

# Impatience, Indeterminacy, and Fiscal Policy for Equilibrium Selection

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## Abstract

This paper studies the role of fiscal policy as an equilibrium selection device under the presence of intertemporal complementarities. To this end, we assume that the individual rate of time preference is endogenously determined by the social level of education. We show how intertemporal complementarities of aggregate human capital can generate multiple equilibria and we examine the role of Ramsey second-best and growth-maximizing fiscal policies in equilibrium selection. We find that fiscal policy can resolve equilibrium indeterminacy because in a Stackelberg environment the government is equipped with additional information and adequate tools, namely the tax rate and the allocation of aggregate endowments, to carry out two tasks, equilibrium selection and welfare (or growth) maximization.

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# 1 Introduction

The literature on equilibrium indeterminacies is organized around the corresponding driving forces, typically characterized by increasing social returns (Caballero and Lyons, 1992; Benhabib and Farmer, 1994; Benhabib and Perli, 1994), monopolistic competition (Farmer and Guo, 1994; Gali, 1994; Evans et al., 1998), positive external effects that lead to constant social returns (Cazzavillan et al., 1998; Evans et al., 1998; Benhabib et al., 2000), and intertemporal complementarities under which dynamic interactions are propagated and magnified over time (Cooper and Haltiwanger, 1996). Although equilibrium indeterminacies have been widely studied in the literature, far less is known on mechanisms in directing the economy towards a desired equilibrium. In models with a continuum of equilibria arising from the presence of animal spirits, learning can act as a selection device for choosing the rational expectations equilibrium that we can expect to observe in practice (Evans et al., 1998; Evans and Honkapohja, 1999). Ennis and Keister (2005) have presented a framework in which search frictions create a coordination problem and generate multiple Pareto-ranked equilibria. The authors show that the desired equilibrium can be chosen by using an exogenously determined equilibrium selection mechanism based on risk dominance.

This paper focuses on the role of standard Ramsey second-best and growth-maximizing fiscal policies in equilibrium selection. We show how multiple equilibria are triggered by intertemporal complementarities of aggregate human capital, which affects private decisions through the endogeneity of time preference. We then introduce the government's objective as an equilibrium selection mechanism. In particular, we highlight the role of the government in selecting a stable equilibrium regime through the endogenous allocation of endowments and then using a feasible tax rate to attain welfare (or growth) maximization. Equilibrium indeterminacy is resolved because in a Stackelberg environment the government is equipped with additional information and adequate tools, namely the tax rate and the allocation of aggregate endowments, to carry out two tasks, equilibrium selection and welfare (or growth) maximization.

The building block of our model is the assumption that the individual rate of time preference

is largely determined by the publicly-provided level of the aggregate human capital stock in the economy. The main idea underlying the paper is that agents are less impatient in a more educated surrounding environment. This point goes back to Strotz (1956), who had noticed that discount functions are formed by teaching and social environment, and was re-raised by Becker and Mulligan (1997), who argued that schooling and other social activities in “future-oriented capital” focus agents’ attention to the future.<sup>1</sup>

We examine the equilibrium properties in the decentralized economy under the assumption that the provision of aggregate human capital, which is financed by a tax rate on output, affects the time preference rate of agents. Following standard literature, surveyed in the next section, we additionally assume that aggregate consumption increases the rate of time preference. We show that there can be one or two positive equilibrium growth rates depending on the parameters of the economy and on fiscal policy instruments, whereas global indeterminacy can be accompanied by local indeterminacy. The central mechanism that drives these results arises from two counterbalancing channels. First, a rise in human capital financed by an increase in the tax rate lowers the rate of time preference, causing savings to increase and as a result the economy can attain higher growth. This in turn increases the tax base, raises public expenditures on education and hence fuels further growth. On the other hand, the rise in taxation decreases private savings, which increases the rate of time preference in the economy due to the rise in aggregate consumption, and lowers growth. A lower growth rate in turn lowers the tax base that finances human capital formation leading to even higher time discounting. The final outcome will depend on the parameters of the economy and we are able to establish parametric conditions under which fiscal policy can generate a unique balanced growth path (BGP), or multiple (two) discrete equilibria.

We then address equilibrium selection by using fiscal policy in the context of Ramsey second-best and growth-maximizing allocations as an equilibrium selection device. We highlight, first,

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<sup>1</sup>Perhaps the most prominent illustration of this issue concerns the causes of the high savings rate observed in Japan during the last decades. Horioka (1990) and Sheldon (1998a,b) attribute this behavior, among other factors, to an array of public policies implemented through educational programmes that promoted the virtues of patience and thrift. See subsection 2.1 for related empirical evidence.

how the government's objective can determine the available set of policy instruments (Atkinson and Stiglitz, 1980) and, second, its importance in the implementation of additional restrictions on private decisions that can lead the decentralized economy to a unique BGP. Typically, the presence of global and local indeterminacy at the decentralized equilibrium implies that the rational expectations equilibria involve random variables, which are unrelated to the economy's fundamentals and are driven by individual beliefs. However, in a Stackelberg environment the government can obtain information through the agents' reaction function and consequently impose additional restrictions on the tax rate and the endowment allocation through commitment, in order to drive the economy to a unique equilibrium. This policy is feasible since intertemporal complementarities that fuel multiplicity of equilibria are external to the agents and are treated as random variables in the decentralized equilibrium, whereas they are internalized under the second-best policy objective. A notable difference with existing papers on indeterminacies related to fiscal policy is therefore that, in our case, multiple equilibria are not the outcome of policy indeterminacy in the decentralized equilibrium in the form of multiple tax rates, as in Park and Philippopoulos (2004) and Park (2009). Instead, the government here not only has to implement a tax rate and aggregate endowment allocation that ensure a positive growth rate, as in standard endogenous growth models with public policy, but also has to implement the tax rate and endowment allocation that guarantee high savings propensity and high growth.

We close the introductory section by stressing two aspects of our work in comparison with the existing literature. First, other studies have also examined the possibility of equilibrium indeterminacy with endogenous time preference. Drugeon (1996) has studied the possibility of multiple steady states, but not multiple BGPs, when the production technology exhibits increasing returns and the investment technology is non-linear. In contrast, the instantaneous utility and production functions adopted here satisfy the standard concavity assumptions and the investment technology is linear. Chen (2007) assumes that time preference depends on individual past consumption through habit formation, which forms an internal, rather than external, intertemporal complementarity resulting in multiple equilibria that arise by the interactions of consumption levels at different time periods. In the present paper we point out

instead the fiscal policy impacts on the optimal dynamic individual choice, which now depends on current and lagged human capital formation decisions that can generate multiple equilibria and propagate growth effects over time.

Second, a by-product of our analysis related to policymaking is that the optimal tax rate now depends not only on the technology parameters of the economy, but also on the endogeneity of time preference. This, in turn, alters the standard effects of taxation change according to the type of equilibrium in the market economy. For instance, if the intertemporal elasticity of substitution is sufficiently low, the tax rate has to be lower than the growth-maximizing taxation rule, put forward by among others Barro (1990), Futagami et al. (1993), Glomm and Ravikumar (1997), which states that the tax rate on output should equal the elasticity of public capital in the production function. This happens because the marginal cost of taxation decreases due to the favorable effect of taxation on human capital financing and, in turn, on the rate of time preference and economic growth as the tax base increases. Our comparative dynamic exercises show that the effects of parameter changes on the variables of the economy depend on the stability property of the selected equilibrium as a manifestation of Samuelson's correspondence principle.

The rest of the paper is structured as follows. Section 2 sets up and solves the optimization problem of households and firms, and studies the steady state and the dynamic properties of the decentralized economy. Sections 3 and 4 analyze the role of Ramsey second-best and growth-maximizing fiscal policies, respectively. Finally, section 5 concludes the paper.

## **2 The Competitive Decentralized Equilibrium**

### **2.1 Related literature and informal description of the model**

What is the relationship between time preference and human capital? Existing evidence suggests that education strongly affects patience by rendering agents less impulsive to choices that tend to overweight rewards in close temporal proximity. Fuchs (1982) was the first study that

attempted to investigate empirically the association between time preference and education, and showed that there is a positive link between patience and years of schooling. Lawrance (1991) has found that nonwhite families without a college education have time preference rates that are about seven percentage points higher than those of white. Similarly, Harrison et al. (2002) have shown on a sample of Danish households that highly educated adults have subjective discount rates that are roughly two thirds compared to those who are less educated. Bauer and Chytilova (2009) report that an additional year of schooling in Ugandan villages has lowered significantly the discount rate, whereas Khwaja et al. (2007) find that the years of education affect negatively the degree of impulsivity defined as the measure of an individual's ability to set goals and to exercise self-control. In addition, an indirect channel regarding the impact of human capital on impatience may come through income and wealth: Hausman (1979), Lawrance (1991) and Samwick (1998) have found that discount rates are inversely related to income level, and Horowitz (1991) and Pender (1996) have reported that discount rates decline with wealth.

In this section we present a model in which the rate of time preference is endogenous to the aggregate human capital stock. Several studies have analyzed the implications of an endogenous time preference for the macroeconomy. Uzawa (1968), Obstfeld (1990), Shin and Epstein (1993), Palivos et al. (1997), Drugeon (1996, 2000), Stern (2006), and Chen (2007) have investigated the effects of individual decisions on the time preference rate. These authors have shown that the endogeneity of time preference is crucial for the dynamics of the economy, the existence of long-run growth, the long-run distribution of capital, and the analysis of income divergence between countries. Epstein and Hynes (1983) and Meng (2006) have endogenized the rate of time preference to the aggregate macroeconomic environment captured by aggregate consumption and income in variants of exogenous and endogenous growth models, and Meng (2006) has also provided general conditions that give rise to local indeterminacy.

We take the analysis on the endogeneity of time preference one step further by assuming that individual patience is determined by the levels of aggregate consumption and publicly-provided human capital, which is financed through a tax rate on individual output as in Glomm and

Ravikumar (1997) and comprises the driving force of income growth. To accommodate this assumption into, an otherwise standard, general equilibrium framework with public policy, we build on Meng (2006) and we assume that the aggregate human capital stock, rather than aggregate income, reduces the subjective discount rate, whereas aggregate consumption has a positive effect.

## 2.2 The basic model

Consider an economy with a large and constant number, normalized to unity, of infinitely-lived agents that consume a single good. We assume that the rate of time preference,  $\rho$ , is not a positive constant, as in standard growth theory, but is endogenously determined by aggregate consumption,  $C$ , and aggregate human capital,  $H$ . Each household seeks to maximize intertemporal discounted utility given by:<sup>2</sup>

$$\int_0^{\infty} u(c) \exp \left[ - \int_0^t \rho(C_v, H_v) dv \right] dt \quad (1)$$

with instantaneous utility function of the form  $u(c_t) = \frac{c^{1-\sigma}}{1-\sigma}$ , where  $0 < \sigma \leq 1$ , subject to the initial asset endowment  $A(0) > 0$  and the income resource constraint:<sup>3</sup>

$$\dot{A} = rA + wl - c \quad (2)$$

where  $A$  denotes financial assets,  $c$  and  $l$  denote individual consumption and labor respectively, and  $r$  and  $w$  denote the market interest rate and the wage rate respectively.

The time preference function has the following properties:

**Assumption 1**  $\rho(C, H) > \check{\rho} > 0$ .

**Assumption 2**  $\rho'_C \geq 0$  and  $\rho'_H \leq 0$ .

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<sup>2</sup>Throughout the rest of the paper, the time subscript  $t$  is omitted for simplicity of notation.

<sup>3</sup>Note that, as pointed out by Meng (2006), positive felicity is guaranteed only for  $0 < \sigma < 1$ . We also include here the logarithmic utility case ( $\sigma = 1$ ) to allow for comparisons in our simulations with this extensively used specification.

**Assumption 3**  $\rho(C, H) = \rho(\frac{C}{H})$ .

Assumption 1 shows that the rate of time preference is positive implying that there exists a lower bound denoted by  $\check{\rho}$ . By Assumption 2 the rate of time preference depends positively on aggregate consumption and negatively on aggregate human capital.<sup>4</sup> As in the rest of the literature with endogenous time preference, we assume that as agents consume more the value of current consumption increases. In addition, we assume that the higher the human capital stock in the economy the more patient is the agent and willing to forego current consumption. Assumption 3 implies homogeneity of the rate of time preference to the ratio of consumption to human capital, which is required for the rate of time preference to be bounded at the steady-state (Palivos et al., 1997; Meng, 2006) and for the utility function to be consistent with balanced growth (Dolmas, 1996). Note that Assumptions 2 and 3 imply that the derivative of time preference to the ratio of consumption to human capital,  $\frac{\partial \rho}{\partial(\frac{C}{H})} \equiv \rho'(\cdot)$ , is positive.

In the supply side of the economy we assume the existence of a continuum of perfectly competitive homogenous firms, normalized to unity, that seek to maximize profits. Each firm  $i$  uses physical capital,  $K_i$ , and labor,  $L_i$ , under the following production technology:

$$Y_i = K_i^a (hL_i)^{1-a} \quad (3)$$

where  $0 < a < 1$  denotes the share of physical capital in the production function,  $Y_i$  denotes individual output, and  $h$  denotes labor productivity. The law of motion for the physical capital stock is given by:

$$\dot{K}_i = I_i - \delta_K K_i \quad (4)$$

where  $I_i$  denote investment in physical capital and  $\delta_K$  denotes the physical capital depreciation rate.

Following Glomm and Ravikumar (1997), we assume that human capital is provided by the public sector and serves as an input in the production function. In particular, labor productivity

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<sup>4</sup>We retain the equality sign in our assumptions to allow, first, for comparisons with the case of constant rate of time preference and, second, for the impatience function to be consistent with a BGP along which the time preference is constant.

depends on the average human capital stock and is given by:

$$h = \frac{H}{L} \quad (5)$$

where  $L$  denotes the aggregate labor force. The law of motion for the human capital stock is then given by:

$$\dot{H} = H^E - \delta_H H \quad (6)$$

where  $H^E$  denotes public expenditures on education and  $\delta_H$  denotes the human capital depreciation rate. The government imposes a flat tax rate on output,  $\tau$ , to finance expenditures on human capital and follows a balanced budget policy given by:<sup>5</sup>

$$H^E = \tau Y \quad (7)$$

### 2.3 The reduced model and balanced growth

We can now define the Competitive Decentralized Equilibrium (CDE) of the economy in order to analyze its properties.

**Definition 1** *The CDE of the economy is defined for the exogenous policy instruments  $\tau$ , factor prices  $r$ ,  $w$ , and aggregate allocations  $K$ ,  $H$ ,  $H^E$ ,  $L$ ,  $C$ , such that*

*i) Individuals solve their intertemporal utility maximization problem by choosing  $c$  and  $A$ , given  $\tau$  and factor prices.*

*ii) Firms choose  $L_i$  and  $K_i$  in order to maximize their profits, given factor prices and aggregate allocations.*

*iii) All markets clear and in the capital market we have  $A \int_0^1 L_i = \int_0^1 K_i$ .*

*iv) The government budget constraint holds.*

*The CDE is then defined by (i)-(iii) under the aggregation conditions  $\int_0^1 K_i = K$ ,  $\int_0^1 L_i = L$ .*

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<sup>5</sup>It is straightforward to show that we could obtain exactly the same results if the flat tax rate was imposed on labour and capital income. This happens because of the Cobb-Douglas production technology.

The per capita growth rate of consumption in the CDE is given by:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma} \left[ r - \rho \left( \frac{C}{H} \right) \right] \quad (8)$$

The first-order conditions of the firms' profit maximization problem are given by  $r = (1 - \tau)a \left( \frac{K_i}{hL_i} \right)^{a-1} - \delta_k$  and  $w = (1 - \tau)(1 - a) \left( \frac{K_i}{L_i} \right)^a h^{1-a}$ , and state that the marginal productivity of capital and labor have to equal respective factor prices. Using the equilibrium conditions for homogenous and symmetric firms  $L_i = L$  and  $K_i = K$ , and assuming for the rest of the paper without loss of generality that  $\delta_K = \delta_H = \delta$ , the equilibrium growth rates of aggregate consumption, aggregate physical and human capital stocks are given by the following equations:

$$\frac{\dot{C}}{C} = \frac{1}{\sigma} \left[ a(1 - \tau) \left( \frac{K}{H} \right)^{a-1} - \rho \left( \frac{C}{H} \right) - \delta \right] \quad (9)$$

$$\frac{\dot{K}}{K} = (1 - \tau) \left( \frac{K}{H} \right)^{a-1} - \frac{C}{K} - \delta \quad (10)$$

$$\frac{\dot{H}}{H} = \tau \left( \frac{K}{H} \right)^a - \delta \quad (11)$$

Equations (9), (10), (11) summarize the dynamics of our economy. The transversality condition for this problem is then given by:

$$\lim_{t \rightarrow \infty} \frac{K(t)}{C(t)} e^{-\rho \left( \frac{C(t)}{H(t)} \right) t} = 0 \quad (12)$$

At the BGP consumption, physical and human capital grow at the same rate,  $\frac{\dot{C}}{C} = \frac{\dot{K}}{K} = \frac{\dot{H}}{H} = g^{CDE}$ . This result can be easily obtained by investigating the equilibrium growth rates of these variables separately. In particular, for the steady-state consumption growth rate, given by (9), to be constant, both  $K$  and  $H$  have to grow at the same constant rate, say  $g_K^{CDE} = g_H^{CDE} = g^{CDE}$ . Since the steady-state ratio of human to physical capital will be constant, the equilibrium growth rate of consumption,  $g_C^{CDE}$ , will be constant too. Then, by inspection of (10), in order for the growth rate of physical capital to be constant we need that

$g_K^{CDE} = g_H^{CDE}$  and  $g_C^{CDE} = g_K^{CDE}$  (for  $\frac{K}{H}$  and  $\frac{C}{K}$  to be constant respectively). These conditions imply that  $g_C^{CDE} = g_K^{CDE} = g_H^{CDE} = g^{CDE}$ , which also satisfies (11) as well as the transversality condition (12). Hence, the necessary condition for the existence of a BGP in this economy is that all variables grow at the same rate,  $g^{CDE}$ .

We can now derive the equilibrium growth rate of the economy. We first define the following auxiliary stationary variables,  $\omega \equiv \frac{C}{K}$  and  $z \equiv \frac{K}{H}$ . It is straightforward to show that the dynamics of (9)-(12) are equivalent to the dynamics of the following system of equations:

$$\frac{\dot{\omega}}{\omega} = \left(\frac{a}{\sigma} - 1\right)(1 - \tau)z^{a-1} + \omega - \frac{1}{\sigma}\rho(z\omega) - \left(\frac{1}{\sigma} - 1\right)\delta \quad (13)$$

$$\frac{\dot{z}}{z} = (1 - \tau)z^{a-1} - \omega - \tau z^a \quad (14)$$

The following Proposition determines the properties (existence and uniqueness) of the BGP at which  $\frac{\dot{\omega}}{\omega} = \frac{\dot{z}}{z} = 0$ .

**Proposition 1** *The BGP of the economy with endogenous time preference to the ratio of consumption to human capital is determined by (11) and, for given parameter values and policy instruments, is given by:*

$$g^{CDE} = \tau \bar{z}^a - \delta$$

*provided that there exists  $\bar{z} > 0 : \Phi(\bar{z}) = \left(\frac{a}{\sigma}\right)(1 - \tau)\bar{z}^{a-1} - \tau \bar{z}^a - \frac{1}{\sigma}\rho(\bar{z} \cdot \bar{\omega}(\bar{z})) - \left(\frac{1}{\sigma} - 1\right)\delta = 0$  and  $\bar{\omega}(\bar{z}) = (1 - \tau)\bar{z}^{a-1} - \tau \bar{z}^a > 0$ , where  $\bar{\omega}$  and  $\bar{z}$  are the steady-state values of  $\omega$  and  $z$  respectively.*

*We distinguish the following cases:*

*Case 1: A necessary condition for the existence of two well-defined physical to human capital ratios, which correspond to two positive equilibrium growth rates, is  $\left(\frac{a}{\sigma} - 1\right)(1 - \tau)^a \tau^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta \geq \frac{1}{\sigma}\check{\rho}$ .*

*Case 2: A sufficient condition for the existence of a unique well-defined physical to human capital ratio, which corresponds to a positive equilibrium growth rate, is  $\left(\frac{a}{\sigma} - 1\right)(1 - \tau)^a \tau^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta < \frac{1}{\sigma}\check{\rho}$ .*

**Proof.** See Appendix 1. ■

Proposition 1 states that when the rate of time preference in the economy depends on the ratio of aggregate consumption to human capital there can be a unique or multiple (two) equilibrium growth rates. Hence, although the instantaneous utility and production technology functions satisfy the standard concavity assumptions, the existence of a unique positive steady-state growth rate is not guaranteed under the assumption that the aggregate human capital affects the impatience rate of agents.

The following numerical example highlights this point for a linear time preference function,  $\rho(\frac{C}{H}) = b * (\frac{C}{H}) + \check{\rho}$ , which follows Meng (2006) and is used for computational tractability.

**Example 1** Consider a linear time preference function that satisfies Assumptions 1-3 with parameter values  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ ,  $\sigma = 0.3$ ,  $\tau = 0.4$ . We find that there exist two equilibria, one with a low growth rate,  $g_1^{CDE} = 0.282$ , high rate of time preference,  $\bar{\rho}_1 = 0.281$ , low physical to human capital ratio,  $\bar{z}_1 = 0.591$ , and low consumption to physical capital ratio,  $\bar{\omega}_1 = 0.473$ , and one with high growth rate,  $g_2^{CDE} = 0.419$ , low rate of time preference,  $\bar{\rho}_2 = 0.119$ , high physical to human capital ratio,  $\bar{z}_2 = 1.234$ , and low consumption to physical capital ratio,  $\bar{\omega}_2 = 0.096$ .

The central mechanism that drives multiplicity arises from two counterbalancing channels. Consider the construction of an equilibrium path through a rise in the tax rate in order to finance human capital accumulation. This decreases private savings and increases the rate of time preference in the economy due to the rise in aggregate consumption, thus lowering growth. A lower growth rate in turn lowers the tax base that finances public investment in human capital leading to even higher time discounting. On the other hand, by increasing the tax rate the government increases the level of human capital expenditures in the economy. Thus, the rate of time preference falls, savings propensity increases and the economy can attain higher growth, which in turn increases the tax base and raises public expenditures on education and growth.

The final outcome will depend on the structural parameters of the economy. Case 2 of Proposition 1 (multiplicity of equilibria) is more likely to arise when the intertemporal elasticity of substitution is sufficiently low and when the elasticity of human capital in the production function and the depreciation rates of human and physical capital are sufficiently high. For instance, assuming a zero depreciation rate of human capital, it is straightforward to show that when  $\sigma < \alpha$  the second channel dominates. In turn, the standard dynamic mechanism of intertemporal substitutability between the savings rate and the rate of return on capital that preserves a unique BGP fails, thus giving rise to equilibrium indeterminacy.

In particular, equation (8) highlights the importance of the intertemporal elasticity of substitution in the determination of the growth rate of the economy. The upper panel of Table 1 provides a sensitivity analysis of the result on multiplicity by showing the response of the CDE allocation to changes in the values of  $\sigma$  under which equilibrium indeterminacy is expected to occur according to the parameter values and functional forms of Example 1. As can be easily verified, the condition for multiplicity holds for all parametric combinations of Table 1. An increasing  $\sigma$  (declining elasticity of substitution of consumption over time) implies that agents are more averse in substituting current consumption for future one. As a result, when the economy is in the “low-growth” equilibrium current consumption is higher, which in turn implies a higher discount rate and less savings leading to lower capital accumulation and a lower value of the physical to human capital ratio,  $z$ . Output will be lower, which implies that for any tax rate there will be less room for human capital financing, and thus the effects stemming from the rise of consumption on the discount rate are reinforced. On the other hand, when the economy is in the “high growth” equilibrium the rise in  $\sigma$ , which raises current consumption, induces a higher  $z$  and a lower consumption to physical capital ratio,  $\omega$ , as long as  $\sigma$  is sufficiently low. Output increases and human capital rises leading to a lower steady-state consumption to human capital ratio mirrored in the fall of the discount rate that in turn induces savings and higher steady-state growth. This happens because in the “high-growth” equilibrium the dynamics of the economy are sufficient to accommodate the initial increase in current consumption with an increase in human capital expenditures through the higher tax base (growth rate), thus leading

to opposite changes in the variable of the economy.<sup>6</sup> In contrast, in the lower panel of Table 1 where  $\sigma$  is sufficiently high and exceeds the elasticity of physical capital in the production function, any rise in  $\sigma$  simply leads to lower capital accumulation and output, which reduces human capital and raises the discount rate, thus precluding any other effects that fuel multiplicity.

This discussion highlights the role of the exogenously set tax rate, used to finance human capital accumulation, in the final outcome in the CDE. To shed some light on this role, Table 2 shows the effects on the same variables under the parameter values and functional forms of Example 1 when the tax rate is in the range between 0.2 and 0.4 for which an equilibrium exists. A higher share of output that goes to human capital financing lowers *ceteris paribus* the discount rate. In the “low-growth” equilibrium the rise in  $\tau$ , which leads to a fall in the after-tax marginal product of physical capital, and the fall in  $\rho$  induce savings and capital accumulation. Current consumption and the discount rate are lower. The marginal cost of taxation (in the form of less available resources for the private sector) is therefore smaller than the benefit arising from higher output, which can finance physical capital accumulation and boost steady-state growth. Instead, when the economy is in the “high-growth” equilibrium the rise in  $\tau$  lowers physical capital accumulation and raises consumption. The discount rate is now higher, but this effect cannot offset the positive impact of higher human capital through the production function, which further increases output and raises steady-state growth.

## 2.4 Transitional dynamics and stability analysis

In this subsection we show that the endogeneity of time preference to the ratio of aggregate consumption to human capital can be a source of local indeterminacy, i.e. there is a continuum of equilibrium paths that converge to the same BGP.

To this end, we examine the transitional dynamics and local stability of the market economy, which are determined by the two-dimensional system of equations (11) and (13). In matrix

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<sup>6</sup>This picture is confirmed in the next subsection, which shows how the transitional dynamics at the “high-growth” BGP differ from those at the “low-growth” BGP leading to different comparative dynamics.

notation we can write:

$$\begin{bmatrix} \dot{\omega} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \left(1 - \frac{\rho'(\cdot)\bar{z}}{\sigma}\right)\bar{\omega} & \left[\left(\frac{a}{\sigma} - 1\right)(1 - \tau)(a - 1)\bar{z}^{a-2} - \frac{1}{\sigma}\rho'(\cdot)\bar{\omega}\right]\bar{\omega} \\ -\bar{z} & [(1 - \tau)(a - 1)\bar{z}^{a-1} - a\tau\bar{z}^a] \end{bmatrix} \begin{bmatrix} \omega - \bar{\omega} \\ z - \bar{z} \end{bmatrix}$$

After some algebra, the determinant of the above system,  $J$ , is given by:

$$J = \bar{\omega}\bar{z} \left[ \underbrace{-\frac{a}{\sigma}(1 - \tau)(1 - a)\bar{z}^{a-2} - a\tau\bar{z}^{a-1}}_{<0} - \frac{\rho'(\cdot)\bar{z}^{a-1}}{\sigma} [(1 - \tau)a - \tau(1 + a)\bar{z}] \right]$$

The sign of  $J$  is ambiguous and depends on the parameters of the economy and the endogeneity of the rate of time preference. One can easily verify the standard result under a constant rate of time preference,  $\rho'(\cdot) = 0$ , where the growth rate is unique and the steady-state ratio of physical to human capital stock is saddle-path stable. However, when the rate of time preference is endogenous the local dynamics of the economy are nontrivial.

**Proposition 2** *If at the steady state  $\bar{z} < \frac{a(1-\tau)}{(1+a)\tau}$  then  $\rho'(\cdot) > 0$  is a sufficient condition for local determinacy. If at the steady state  $\bar{z} > \frac{a(1-\tau)}{(1+a)\tau}$  then  $\rho'(\cdot) > 0$  is a necessary condition for local indeterminacy.*

**Proof.** Follows straightforward from the sign of the determinant,  $J$ . ■

Proposition 2 shows that for a sufficiently high steady-state ratio of physical to human capital stock,  $\bar{z}$ , the steady-state can be indeterminate when  $\rho'(\cdot) > 0$ . The intuition on how the endogeneity of time preference to the ratio of aggregate consumption to aggregate human capital may generate an intertemporal complementarity between savings and their return is as follows. Consider what happens if agents decide to increase savings. In the Ramsey–Cass–Koopmans model current consumption jumps downwards, capital accumulates at a faster rate and the economy achieves a higher equilibrium stock of capital. During the transition the rate of return on capital,  $r$ , falls and the growth rate, driven by  $(r - \rho)$ , falls under constant time discounting, thus acting as a stabilizing force to the initial increase in savings. This also happens here for low values of physical to human capital ratio,  $\bar{z} < \frac{a(1-\tau)}{(1+a)\tau}$ , where the rate of

return on capital falls and the subjective discount rate is an increasing function of  $z$  (see Proof of Proposition 1) preserving the stabilizing force towards the BGP. However, for sufficiently large values of  $\bar{z} > \frac{a(1-\tau)}{(1+a)\tau}$  the rate of time preference becomes a decreasing function of  $z$  and the standard counteracting force do not guarantee a unique path. The reason is that although the rate of return on capital falls, the growth rate of consumption, determined by  $r - \rho(\cdot)$ , can increase due to the fall in impatience. Thus, a continuum of equilibrium paths towards the steady-state can occur depending on the parameters of the model.

Due to the complexity for the computation of the determinant sign of the dynamic system we will numerically characterize the stability and dynamic properties using the assumptions of Example 1.

**Example 2** Consider a linear time preference function,  $\rho(\frac{C}{H}) = b * (\frac{C}{H}) + \check{\rho}$ , that satisfies Assumptions 1-3 with parameter values  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ ,  $\sigma = 0.3$ ,  $\tau = 0.4$ . We can characterize the stability and dynamics of the system by computing its determinant and trace. We get that both steady-state values of  $z$  and  $\omega$  are locally stable. However, the first is saddle-path and thus determinate, whereas the second is a node and thus indeterminate. In particular, in the first steady state we have  $z_1 = 0.59$ ,  $\omega_1 = 0.47$ , and the determinant of the system is negative ( $J = -0.30$ ), which implies saddle-path stability. In the second steady state we get,  $z_2 = 1.23$ ,  $\omega_2 = 0.09$ , and the determinant is positive ( $J = 0.08$ ). The type of stability given by  $\text{trace}^2 - 4J = 0.24 > 0$  is a node and the trace is negative ( $\text{trace} = -0.77$ ), which implies that the steady state is stable.

Example 2 shows that when there are two BGPs, the one with the higher growth rate corresponds to the lower steady-state value of the consumption to capital ratio, higher physical to human capital stock ratio and is locally indeterminate, whereas the other one with the lower growth rate corresponds to the lower steady-state value of physical to human capital stock ratio and is locally determinate. Thus, consistent with the conditions of Propositions 1 and 2 we can have both local and global indeterminacy, which implies that given an initial condition it is not possible to choose an initial value of consumption to either place the economy in the

determinate BGP with the low growth rate or in the neighborhood of the indeterminate BGP with the high growth rate.<sup>7</sup>

### 3 Impatience and Ramsey Second-Best Fiscal Policy

In the present model the human capital stock in the economy is determined by government policies and is crucial for the results on indeterminacy. Previous studies of taxation and spending policies have assumed that a government is endowed with nondistortionary policy instruments (e.g. lump-sum taxes or transfers). In turn, the public finance literature has assumed that the government has a comprehensive mechanism (e.g. Pigouvian taxation) for fully internalizing any market failures from externalities. Appendix 2 shows that in our context the market economy cannot replicate the Pareto optimal allocation. In this section we endogenize government policy and we examine the equilibrium selection mechanism of the governments' objective in the context of Ramsey-Second-Best (RSB) fiscal policy.

In his seminal work, Ramsey (1927) recognized that whenever a government, in the form of a Stackelberg game, introduces distortionary economic policies, agents react in their own interests through a decentralized competitive market. The Ramsey approach took into account agents' equilibrium reactions to the government's intended optimal fiscal policy and has been extensively revitalized in capital accumulation models (Lucas and Stokey, 1983; Chamley, 1986; Judd, 1987; Lucas, 1990; Jones et al., 1993).

In the current setup, there exists a range of the initial endowments of the aggregate physical and human capital stocks in the CDE, under which the economy will exhibit multiplicity for any tax rate. We examine whether second-best policy can act as an equilibrium selection device by investigating if, and how, the government's objective can impose restrictions and allocate the initial endowment to solve the indeterminacy problem. In particular, we investigate the impact of fiscal policy that leads to RSB allocation under which the government maximizes aggregate welfare subject to the response of the CDE.

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<sup>7</sup>These results are robust to alternative parameter values. Notice that in the case of uniqueness, e.g. for a logarithmic utility function with  $\sigma = 1$ , the steady-state ratio of physical to human capital is saddle-path stable.

**Definition 2** *The RSB fiscal policy is given under Definition 1 when (i) the government chooses the tax rate and aggregate allocations in order to maximize the welfare of the economy by taking into account the aggregate optimality conditions of the CDE, and (ii) the government budget constraint and the feasibility and technological conditions are met.*

The government seeks to maximize welfare of the economy subject to the outcome of the decentralized equilibrium summarized by (9)-(11). The Hamiltonian of this problem is given by:

$$\begin{aligned} \Lambda^{RSB} = & u(C)e^{-\Delta} + \frac{1}{\sigma}\tilde{\mu}_C C \left[ a(1-\tau) \left( \frac{K}{H} \right)^{a-1} - \rho\left(\frac{C}{H}\right) - \delta \right] + \\ & \tilde{\mu}_K [(1-\tau)K^a (H)^{1-a} - C - \delta K] + \tilde{\mu}_H [\tau K^a H^{1-a} - \delta H] + \tilde{\mu}_\rho [\rho\left(\frac{C}{H}\right)] \end{aligned}$$

where  $\tilde{\mu}_C, \tilde{\mu}_K, \tilde{\mu}_H$  are the dynamic multipliers associated with (9), (10), (11) respectively and  $\Delta = \int_0^t \rho(C_v, H_v) dv$ . For computational tractability we use a monotonic transformation of the instantaneous utility function of the household given by  $\ln u(C) = \ln\left(\frac{C^{1-\sigma}}{1-\sigma}\right) = (1-\sigma) \ln C - \ln(1-\sigma)$ , which preserves its cardinal properties and does not affect the qualitative outcome of the maximization problem (Mas-Colell et al., 1995).

The first-order conditions of the RSB problem include the constraints (9)-(11) and the optimality conditions with respect to  $C, H, K, \tau$ :

$$\dot{\tilde{\mu}}_C = - \left[ \frac{(1-\sigma)}{C} \right] e^{-\Delta} - \frac{\tilde{\mu}_C}{\sigma} \left[ a(1-\tau) \left( \frac{K}{H} \right)^{a-1} - \rho\left(\frac{C}{H}\right) - \delta \right] + \left[ \frac{\tilde{\mu}_C C}{\sigma} - \tilde{\mu}_\rho \right] \rho'(\cdot) \frac{1}{H} + \tilde{\mu}_K \quad (15)$$

$$\dot{\tilde{\mu}}_K = \frac{\tilde{\mu}_C C}{\sigma} \left[ a(1-a)(1-\tau) \left( \frac{K}{H} \right)^{a-1} K^{-1} \right] - \tilde{\mu}_K \left[ a(1-\tau) \left( \frac{K}{H} \right)^{a-1} - \delta \right] - \tilde{\mu}_H \tau a \left( \frac{K}{H} \right)^{a-1} \quad (16)$$

$$\begin{aligned} \dot{\tilde{\mu}}_H = & -\frac{\tilde{\mu}_C C}{\sigma} \left[ a(1-a)(1-\tau) \left( \frac{K}{H} \right)^a K^{-1} + \rho'(\cdot) \frac{C}{H^2} \right] \\ & - \tilde{\mu}_K (1-a)(1-\tau) \left( \frac{K}{H} \right)^a - \tilde{\mu}_H \left[ (1-a)\tau \left( \frac{K}{H} \right)^a - \delta \right] + \tilde{\mu}_\rho \rho'(\cdot) \frac{C}{H^2} \end{aligned} \quad (17)$$

$$\frac{\tilde{\mu}_C a C}{\sigma} \left(\frac{K}{H}\right)^{a-1} + (\tilde{\mu}_K - \tilde{\mu}_H) \left(\frac{K}{H}\right)^a H = 0 \quad (18)$$

$$\dot{\tilde{\mu}}_\rho = [(1 - \sigma) \ln C - \ln(1 - \sigma)] e^{-\Delta} = -\tilde{\mu}_C \dot{C} - \tilde{\mu}_K \dot{K} - \tilde{\mu}_H \dot{H} - \tilde{\mu}_\rho \left[\rho\left(\frac{C}{H}\right)\right] \quad (19)$$

Equations (15)-(19), the transversality condition  $\lim_{t \rightarrow \infty} \Lambda^{RSB} = 0$ , and equations (9), (10), (11) characterize the solution of the Ramsey problem. The methodology to derive the stationary RSB allocation is similar to that of the social planner's problem (see Appendix 2). Let us define  $\mu_j \equiv \tilde{\mu}_j e^{\Delta(t)}$  where  $j = C, K, H, \rho$ . We can now transform the variables by defining  $\omega \equiv \frac{C}{K}$ ,  $z \equiv \frac{K}{H}$ ,  $\psi \equiv \mu_C C$ ,  $\phi \equiv \mu_K K$ ,  $\chi \equiv \mu_H H$ . Thus,  $\psi$ ,  $\phi$ ,  $\chi$  measure respectively the social value of economy-wide consumption, physical and human capital stocks, and  $\omega$  and  $z$  are as in the analysis of the CDE above. It is straightforward to show that the dynamics of (15)-(19) and (9)-(11) are equivalent to the dynamics presented below:

$$\frac{\dot{\omega}}{\omega} = \left(\frac{a}{\sigma} - 1\right)(1 - \tau)z^{a-1} + \omega - \frac{1}{\sigma}\rho(\omega z) - \left(\frac{1}{\sigma} - 1\right)\delta \quad (20)$$

$$\frac{\dot{z}}{z} = (1 - \tau)z^{a-1} - \omega - \tau z^a \quad (21)$$

$$-\frac{1}{\sigma}\psi a z^{a-1} - \phi z^{a-1} + \chi z^a = 0 \quad (22)$$

$$\frac{\dot{\chi}}{\chi} = \tau a z^a - \frac{(1 - \tau)(1 - a)z^{a-1}}{\chi} \left[\frac{a\psi}{\sigma} + \phi\right] + \rho(\omega z) - \frac{\rho'(\cdot)\omega z}{\chi} \left[\frac{\psi}{\sigma} - \mu_\rho\right] \quad (23)$$

$$\frac{\dot{\phi}}{\phi} = (1 - \tau)(1 - a)z^{a-1} \left(1 + \frac{a\psi}{\sigma\phi}\right) - \omega - \frac{\chi}{\phi}\tau a z^a + \rho(\omega z) \quad (24)$$

$$\frac{\dot{\psi}}{\psi} = -\frac{1 - \sigma}{\psi} + \frac{\omega\phi}{\psi} + \rho(\omega z) + \rho'(\cdot)\omega z \left[\frac{1}{\sigma} - \frac{\mu_\rho}{\psi}\right] \quad (25)$$

$$\dot{\mu}_\rho = -(1 - \tau)z^{a-1} \left[\frac{\psi a}{\sigma} + \phi\right] + \delta \left[\frac{\psi}{\sigma} + \phi + \chi\right] + \omega\phi - \tau\chi z^a + \frac{\psi\rho(\omega z)}{\sigma} \quad (26)$$

In the long run,  $\frac{\dot{\omega}}{\omega} = \frac{\dot{z}}{z} = \frac{\dot{\chi}}{\chi} = \frac{\dot{\phi}}{\phi} = \frac{\dot{\psi}}{\psi} = \frac{\dot{\tau}}{\tau} = \dot{\mu}_\rho = 0$ . Since the impatience function is endogenous to the ratio of consumption to human capital and the economy as described in the CDE accepts a BGP at which  $K$  and  $H$  grow at the same rate for any government policy, then

along such a path the rate of time preference is constant over time,  $\rho'(\cdot) = 0$ , and the value of  $\mu_\rho$  does not affect the endowment allocation in the economy (Palivos et al., 1997). Solving (22)-(25) for  $\tilde{\tau}$  yields the second-best tax rate as a function of  $\omega$  and  $z$ .

It is straightforward to show that the RSB environment is characterized by the following system of equations:

$$\tilde{\tau} = (1 - a) - \rho(\tilde{\omega}\tilde{z})\tilde{z}^{-a} \quad (27)$$

$$\left(\frac{a}{\sigma} - 1\right)(1 - \tau)\tilde{z}^{a-1} + \tilde{\omega} - \frac{1}{\sigma}\rho(\tilde{\omega}\tilde{z}) - \left(\frac{1}{\sigma} - 1\right)\delta = 0 \quad (28)$$

$$(1 - \tau)\tilde{z}^{a-1} - \tilde{\omega} - \tau\tilde{z}^a = 0 \quad (29)$$

Equations (27)-(29) yield the RSB tax rate,  $\tilde{\tau}$ , and  $\tilde{\omega}$  and  $\tilde{z}$  as functions of the parameters. The following Proposition summarizes the properties of the equilibrium in the RSB environment.

**Proposition 3** *The equilibrium tax rate and growth rate in the RSB environment are unique.*

**Proof.** See Appendix 3. ■

Proposition 3 states that, in the presence of multiple equilibria at the CDE, fiscal policy under a second best environment can act as an equilibrium selection device. Given the complexity of the system we present below numerical solutions in order to analyze the behavior of the economy under the RSB allocation.

**Example 3** *Consider a linear time preference function,  $\rho\left(\frac{C}{H}\right) = b * \left(\frac{C}{H}\right) + \check{\rho}$ , that satisfies Assumptions 1-3 with parameter values  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ ,  $\sigma = 0.3$ . Under the RSB allocation we find a unique equilibrium growth rate given by  $g^{RSB} = 0.399$  corresponding to a tax rate given by  $\tilde{\tau} = 0.395$ .*

Example 3 shows that for the parameter values under which the CDE exhibits multiple equilibria (see Example 1), the government can attain a unique equilibrium by implementing the appropriate allocation and conditions in the RSB environment. Hence, in this model the objective of fiscal policy acts as a selection device when multiple equilibria are present in the

CDE. It is well known that global and local indeterminacy at the CDE implies that the rational expectations equilibria involve random variables, which are unrelated to the economy's fundamentals and are driven by individual beliefs. However, in a Stackelberg environment the government can obtain information through the agents' reaction function and consequently impose additional restrictions on the tax rate and endowment allocation through commitment in order to drive the CDE to a unique equilibrium. This policy is feasible since intertemporal complementarities that fuel multiplicity are external to the agents and treated as random variables in the CDE, whereas they are internalized under the RSB policy objective. Hence, in accordance with our analysis of local stability, the government can allocate endowments so as to select a stable equilibrium regime and then use a feasible tax rate to attain welfare maximization.

Following the rationale of Table 1, in Table 3 we present the response of the RSB allocation to changes in the value of  $\sigma$  under which the CDE is characterized by multiplicity. The RSB allocation implements a unique equilibrium by selecting the "high-growth" CDE. Table 3 also shows that the tax rate and the growth rate are increasing functions of the intertemporal elasticity of substitution, while the ratio of physical to human capital stock and the subjective discounting are decreasing. Intuitively, following an increase in  $\sigma$  agents are more averse in substituting current consumption for future one, which in turn implies a higher discount rate and less savings leading to lower capital accumulation and a lower value of the physical to human capital ratio,  $z$ . At the same time a lower  $z$  decreases the marginal cost of public funds, lowers the tax base of the economy and results, at the BGP, in an increase in the tax rate. The endogenous change in the tax rate increases public expenditures and human capital, lowers the rate of time preference and thus results in an increase of the growth rate. These channels will be studied in more detail below when we analyze the growth-maximizing fiscal policy allocation, which has been extensively studied in the context of policy rules in the public finance literature.

## 4 Impatience and Growth-Maximizing Fiscal Policy

In this section we analyze growth-maximizing fiscal policy rules. Although earlier papers, like Barro (1990), have mostly considered welfare and growth-maximizing policies under a unified perspective, subsequent studies have emphasized the role of growth maximization as an independent policy target.<sup>8</sup>

**Definition 3** *A growth-maximizing (GM) allocation in the competitive equilibrium of the aggregate economy is given under Definition 1 when (i) the government acts a Stackelberg player and chooses the tax rate  $\tau$  in order to maximize the long-run growth rate of the economy by taking into account the aggregate maximizing behavior of the competitive equilibrium, and (ii) the government budget constraint and the feasibility and technological conditions are met.*

The government seeks to maximize the growth rate of the economy,  $g$ , given by:

$$\max_{z, \tau} g = \tau z^a - \delta$$

subject to the equilibrium CDE response summarized by  $(\frac{a}{\sigma})(1 - \tau)z^{a-1} - \tau z^a - \frac{1}{\sigma}\rho(\omega(z)z) - (\frac{1}{\sigma} - 1)\delta = 0$  and  $\omega(z) = (1 - \tau)z^{a-1} - \tau z^a$ .

The first-order conditions with respect to  $z$  and  $\tau$  are:

$$a\hat{\tau}\hat{z}^{a-1} + \hat{\lambda}\left(\frac{a}{\sigma}\right)(a-1)(1-\hat{\tau})\hat{z}^{a-2} - \hat{\lambda}\hat{\tau}a\hat{z}^{a-1} - \hat{\lambda}\frac{1}{\sigma}\rho'(\cdot) [a(1-\hat{\tau})\hat{z}^{a-1} - (a+1)\hat{\tau}\hat{z}^a] = 0 \quad (30)$$

$$\hat{z}^a - \left(\frac{a}{\sigma}\right)\hat{\lambda}\hat{z}^{a-1} - \hat{\lambda}\hat{z}^a + \hat{\lambda}\frac{1}{\sigma}\rho'(\cdot) [\hat{z}^a + \hat{z}^{a+1}] = 0 \quad (31)$$

where  $\hat{\lambda}$  is the associated Lagrange multiplier, and  $\hat{z}$  and  $\hat{\tau}$  are the GM values of  $z$  and  $\tau$  respectively. Solving (31) for  $\hat{\lambda}$  and substituting in (30) we can obtain the following system of equations that characterize the GM policy rules:

$$\hat{\tau} = \frac{a(1-a + \rho'(\cdot)\hat{z})}{a + \rho'(\cdot)\hat{z}^2} > 0 \quad (32)$$

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<sup>8</sup>See Economides et al. (2007) for a similar approach.

$$\left(\frac{a}{\sigma}\right)(1 - \hat{\tau})\hat{z}^{a-1} - \hat{\tau}\hat{z}^a - \frac{1}{\sigma}\rho(\omega(\hat{z})\hat{z}) - \left(\frac{1}{\sigma} - 1\right)\delta = 0 \quad (33)$$

Equation (32) yields the GM tax rate,  $\hat{\tau}$ . Notice that when the rate of time preference is constant ( $\rho'(\cdot) = 0$ ) the government has to implement a marginal tax rate that is equal to the elasticity of publicly provided human capital in the production function,  $\tau = (1 - a)$ , as in Barro (1990), Futagami et al. (1993) and Glomm and Ravikumar (1997). However, under endogenous time preference ( $\rho'(\cdot) \neq 0$ ) the GM tax rate can be lower or higher than the elasticity of human capital in the production function since the tax policy also depends on demand-driven parameters.

To highlight these points we provide some numerical examples for a range of parameter values to check equilibrium selection under the GM allocation and how the comparative statics evolve. For comparison purposes we use in the following Example the parameter values of Example 1 for which the CDE is multiple.

**Example 4** Consider a linear time preference function,  $\rho\left(\frac{C}{H}\right) = b * \left(\frac{C}{H}\right) + \check{\rho}$ , that satisfies Assumptions 1-3 with parameter values  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ ,  $\sigma = 0.3$ . The GM tax rate is given by  $\hat{\tau} = 0.367$  with respective growth rate  $g^{GM} = 0.42$ , physical to human capital ratio  $\hat{z} = 1.48$ , rate of time preference  $\hat{\rho} = 0.108$ , consumption to physical capital ratio  $\hat{\omega} = 0.072$  and consumption to human capital ratio  $\left(\frac{\hat{C}}{\hat{H}}\right) = 0.1$ .

Example 4 shows that under the parameter values that produce multiplicity in the CDE, the GM allocation can act as an equilibrium selection device and impose the allocation restrictions and the tax rate that guarantee a unique equilibrium growth. Notice that in the GM allocation the “high-growth” BGP is selected, a result that is consistent with the government’s objective.<sup>9</sup>

In Table 4 we examine the sensitivity of equilibrium selection to changes in key parameters and the implications for the tax rate in comparison with the standard Barro (1990) taxation rule. As can be readily seen, the equilibrium selection property for the GM allocation is robust to changes in  $\sigma$ , for which there is multiplicity of equilibria in the CDE. Table 4 then shows the

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<sup>9</sup>For the parameter values used in Example 4 and the growth maximizing tax rate  $\hat{\tau} = 0.367$ , the CDE gives two equilibrium growth rates  $\bar{g}_1 = 0.25$  and  $\bar{g}_2 = 0.42$ .

response of the tax rate to changes in  $\sigma$  and the associate change in the comparative statics. For  $0.2 < \sigma < 0.4$  the GM tax rate is lower than the one dictated by the Barro (1990) taxation rule (0.5), whereas for  $0.6 < \sigma < 1$  the corresponding GM tax rate is higher. According to Table 4 for sufficiently low levels of  $\sigma$ ,  $0.2 < \sigma < 0.4$ , an increase in the intertemporal elasticity of substitution increases the GM tax rate and the growth rate of the economy and results in a lower physical to human capital ratio, lower consumption to human capital ratio and lower rate of time preference. Intuitively, following an increase in  $\sigma$  agents are more averse in substituting current consumption for future one, which in turn implies a higher discount rate and less savings leading to lower capital accumulation and a lower value of the physical to human capital ratio,  $z$ . This lowers the marginal cost of public funds and the tax base of the economy, thus resulting in an endogenous increase in the tax rate at the BGP that activates two opposing channels. On the one hand, the increase in the tax rate lowers capital accumulation, increases consumption (depending on the level of  $\sigma$ ), the rate of time preference and in turn decreases the growth rate. On the other hand, an increase in the tax rate increases public expenditures and human capital, lowers the rate of time preference and raises the growth rate of the economy. Hence, for a low level of  $\sigma$  the effect of the endogenous change in the tax rate on consumption and the implied positive effect on the rate of time preference are low and the second channel dominates, resulting in a lower  $\frac{C}{H}$  that in turn lowers time preference and raises growth (see the effects of  $\sigma$  on  $\hat{g}$ ,  $\frac{\hat{C}}{H}$ ,  $\hat{\rho}$  for sufficiently low values of  $\sigma$ ,  $0.2 < \sigma < 0.4$ ). In this case, a higher growth rate forms a higher tax base, the marginal benefit of public funds increases and the growth-maximizing tax rate has to be lower than the elasticity of human capital in the production function. In contrast, for sufficiently high levels of  $\sigma$ ,  $0.6 < \sigma < 1$ , the first channel dominates, resulting in a lower growth rate, a lower tax base and marginal benefit of public funds leading to a tax rate that is higher than the Barro (1990) taxation rule.

An immediate implication of the results presented in Table 4 is that the changes of the endogenous variables depend qualitatively on the local stability properties of the CDE. Recall that for  $0.2 < \sigma < 0.4$  the CDE is multiple, while the GM allocation selects the “high-growth” equilibrium. For  $0.2 < \sigma < 0.4$  the “high-growth” BGP is locally indeterminate, whereas

for  $0.6 < \sigma < 1$  the unique CDE equilibrium is locally determinate. In these two parametric ranges the responses of the variables are in the opposite direction, an outcome that is consistent with Samuelson's correspondence principle, which states that the comparative statics depend crucially on the local properties of the equilibrium. In particular, the results derived in Table 5 indicate, first, that an increase in  $\sigma$  for  $0.2 < \sigma < 0.4$  raises the tax rate and the growth rate, whereas an increase in  $\sigma$  for  $0.6 < \sigma < 1$  lowers the tax rate and growth. The effect of an increasing low (high)  $\sigma$  on the growth-maximizing tax rate is lower (higher) than the Barro (1990) tax rate.

The preceding discussion highlights the role of the slope of impatience function,  $b$ , in the qualitative response of the economy. Table 5 shows the response of the economy to changes in  $b$ . The upper panel of Table 5 indicates that if we set parameter values where the CDE is characterized by multiple equilibria, the equilibrium selection property of the GM allocation is not affected by changes in  $b$ . In turn, the lower panel of Table 5 checks the response of the Barro (1990) taxation rule to changes in  $b$  and, in conjunction with the upper panel, provides a picture of the response of the endogenous allocation of the GM allocation problem to changes in the slope of impatience function. In particular, an increase in  $b$  leads to an increase in the tax rate and to a decrease in the physical to human capital stock ratio, whereas the effects on the rate of time preference and the growth rate are ambiguous and depend on the level of the intertemporal elasticity of substitution and the initial level of  $b$ . Intuitively, an increase in the slope of the impatience function increases *ceteris paribus* the rate of time preference, which lowers savings and capital accumulation and in turn decreases the physical to human capital ratio. Also, by the Euler equation an increase in the rate of time preference lowers the growth rate and the tax base of the economy, and generates an endogenous increase in the tax rate to finance public expenditures at the BGP. In turn, the endogenous increase in the tax rate activates the previously analyzed mechanism. For a sufficiently high level of the intertemporal elasticity of substitution (e.g.  $\sigma = 1$ ), the rise in the tax rate increases consumption more than human capital expenditures and reinforces the initial increase in the rate of time preference leading to an additional decrease in the growth rate of the economy. In contrast, when  $\sigma$  is

sufficiently low (e.g.  $\sigma = 0.3$ ) an increase in the tax rate increases human capital expenditures leading to an increase in the growth rate of the economy which counteracts the initial decrease. Also, in the latter case the increase in the tax rate lowers the consumption to human capital ratio, since human capital expenditures increase more than consumption for low  $\sigma$ , with the final effect on the rate of time preference depending on the magnitude of  $b$ . As summarized in Table 6, for low values of  $b$  the response of the rate of time preference to the decrease in the consumption to human capital ratio,  $\frac{C}{H}$ , is low and is dominated by the initial exogenous increase of the rate of time preference caused by  $b$ , whereas for high values of  $b$  the initial increase in  $\rho$  due to  $b$  is lower than the endogenous decrease driven by  $\frac{C}{H}$ .

## 5 Concluding Remarks

This paper studied the macroeconomic implications of the endogeneity of time preference to aggregate human capital provided by the public sector. We derived the long-run behavior of the economy and analyzed the impact of fiscal policy. The main findings are that multiple BGPs emerge in the decentralized economy and that second-best (Ramsey and growth-maximizing) government policy can act as an equilibrium selection device in order to lead the economy to a desired BGP. In addition, we also challenged the standard fiscal policy rules of the public finance literature and showed that their qualitative properties depend on demand-side parameters through intertemporal preferences.

These results have some novel policy implications for economic performance as it is argued that active public policies in sectors like education are crucial in boosting growth, particularly in countries that face development traps. Given that countries with similar structural characteristics often seem to display divergent economic behavior, our findings suggest an additional generating mechanism of “low-growth” equilibria. This stems from the linkage between endogenous time discounting and productive fiscal policy, with the latter now operating through the demand, rather than the supply, side of the economy by forming the patience of consumers. In turn, our evidence on the role of second-best fiscal policy in driving the economy to a

“high-growth” path, albeit highly stylized, indicate the importance of active policymaking in determining the long-run performance of the economy by affecting individual patience. Therefore, to the extent that government policies affect time discounting by enhancing education and other “future-oriented” policies, our findings suggest a channel for the impact of fiscal policy on long-run growth that has been left unnoticed in existing studies and warrants further analysis.

We close the paper by noting that in light of these results the importance of an empirical assessment of the magnitude of the linkage between individual time preference and the social level of education cannot be underestimated. Existing studies that have empirically investigated the determinants of time preference at the country level, surveyed by Becker and Mulligan (1997), normally indicate that wealth, which is higher for more educated individuals, is positively associated with patience. However, there is no cross-country evidence on the magnitude of time preference and its association with human capital and public policies. It appears therefore that the empirical studies on the determinants of the rate of time preference that will take into account the role of education in patience formation can be important for the design of optimal tax policy over the long run.

## Appendix 1: Proof of Proposition 1

The method will be to separate function  $\Phi(z)$  in two functions and find their intersection to solve it. We define  $\Gamma(z) \equiv (\frac{a}{\sigma})(1-\tau)(z)^{a-1} - (z)^a\tau - (\frac{1}{\sigma} - 1)\delta$  and  $\Lambda(z) \equiv \frac{1}{\sigma}\rho(z \cdot \omega(z))$ . Both  $\Gamma(z)$  and  $\Lambda(z)$  are continuous in  $z$ . In order for  $\omega(z) > 0$  to hold we must have  $z < \frac{1-\tau}{\tau}$ .

Equation  $\Gamma(z)$  has the following properties:

1.  $\lim_{z \rightarrow 0} \Gamma(z) = +\infty$ ,  $\lim_{z \rightarrow \frac{1-\tau}{\tau}} \Gamma(z) = (\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta$ .
2.  $\frac{\partial \Gamma(z)}{\partial z} < 0$ ,  $\frac{\partial^2 \Gamma(z)}{\partial z^2} > 0$ .

From the properties of  $\Gamma(z)$  it follows that it is a strictly decreasing and convex function in its domain, starts from  $+\infty$  and ends at  $(\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta$ .

Equation  $\Lambda(z)$  has the following properties:

1.  $\lim_{z \rightarrow 0} \Lambda(z) = \frac{1}{\sigma}\rho(0) = \frac{1}{\sigma}\check{\rho}$ ,  $\lim_{z \rightarrow \frac{1-\tau}{\tau}} \Lambda(z) = \frac{1}{\sigma}\rho(0) = \frac{1}{\sigma}\check{\rho}$ .
2.  $\frac{\partial \Lambda(z)}{\partial z} = \frac{1}{\sigma}\rho'(\cdot) [a(1-\tau)z^{a-1} - \tau(1+a)z^a]$ . We have  $\frac{\partial \Lambda(z)}{\partial z} > 0$  for  $a(1-\tau)z^{a-1} - (1+a)\tau z^a > 0 \Rightarrow z < \frac{a(1-\tau)}{(1+a)\tau}$  and  $\frac{\partial \Lambda(z)}{\partial z} < 0$  for  $z > \frac{a(1-\tau)}{(1+a)\tau}$ . Thus,  $\Lambda(z)$  has a maximum at  $z = \frac{a(1-\tau)}{(1+a)\tau}$ .

From the properties of  $\Lambda(z)$  it follows that it is an inverse U-shaped curve starting from  $\frac{1}{\sigma}\check{\rho}$  and ending at  $\frac{1}{\sigma}\check{\rho}$ .

Assuming equilibrium existence, from the properties of  $\Lambda(z)$  and  $\Gamma(z)$  it follows that there exist one or two positive equilibrium growth rates. For low values of  $z$ , since  $+\infty > \frac{1}{\sigma}\check{\rho}$  we get that  $\Gamma(z)$  lies above  $\Lambda(z)$ . Also, for the upper bound value of  $z$ ,  $\Gamma(z) = (\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta$  and  $\Lambda(z) = \frac{1}{\sigma}\check{\rho}$ . Since both functions are continuous if  $(\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta < \frac{1}{\sigma}\check{\rho}$ , which means that  $\Gamma(z)$  ends below  $\Lambda(z)$ , then, since  $\Gamma(z)$  starts above  $\Lambda(z)$ ,  $\Gamma(z)$  will cross  $\Lambda(z)$  once and thus there will exist a unique equilibrium growth rate. Thus,  $(\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta < \frac{1}{\sigma}\check{\rho}$  is a sufficient parametric condition for a unique equilibrium growth rate in the economy.

If  $(\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$  then there can exist two equilibrium growth rates because  $\Lambda(z)$  is an inverse U-shaped curve while  $\Gamma(z)$  strictly monotone and decreasing, so  $\Gamma(z)$  can cross  $\Lambda(z)$  at most two times. Thus,  $(\frac{a}{\sigma} - 1)(1-\tau)^a\tau^{1-a} - (\frac{1}{\sigma} - 1)\delta \geq \frac{1}{\sigma}\check{\rho}$  is a necessary parametric condition for multiplicity (two positive equilibrium growth rates). ■

## Appendix 2: Endogenous Impatience and First-Best Fiscal Policy

In this model the social planner (SP) seeks to maximize aggregate utility subject to the production technology, the aggregate budget constraint and the physical and the human capital accumulation technology.

**Definition A1** Let a path  $\{K(t), H(t), \lambda_i(t)\}$  for  $t \geq 0$  be the solution to the social planner problem, where  $\lambda_i(t)$  are the costate variables associated with the constraints. We call it a BGP in the SP problem if the growth rates of these variables,  $g_K^{SP}$ ,  $g_H^{SP}$ ,  $g_{\lambda_i}^{SP}$ , are constant over time.

Formally, the SP problem is to maximize aggregate discounted utility given by:

$$\int_0^{\infty} u(C) \exp[-\Delta(t)] dt \quad (\text{A.1})$$

$$\dot{K} = K^a H^{1-a} - C - H^E - \delta K \quad (\text{A.2})$$

$$\dot{\Delta} = \rho(C, H) \quad (\text{A.3})$$

and the human capital accumulation equation (6) and  $K(0) > 0$ ,  $H(0) > 0$ , where  $\Delta = \int_0^t \rho(C_v, H_v) dv$ . The present value Hamiltonian of the problem is given by:

$$\Lambda^{SP} = \frac{C^{1-\sigma}}{1-\sigma} e^{-\Delta} + \tilde{\lambda}_K [K^a H^{1-a} - C - H^E - \delta K] + \tilde{\lambda}_H [H^E - \delta H] + \tilde{\lambda}_\rho [\rho(\frac{C}{H})]$$

The first-order conditions are given by:

$$C^{-\sigma} e^{-\Delta} - \tilde{\lambda}_K + \tilde{\lambda}_\rho \rho'(\cdot) \frac{1}{H} = 0 \quad (\text{A.4})$$

$$\tilde{\lambda}_K = \tilde{\lambda}_H \quad (\text{A.5})$$

$$-\frac{\dot{\tilde{\lambda}}_K}{\tilde{\lambda}_K} = a \left( \frac{K}{H} \right)^{a-1} - \delta \quad (\text{A.6})$$

$$\tilde{\lambda}_K \left[ (1-a) \left( \frac{K}{H} \right)^a \right] - \tilde{\lambda}_H \delta - \tilde{\lambda}_\rho \rho'(\cdot) \frac{C}{H^2} = -\dot{\tilde{\lambda}}_H \quad (\text{A.7})$$

$$\dot{\tilde{\lambda}}_\rho = \frac{C^{1-\sigma}}{1-\sigma} e^{-\Delta} \quad (\text{A.8})$$

$$\lim_{t \rightarrow \infty} \Lambda^{SP}(t) = 0 \quad (\text{A.9})$$

Defining  $\lambda_i \equiv \tilde{\lambda}_i e^{\Delta(t)}$  where  $i = K, H, \rho$  and using (A.5) and (A.3) it follows that

$$-\frac{\dot{\lambda}_K}{\lambda_K} = a \left( \frac{K}{H} \right)^{a-1} - \delta - \rho \left( \frac{C}{H} \right) \quad (\text{A.10})$$

Let  $g_\lambda$  denote the common growth rate of  $\lambda_K$  and  $\lambda_H$ . From (A.10) we have that  $\rho \left( \frac{C}{H} \right) = g_\lambda + a \left( \frac{K}{H} \right)^{a-1} - \delta$ , which for equal growth rates of  $K$  and  $H$  implies a constant rate of time preference at the BGP (Palivos et al., 1997).

**Result A1** *Under Assumptions 1-3 the rate of time preference in the SP economy is constant over time at the BGP and fiscal policy cannot replicate the first-best environment.*

Under Result A1 we have that (A.4) is given by:

$$C^{-\sigma} - \lambda_K = 0 \quad (\text{A.11})$$

Combined with (A.10) the social planner growth rate of consumption,  $g_C^{SP}$ , is:

$$g_C^{SP} = \frac{1}{\sigma} \left[ a \left( \frac{K}{H} \right)^{a-1} - \delta - \rho \left( \frac{C}{H} \right) \right] \quad (\text{A.12})$$

Along the optimal trajectory the Hamiltonian is independent of time and, together with the transversality condition (A.9), we have that  $\Lambda^{SP} = 0$ , which implies that:

$$\lambda_\rho = -\frac{1}{\rho \left( \frac{C}{H} \right)} \left[ \frac{C^{1-\sigma}}{1-\sigma} + \lambda_K [K^a H^{1-a} - C - H^E - \delta K] + \lambda_H [H^E - \delta_h H] \right] \quad (\text{A.13})$$

From (A.3), (A.7) and the definitions of the costate variables we can get that:

$$-\frac{\dot{\lambda}_H}{\lambda_H} = (1-a) \left( \frac{K}{H} \right)^a - \delta - \rho \left( \frac{C}{H} \right) + \frac{\lambda_\rho}{\lambda_H} \rho'(\cdot) \frac{C}{H^2} \quad (\text{A.14})$$

Combining (A.5), (A.10), (A.13), and (A.14), yields under Proposition 3 the costate variables  $\lambda_K, \lambda_H, \lambda_\rho$ , as well as the physical to human capital stock in the socially planned economy at the BGP as  $\frac{K}{H} = \frac{a}{1-a}$ .

Suppose that there exists  $\tau > 0$  such that the CDE replicates the allocations and thus the growth rates of the SP, i.e. that is  $(\frac{K}{H})^{CDE} = (\frac{K}{H})^{SP} = \frac{a}{1-a}$ ,  $(\frac{C}{H})^{CDE} = (\frac{C}{H})^{SP}$ , and  $g_C^{CDE} = g_C^{SP}$ . The growth rate under the SP and the CDE will be given respectively by  $g_C^{SP} = a \left( (\frac{K}{H})^{SP} \right)^{a-1} - \delta - \rho (\frac{C}{H})^{SP}$  and  $g_C^{CDE} \equiv \frac{\dot{C}}{C} = (1-\tau)a \left( (\frac{K}{H})^{CDE} \right)^{a-1} - \delta - \rho (\frac{C}{H})^{SP}$ . The CDE can replicate the social planner solution,  $g_C^{SP} = g_C^{CDE}$ , only for  $\tau = 0$ . ■

The market economy cannot replicate the SP solution that requires to allocate initial endowments so that  $\frac{K_0}{H_0} = \frac{a}{1-a}$  and finance human capital expenditures with a lump-sum tax.

### Appendix 3: Proof of Proposition 3

Equations (27)-(29) form a system of 3 equations with three positive unknowns,  $\tilde{z}, \tilde{\omega}$  and  $\tilde{\tau}$  that give the solution of the RSB problem. To establish the uniqueness of the solution we solve (27) for  $\rho(\tilde{\omega}\tilde{z}) > 0$ , which holds for  $\tilde{\tau} < 1-a$ , and (29) for  $\tilde{\omega}$ , and substitute in (28) to obtain:

$$V(\tilde{z}, \tilde{\tau}) \equiv \left(\frac{a}{\sigma}\right)(1-\tilde{\tau})(\tilde{z})^{a-1} - (\tilde{z})^a \tilde{\tau} - \left(\frac{1}{\sigma} - 1\right)\delta - \frac{1}{\sigma} \frac{(1-a) - \tilde{\tau}}{\tilde{z}^{-a}} = 0$$

Since we have used all three equilibrium equations to obtain the last equation, it is sufficient to characterize the existence and uniqueness of equilibrium bundle  $(\tilde{z}, \tilde{\tau})$  if there exists a bundle  $(\tilde{z}^{RSB}, \tilde{\tau}^{RSB})$  such that  $V(\tilde{z}^{RSB}, \tilde{\tau}^{RSB}) = 0$  for any  $\tilde{\tau} < 1-a$  and  $\tilde{z}$  that satisfies the interior solution of the problem ( $\tilde{z} > 0, \tilde{\omega} > 0, \tilde{\tau} > 0, \rho > 0$ ). The method will be to separate function  $V(\tilde{z}, \tilde{\tau})$  in two functions and find their intersection to solve it. We define  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau}) \equiv \left(\frac{a}{\sigma}\right)(1-\tilde{\tau})(\tilde{z})^{a-1} - (\tilde{z})^a \tilde{\tau} - \left(\frac{1}{\sigma} - 1\right)\delta$  and  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau}) \equiv \frac{1}{\sigma} \left(\frac{(1-a)-\tilde{\tau}}{\tilde{z}^{-a}}\right)$ . Both  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$  and  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  are continuous in  $\tilde{z}$  and  $\tilde{\tau}$ . Also, from (29) in order for  $\tilde{\omega}(\tilde{z}) > 0$  to hold we must have  $\tilde{z} < \frac{1-\tilde{\tau}}{\tilde{\tau}}$ .

Equation  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$  has the following properties:

1.  $\lim_{\tilde{z} \rightarrow 0} \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = +\infty$ ,  $\lim_{\tilde{z} \rightarrow \frac{1-\tilde{\tau}}{\tilde{\tau}}} \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = \left(\frac{a}{\sigma} - 1\right)(1-\tilde{\tau})a\tilde{\tau}^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta$
2.  $\lim_{\tilde{\tau} \rightarrow 0} \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = \left(\frac{a}{\sigma}\right)(\tilde{z})^{a-1} - \left(\frac{1}{\sigma} - 1\right)$ ,  $\lim_{\tilde{\tau} \rightarrow 1-a} \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = \left(\frac{a}{\sigma}\right)a(\tilde{z})^{a-1} - (\tilde{z})^a(1-a) - \left(\frac{1}{\sigma} - 1\right)\delta$

3.  $\frac{\partial \Gamma^{RSB}(\tilde{z}, \tilde{\tau})}{\partial \tilde{z}} = \left(\frac{a}{\sigma}\right)(1 - \tilde{\tau})(a - 1)(\tilde{z})^{a-2} - a(\tilde{z})^{a-1}\tilde{\tau} < 0$
4.  $\frac{\partial \Gamma^{RSB}(\tilde{z}, \tilde{\tau})}{\partial \tilde{\tau}} = -\left(\frac{a}{\sigma}\right)(\tilde{z})^{a-1} - (\tilde{z})^a < 0.$

Equation  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  has the following properties:

1.  $\lim_{\tilde{z} \rightarrow 0} \Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = 0, \quad \lim_{\tilde{z} \rightarrow \frac{1-\tilde{\tau}}{\tilde{\tau}}} \Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = \frac{1}{\sigma} \left( \frac{(1-a)-\tilde{\tau}}{\left(\frac{1-\tilde{\tau}}{\tilde{\tau}}\right)^{-a}} \right) > 0$  (since  $\tilde{\tau} < 1 - a$ )
2.  $\lim_{\tilde{\tau} \rightarrow 0} \Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = \frac{(1-a)}{\sigma \tilde{z}^{-a}} > 0, \quad \lim_{\tilde{\tau} \rightarrow 1-a} \Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = 0$
3.  $\frac{\partial \Lambda^{RSB}(\tilde{z}, \tilde{\tau})}{\partial \tilde{z}} = \frac{(1-a)-\tilde{\tau}}{\sigma} \left( \frac{a \tilde{z}^{-a-1}}{\tilde{z}^{-2a}} \right) > 0$  (since  $\tilde{\tau} < 1 - a$ )
4.  $\frac{\partial \Lambda^{RSB}(\tilde{z}, \tilde{\tau})}{\partial \tilde{\tau}} = -\frac{\tilde{\tau}}{\tilde{z}^{-a}} < 0$  (since  $\tilde{\tau} < 1 - a$ )

From the properties of  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$  and  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  it follows that they are monotone functions in both arguments for the domain of the variables.

Thus, using all the information from equilibrium equations (27)-(29) and since both functions,  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$ ,  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$ , are monotone in both arguments if an intersection exists (see Proposition 1) this will be unique and only one bundle  $(\tilde{z}, \tilde{\tau}) = (\tilde{z}^{RSB}, \tilde{\tau}^{RSB})$  can solve  $V(\tilde{z}, \tilde{\tau})$  for any feasible tax rate. In turn, a unique solution for  $\tilde{z}$  gives from and  $\tilde{\omega}$  by (29) a unique solution for  $\tilde{\omega}$ . Thus, RSB policy objective acts as an equilibrium selection mechanism.

Now, in order to provide the necessary and sufficient conditions for existence we need to analyze the intersection of  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  and  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$ . In order for these functions to intersect in  $\tilde{z}$ -space we have the following:  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  starts from 0 and ends at  $\frac{1}{\sigma} \left( \frac{(1-a)-\tilde{\tau}}{\left(\frac{1-\tilde{\tau}}{\tilde{\tau}}\right)^{-a}} \right) > 0$  and is increasing.  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$  starts from  $+\infty$ , ends at  $\left(\frac{a}{\sigma} - 1\right)(1 - \tilde{\tau})^a \tilde{\tau}^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta$ . Thus, for  $\tilde{z} \rightarrow 0$  then  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau}) > \Lambda^{RSB}(\tilde{z}, \tilde{\tau})$ . As  $\tilde{z} \rightarrow \frac{1-\tilde{\tau}}{\tilde{\tau}}$  and  $\left(\frac{a}{\sigma} - 1\right)(1 - \tilde{\tau})^a \tilde{\tau}^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta < \frac{1}{\sigma} \left( \frac{(1-a)-\tilde{\tau}}{\left(\frac{1-\tilde{\tau}}{\tilde{\tau}}\right)^{-a}} \right)$  then  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau}) < \Lambda^{RSB}(\tilde{z}, \tilde{\tau})$ . Since, both functions are continuous then  $\left(\frac{a}{\sigma} - 1\right)(1 - \tilde{\tau})^a \tilde{\tau}^{1-a} - \left(\frac{1}{\sigma} - 1\right)\delta < \frac{1}{\sigma} \left( \frac{(1-a)-\tilde{\tau}}{\left(\frac{1-\tilde{\tau}}{\tilde{\tau}}\right)^{-a}} \right)$  is a sufficient condition for the existence of a solution in  $\tilde{z}$ -space which holds for any  $\tilde{\tau}$  if  $\sigma < a$ .

A similar approach holds for the  $\tilde{\tau}$ -space. In order for the above functions to intersect in  $\tilde{\tau}$ -space we have the following:  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau})$  starts from  $\frac{(1-a)}{\sigma \tilde{z}^{-a}}$ , ends at zero and is decreasing.  $\Gamma^{RSB}(\tilde{z}, \tilde{\tau})$  starts from  $\left(\frac{a}{\sigma}\right)(\tilde{z})^{a-1} - \left(\frac{1}{\sigma} - 1\right)$ , ends at  $\left(\frac{a}{\sigma}\right)a(\tilde{z})^{a-1} - (\tilde{z})^a(1 - a) - \left(\frac{1}{\sigma} - 1\right)\delta$  and is increasing. Thus, for  $\tilde{\tau} \rightarrow 0$  then  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = \frac{(1-a)}{\sigma \tilde{z}^{-a}} > \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = 0$ . As  $\tilde{\tau} \rightarrow 1 - a$  we have  $\Lambda^{RSB}(\tilde{z}, \tilde{\tau}) = 0 < \Gamma^{RSB}(\tilde{z}, \tilde{\tau}) = \frac{1}{\sigma} \left( \frac{(1-a)-\tilde{\tau}}{\left(\frac{1-\tilde{\tau}}{\tilde{\tau}}\right)^{-a}} \right)$ . ■

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**Table 1.** Changes in  $\sigma$  and CDE properties

$\sigma$	$g_1^{CDE}$	$g_2^{CDE}$	$\bar{z}_1$	$\bar{z}_2$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\rho}_1$	$\bar{\rho}_2$
Multiplicity								
<i>0.2</i>	0.314	0.376	0.719	1.008	0.368	0.195	0.265	0.198
<i>0.25</i>	0.295	0.401	0.640	1.136	0.429	0.136	0.276	0.156
<i>0.3</i>	0.282	0.419	0.591	1.234	0.473	0.096	0.281	0.119
<i>0.35</i>	0.273	0.434	0.554	1.319	0.508	0.063	0.283	0.084
<i>0.4</i>	0.265	0.448	0.524	1.397	0.539	0.035	0.284	0.050
Uniqueness								
<i>0.5</i>	0.251	-	0.477	-	0.592	-	0.284	-
<i>0.6</i>	0.241	-	0.441	-	0.638	-	0.282	-
<i>0.7</i>	0.232	-	0.412	-	0.678	-	0.280	-
<i>0.8</i>	0.224	-	0.387	-	0.715	-	0.278	-
<i>0.9</i>	0.217	-	0.366	-	0.749	-	0.275	-

Notes:  $a = 0.5$ ,  $\delta = 0.025$ ,  $\tau = 0.4$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ .

**Table 2.** Changes in  $\tau$  and multiplicity in the CDE

$\tau$	$g_1^{CDE}$	$g_2^{CDE}$	$\bar{z}_1$	$\bar{z}_2$	$\bar{\omega}_1$	$\bar{\omega}_2$	$\bar{\rho}_1$	$\bar{\rho}_2$
<i>0.2</i>	0.118	0.366	0.510	3.824	0.977	0.018	0.499	0.069
<i>0.25</i>	0.155	0.394	0.520	2.810	0.859	0.028	0.448	0.080
<i>0.3</i>	0.194	0.413	0.534	2.127	0.738	0.042	0.395	0.091
<i>0.35</i>	0.235	0.422	0.555	1.628	0.611	0.063	0.341	0.103
<i>0.4</i>	0.282	0.419	0.591	1.234	0.473	0.096	0.281	0.119

Notes:  $a = 0.5$ ,  $\delta = 0.025$ ,  $\sigma = 0.3$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ .

**Table 3.** Changes in  $\sigma$  and RSB equilibrium selection

$\sigma$	$g^{RSB}$	$\tilde{\tau}$	$\tilde{z}$	$\tilde{\omega}$	$\tilde{\rho}$
Multiplicity in CDE					
<i>0.2</i>	0.399	0.359	1.395	0.119	0.166
<i>0.25</i>	0.409	0.376	1.332	0.107	0.143
<i>0.3</i>	0.420	0.395	1.266	0.092	0.117
<i>0.35</i>	0.433	0.418	1.197	0.074	0.089
<i>0.4</i>	0.448	0.446	1.123	0.050	0.057

Notes:  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ .

**Table 4.** Changes in  $\sigma$  and GM equilibrium selection

$\sigma$	$g^{GM}$	$\hat{\tau}$	$\hat{z}$	$\hat{\omega}$	$\hat{\rho}$
<i>0.2</i>	0.401	0.330	1.669	0.092	0.154
<i>0.3</i>	0.422	0.366	1.486	0.072	0.108
<i>0.4</i>	0.448	0.422	1.259	0.042	0.054
<i>0.5</i>	0.392	0.683	0.373	0.102	0.039
<i>0.6</i>	0.342	0.677	0.294	0.229	0.068
<i>0.7</i>	0.314	0.669	0.257	0.313	0.082
<i>0.8</i>	0.294	0.661	0.233	0.381	0.090
<i>0.9</i>	0.279	0.655	0.216	0.439	0.095
<i>1</i>	0.266	0.649	0.201	0.491	0.099

Notes:  $a = 0.5$ ,  $\delta = 0.025$ ,  $b = 1$ ,  $\check{\rho} = 0.001$ .

**Table 5.** Changes in  $b$  and GM equilibrium selection

$b$	$g^{GM}$	$\hat{\tau}$	$\hat{z}$	$\hat{\omega}$	$\frac{\hat{C}}{\hat{H}}$	$\hat{\rho}$
Multiplicity ( $\sigma = 0.3$ )						
<i>0.2</i>	0.277	0.134	5.08	0.082	0.419	0.084
<i>0.4</i>	0.348	0.223	2.798	0.092	0.256	0.104
<i>0.6</i>	0.385	0.287	2.046	0.088	0.181	0.109
<i>1</i>	0.422	0.367	1.486	0.072	0.107	0.108
<i>1.4</i>	0.439	0.410	1.283	0.058	0.074	0.105
<i>1.8</i>	0.448	0.433	1.189	0.047	0.056	0.101
<i>10</i>	0.471	0.491	1.019	0.008	0.008	0.086
Uniqueness ( $\sigma = 1$ )						
<i>0.2</i>	0.312	0.545	0.382	0.400	0.153	0.031
<i>0.4</i>	0.298	0.579	0.310	0.432	0.134	0.055
<i>0.6</i>	0.285	0.607	0.262	0.456	0.120	0.072
<i>1</i>	0.266	0.649	0.201	0.491	0.099	0.100
<i>1.4</i>	0.251	0.679	0.165	0.515	0.085	0.120
<i>1.8</i>	0.238	0.702	0.140	0.533	0.075	0.135
<i>10</i>	0.138	0.847	0.037	0.634	0.023	0.235

Notes:  $a = 0.5$ ,  $\delta = 0.025$ ,  $\check{\rho} = 0.001$ .**Table 6.** Changes in  $\sigma$  and  $b$  and summarized GM allocation responses

	$g^{GM}$	$\hat{\tau}$	$\hat{z}$	$\hat{\omega}$	$\frac{\hat{C}}{\hat{H}}$	$\hat{\rho}$
$\sigma$ low	(+)	(+)	(-)	(-)	(-)	(-)
$\sigma$ high	(-)	(+)	(-)	(+)	(+)	(+)
$\sigma = 0.3$ and low $b$	(+)	(+)	(-)	(+)	(-)	(+)
$\sigma = 0.3$ and high $b$	(+)	(+)	(-)	(-)	(-)	(-)
$\sigma = 1$ and low $b$	(-)	(+)	(-)	(+)	(-)	(+)
$\sigma = 1$ and high $b$	(-)	(+)	(-)	(+)	(-)	(+)