re} H_d^0 - v_d) \cos \alpha + (\sqrt{2} \text{Re} H_u^0 - v_u) \sin \alpha, \tag{B.9}
\end{align*}
\]

(with \( m_h \leq m_H \)). The angle \( \alpha \) arises when the CP-even Higgs squared-mass matrix (in the \( H_d^0 - H_u^0 \) basis) is diagonalized to obtain the physical CP-even Higgs states.

The supersymmetric structure of the theory imposes constraints on the Higgs sector of the model. For example, the Higgs self-interactions are not independent parameters; they can be expressed in terms of the electroweak gauge coupling constants. As a result, all Higgs sector parameters at tree-level are determined by two free parameters: the ratio of the two neutral Higgs field vacuum expectation values, \( \tan \beta \), and one Higgs mass, conveniently chosen to be \( m_A \) (the mass of the CP-odd Higgs boson). In particular, the masses of the other Higgs bosons are given by

\[
m_{H^\pm}^2 = m_A^2 + m_W^2 \tag{B.10}
\]
and the CP-even Higgs bosons \( h \) and \( H \) are eigenstates of the following squared-mass matrix

\[
\mathcal{M}_0^2 = \begin{pmatrix}
  m_A^2 \sin^2 \beta + m_Z^2 \cos^2 \beta & -(m_A^2 + m_Z^2) \sin \beta \cos \beta \\
  -(m_A^2 + m_Z^2) \sin \beta \cos \beta & m_A^2 \cos^2 \beta + m_Z^2 \sin^2 \beta
\end{pmatrix}.
\] (B.11)

The eigenvalues of \( \mathcal{M}_0^2 \) are the squared-masses of the two CP-even Higgs scalars

\[
m_{H,h}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 \pm \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2m_Z^2 \cos^2(2\beta)} \right].
\] (B.12)

This leads to the important upper bound

\[
m_h \leq m_Z |\cos(2\beta)| \leq m_Z.
\] (B.13)

The MSSM seems to predict that one of the neutral Higgs scalars must be lighter than the \( Z \) boson! This is in marked contrast to the Standard Model, in which the theory does not constrain the value of \( m_{h_{SM}} \) at tree-level. The origin of this strong bound can be traced back to the fact that the only Higgs self couplings in eq.(B.5) are electroweak gauge couplings. In contrast, in the Standard Model, \( m_{h_{SM}}^2 = \frac{1}{2} \lambda v^2 \) is proportional to the Higgs self-coupling \( \lambda \), which is a free parameter. See e.g. the review [78] for further details of the Higgs mass spectrum at tree-level and higher-loop order.

### B.2 Radiative Electroweak Symmetry Breaking

Arguably the most important success of supersymmetry is that it can provide a natural mechanism for understanding Higgs physics and electroweak symmetry breaking. Therefore, this section is devoted to a basic explanation of this mechanism. The main result is that this mechanism requires basic correlations among the Higgs soft supersymmetry-breaking parameters and the supersymmetric Higgs mass parameter \( \mu \), which leads naturally into a discussion of the \( \mu \) problem of the MSSM.

After replacing the Higgs doublets in the potential Eq.(B.5) by their VEVs, the potential takes the form

\[
V_{\text{higgs}} = (|\mu|^2 + m_{H_u}^2)v_u^2 + (|\mu|^2 + m_{H_d}^2)v_d^2 - 2Bv_u v_d + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)^2.
\] (B.14)
Let us now consider the phenomenologically viable situation in which the soft supersymmetry-breaking terms $m_{H_u}^2$, $m_{H_d}^2$, $B$ and $\mu$ are nonzero. The minimum of the potential must break $SU(2)_L \times U(1)_Y$; i.e., the minimum of the potential should not occur for $v_{u,d} = 0$. This leads to the condition

$$ (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2) < B^2. \quad (B.15) $$

The potential must be also bounded from below along $D$ flat directions (i.e., with vanishing $D$ terms), yielding the constraint

$$ 2|\mu|^2 + m_{H_d}^2 + m_{H_u}^2 \geq 2|B|. \quad (B.16) $$

The minimization conditions for this potential are as follows:

$$ |\mu|^2 + m_{H_d}^2 = B \tan \beta - \frac{m_Z^2}{2} \cos 2\beta \quad (B.17) $$

$$ |\mu|^2 + m_{H_u}^2 = B \cot \beta + \frac{m_Z^2}{2} \cos 2\beta. \quad (B.18) $$

The minimization conditions demonstrate explicitly that the soft parameters $m_{H_u}^2$, $m_{H_d}^2$, $B$ and the supersymmetric parameter $\mu$ all must be of approximately the same order of magnitude as $m_Z$ for the electroweak symmetry breaking to occur in a natural manner, i.e. without requiring large cancellations.

The minimization conditions for an $SU(2)_L \times U(1)_Y$ breaking vacuum suggest that one or both of the Higgs doublets has a negative mass-squared at $v_d = v_u = 0$, like the negative mass-squared in the SM. Nevertheless, a celebrated feature of the MSSM is that the up-type Higgs soft mass-squared parameter does get driven negative via renormalization group running due to the large top quark Yukawa coupling. This can be seen upon an inspection of the renormalization group equations for the relevant soft parameters. Retaining only the top quark Yukawa coupling, one can see that the $m_{H_u}^2$ parameter is driven down by the large top Yukawa terms as one runs down from the high scale to the low scale. In the large $\tan \beta$ regime in which the bottom and tau Yukawas are also large, there is a similar effect for $m_{H_d}^2$. Other masses such as the stop mass-squared parameters also are driven down by the Yukawa terms; however, they also receive large positive contributions from gluino loops, so they do not usually run negative, although they can. Therefore, the Higgs soft mass-squared parameters can be driven to negative values near the electroweak scale due to perturbative logarithmic running.
The running of the Higgs masses leads to the phenomenon known as \textit{radiative electroweak symmetry breaking}. By this we mean the following: at the GUT energy scale both the Higgs soft mass parameters are positive, and the Higgs potential has no nontrivial minima. However, when running down to the EW scale due to the radiative corrections they may change the sign so that the potential develops a nontrivial minimum. At this minimum the electroweak symmetry happens to be spontaneously broken. The vacuum expectations of the Higgs fields acquire nonzero values and provide masses to quarks, leptons and $SU(2)$ gauge bosons, and additional masses to their superpartners. Thus, contrary to the SM, where one has to choose the negative sign of the Higgs mass squared ‘by hand’, in the MSSM the effect of spontaneous symmetry breaking is triggered by the radiative corrections.

\section*{B.3 The $\mu$ Problem}

Electroweak symmetry breaking can thus take place in a natural way in the MSSM via a radiative mechanism by which the soft mass-squared parameter of the up-type Higgs doublet (and also that of the down-type Higgs when $\tan \beta$ is large) approaches or becomes zero, provided that $\mu$ and $b$ are nonzero and take values roughly of the same order as $m_Z$. To see this correlation let us demonstrate it explicitly for the $\mu$ parameter. Rewriting the minimization conditions yields the following expression:

$$
\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \frac{1}{2} m_Z^2.
$$

(B.19)

This correlation leads to a puzzle. Just as we are ignorant of the origin and dynamical mechanism of supersymmetry breaking, we do not know why the supersymmetric mass parameter $\mu$ should be of the order of the electroweak scale, and of the same order as the supersymmetry breaking parameters (or else there would be a chargino lighter than the $W$ boson, which has been excluded experimentally). Given that $\mu$ is a superpotential parameter one might expect $\mu \sim \mathcal{O}(M_X)$, where $M_X$ is a high scale, \textit{e.g.} the unification or GUT scale. If this were true, the hierarchy problem is clearly restored. This puzzle, known as the $\mu$ \textit{problem}, was first pointed out in [79], for further analysis see ref. [80]
B.4 Gaugino Masses and Mixings

- **Charginos**: The charginos are linear combinations of the charged gauge bosons $\tilde{W}^\pm$ and of the charged Higgsinos $\tilde{H}_u^+, \tilde{H}_d^-$. Their mass terms are given by

$$\begin{pmatrix} \tilde{W}^- & \tilde{H}_d^- \end{pmatrix} M_{\tilde{\chi}^\pm} \begin{pmatrix} \tilde{W}^+ \\ \tilde{H}_u^+ \end{pmatrix} + \text{H.c.}$$

(B.20)

Their mass matrix

$$M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & gv_u \\ gv_d & \mu \end{pmatrix}$$

(B.21)

is diagonalized by the linear combinations

$$\begin{align*}
\tilde{\chi}_i^- &= U_{i1} \tilde{W}^- + U_{i2} \tilde{H}_d^- , \\
\tilde{\chi}_i^+ &= V_{i1} \tilde{W}^+ + V_{i2} \tilde{H}_u^+ .
\end{align*}$$

(B.22)

We choose $\det(U) = 1$ and $U^* M_{\tilde{\chi}^\pm} V = \text{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm})$ with non-negative chargino masses $m_{\tilde{\chi}_i^\pm} \geq 0$. We do not include any one-loop corrections to the chargino masses since they are negligible compared to the corrections introduced below for the neutralino masses.

- **Neutralinos**: In the MSSM the neutral gauginos $\tilde{B}$, $\tilde{W}_3$ and the neutral components of the higgsinos $\tilde{H}_d^0$, $\tilde{H}_u^0$ have the same quantum numbers and, therefore, mix into four mass eigenstates, $\tilde{\chi}_i^0$, called neutralinos. The neutralino mass matrix in the $\tilde{B} - \tilde{W}_3 - \tilde{H}_d^0 - \tilde{H}_u^0$ basis is given by

$$M_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -M_2 \cos \beta \sin \theta_W & M_2 \sin \beta \sin \theta_W \\ 0 & M_2 & -M_2 \cos \beta \cos \theta_W & -M_2 \sin \beta \cos \theta_W \\ -M_2 \cos \beta \sin \theta_W & M_2 \cos \beta \cos \theta_W & M_2 \cos \beta \cos \theta_W & -M \sin \beta \sin \theta_W \\ M_2 \sin \beta \sin \theta_W & M_2 \cos \beta \cos \theta_W & -M_2 \sin \beta \cos \theta_W & M_2 \sin \beta \sin \theta_W \end{pmatrix} ,$$

where $M_1$, $M_2$ and $\mu$ are the bino, wino and higgsino mass parameters, respectively, $\theta_W$ is the Weinberg angle, $\delta_{33}$ and $\delta_{44}$ are the most important one-loop corrections. These can change the neutralino masses by a few GeV up or down and are only important when there is a severe mass degeneracy of the lightest neutralinos and/or charginos. This matrix can be diagonalized by the matrix, $N$

$$M_{\tilde{\chi}^0}^{\text{diag}} = N^\dagger M_{\tilde{\chi}^0} N ,$$

(B.24)

where $N$ is a $4 \times 4$ unitary matrix, to give four neutral Majorana states:

$$\tilde{\chi}_i^0 = N_{i1} \tilde{B} + N_{i2} \tilde{W}_3^\pm + N_{i3} \tilde{H}_u^0 + N_{i4} \tilde{H}_d^0 .$$

(B.25)
The lightest of which, to be called $\chi_1$, is then the candidate for the particle making up the dark matter in the Universe. The $N_{ij}$ are the mixture parameters. The gaugino fraction of $\chi_1$ is then defined as

$$f_g = N^2_{11} + N^2_{12}$$  \hspace{1cm} (B.26)

and its higgsino fraction as

$$f_H = N^2_{13} + N^2_{14}.$$  \hspace{1cm} (B.27)

We will call the neutralino gaugino-like if $f_g > 0.99$, Higgsino-like if $f_g < 0.01$ and mixed if $0.01 \leq f_g \leq 0.99$. Note that the boundaries for what we call gaugino-like and Higgsino-like are somewhat arbitrary and may differ from those of other authors.

## B.5 Neutralino Annihilations

In this section, we present the most important neutralino annihilation channels. For a more complete list, with all $S$ and $P$-wave tree level annihilation amplitudes, see Refs. [81, 82, 83, 84, 85].

\[ \to \text{Annihilation into fermions.} \] Neutralinos can annihilate to fermion pairs by four tree level diagrams. These processes consist of s-channel exchange of pseudoscalar Higgs and $Z^0$-bosons, u-channel exchange of sfermions and t-channel exchange of sfermions (see Fig. B.1).

\[ \to \text{Annihilation into gauge bosons.} \] Generally, neutralinos can annihilate into gauge boson pairs via several processes (see Fig. B.2). In the low velocity limit, however, only t-channel processes via chargino or neutralino exchange are non-vanishing.

\[ \to \text{Annihilation into Higgs bosons.} \] There are many tree level diagrams which contribute to neutralino annihilation into Higgs boson pairs or a Higgs boson and a gauge boson (see Fig. B.3).

Finally in the Table B.1 we present all Feynman diagrams for which we can calculate all two-body final state cross sections at the tree level for neutralino-neutralino annihilation. A complete list, which include neutralino-chargino, and chargino-chargino annihilation you can find in Ref. [30].
B.5. NEUTRALINO ANNIHILATIONS

Figure B.1: Tree level diagrams for neutralino annihilation into fermion pairs. From Ref. [82].

Figure B.2: Tree level diagrams for neutralino annihilation into gauge boson pairs. From Ref. [82].

Figure B.3: Tree level diagrams for neutralino annihilation into a Z and a Higgs boson. From Ref. [82].
Table B.1: All Feynman diagrams for which we calculate the annihilation cross section for neutralino-neutralino annihilation. $s(x), t(x),$ and $u(x)$ denote a tree-level Feynman diagram in which particle $x$ is exchanged in the $s, t,$ and $u$ channel, respectively. Indices $i, j, k$ run from 1 to 4, and indices $c, d, e$ from 1 to 2. A sum of diagrams over (s)fermion generation indices and over the neutralino indices $k$ and $e$ is understood (no sum over indices $i, j, c, d$).

<table>
<thead>
<tr>
<th>Initial state</th>
<th>Final state</th>
<th>Feynman diagrams</th>
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<td>$t(\chi_0^0), u(\chi_k^0), s(H_{1,2})$</td>
<td>$t(\chi_0^0), u(\chi_k^0), s(H_{1,2})$</td>
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<tr>
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<td>$t(\chi_0^0), u(\chi_k^0), s(Z_0)$</td>
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<td>$t(\chi_0^0), u(\chi_k^0), s(H_{1,2})$</td>
<td>$t(\chi_0^0), u(\chi_k^0), s(H_{1,2})$</td>
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<tr>
<td>$W^-H^+, W^+H^-$</td>
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<tr>
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</tr>
</tbody>
</table>
Bibliography


[70] E. Brubaker et al. Combination of CDF and D0 Results on the Mass of the Top Quark. 2006.


