

This page may be removed to conceal the identities of the authors

An intertemporal model of the real exchange rate, stock market, and international debt dynamics: policy simulations

Saziye Gazioglu
and
W. David McCausland¹

Department of Economics, University of Aberdeen, Edward Wright Building,
Old Aberdeen, AB24 3QY. Tel: +44 (0)1224 272182/0
Fax: +44 (0)1224 272181 Email: pec086/pec140@abdn.ac.uk

J. E. Lit. Nos.: E60, F40, G15, H60

Key words: stock market, international debt, the real exchange rate

Abstract

This paper develops an open economy intertemporal optimising model that seeks to analyse the effect of bill financed government expenditure on several key financial markets. The main results suggest that an increase in bill financed government expenditure leads to a rise in net international debt, a fall in domestic the real exchange rate and a fall in the stock market value. Furthermore, due to the presence of non-linearities in the model, reversing the deficit financing policy doesn't restore the initial net international credit, high stock market value state. Instead, the country finds itself stuck in an international debt and low stock market value trap.

© Copyright: Department of Economics, University of Aberdeen

E286 11/03/2004 12:48 PM

¹ Comments received on an earlier version of the paper presented at Louisiana State University and Bilkent University are gratefully acknowledged; all remaining errors are our own.

An Intertemporal Model of The real exchange rate, Stock Market, and International Debt Dynamics

J. E. Lit. Nos.: E60, F40, G15, H60

Key words: stock market, international debt, the real exchange rate

Abstract

This paper develops an open economy intertemporal optimising model that seeks to analyse the effect of bill financed government expenditure on several key financial markets. The main results suggest that an increase in bill financed government expenditure leads to a rise in net international debt, a fall in domestic the real exchange rate and a fall in the stock market value. Furthermore, due to the presence of non-linearities in the model, reversing the deficit financing policy doesn't restore the initial net international credit, high stock market value state. Instead, the country finds itself stuck in an international debt and low stock market value trap.

1 Introduction

Our motivation in this paper is to explore the effect on international debt of widening the portfolio choice to include shares as well as government bills. Bill financing the budget deficit, through portfolio reallocations, affects the real exchange rate and net international debt. Implicitly, there is a transfer of debt from being domestic to being international. Evidence from developing countries seems to confirm this mechanism. Our results show that an increase in bill financed government expenditure leads to a rise in net international debt, and, due to the presence of non-linearities in the model, reversing the deficit financing policy doesn't restore the initial state of net international credit, but rather, the country finds itself stuck in an international debt trap.

This paper constructs an open economy intertemporal optimising model that extends the approach developed by Obstfeld and Rogoff (1995) by considering the dynamics of the real exchange rate in a Ramsey (1928) type continuous-time framework. Karayalcen (1996) adopts this approach with adjustment costs of investment with free international mobility of capital and instantaneous mobility of ownership claims to capital stocks. In this paper we develop this approach to include the real exchange rate, stock market prices and debt dynamics, with non-linearities in international debt. The model is based around the behaviour of representative utility maximising agents in an environment characterised by a number of intertemporal stock-flow constraints. On the firm side, we consider the behaviour of a representative profit maximising perfectly competitive firm subject to a standard neo-classical production function. Some units of output are used to augment the capital stock - the quantity of such capital investment is determined optimally by investing up until the point where the cost of investment exactly matches the real return. Interestingly, the approach developed here gives a strong micro-foundations justification for the use of the arbitrage conditions that result from this technique in new macro models.

Our key innovation is the integration of the dynamics of the real exchange rate, net international debt and the stock market. Although the model developed in this paper is intrinsically Ricardian, the addition of the analysis of the stock market allows us to make a bridge between the firm and the financial sector (this link is formally represented by the combined constraint in equation (10)). Since shares and government bills are substitutes, the

issue of bills has dynamic effects in the stock market, as well as on net international debt and domestic the real exchange rate. We fully integrate the dynamics of the real exchange rate with the dynamics of net international debt, and uniquely, the stock market. Although Obstfeld and Rogoff (1995) do deal with the dynamics of the stock market, for the first time in the literature we integrate these with the dynamics of net international debt and the real exchange rate. Finally, a very important product of the use of our non linear framework is that we can capture hysteresis effects in the stock market, the real exchange rate and net international debt dynamics.

In view of the importance attached to the control of budget deficits in contemporary policy making debates, this paper seeks to analyse the effects of changes in such deficit expenditure on other key financial markets, such as the stock market. Although there may be a short term incentive to run large deficits, we investigate the possibility that there exists unforeseen long run consequences to this course of action.

In section two we develop the model. In section three we begin our policy analysis. Section four presents our findings and our simulation results. Concluding comments are offered in section five.

2 The Model

We consider first the behaviour of firms, then of households, before proceeding to solve the model.

2.1 Firms

We assume there are a large number of perfectly competitive firms with constant returns to scale, each earning zero long run profit. Each firm produces both non-traded goods (goods for domestic consumption) and traded goods (goods for export). The price of non-traded goods is normalised to unity, and so therefore, due to purchasing power parity, the real exchange rate determines the price of traded goods. At each point in time they employ the given stocks of labour and capital, pay them their marginal product, and sell the resulting output. The representative firm's constraint is therefore

$$Y = WN + X^d D^d + \dot{K} \quad (1)$$

which states that the revenue of the firm, Y , which is equal to the revenue from traded and non-traded goods, is divided between payment to labour, WN , dividend payments, $X^d D^d$, and physical capital augmentation, $\dot{K} \equiv I$. This profit maximising, perfectly competitive firm operates subject to a standard neo-classical production function of the form given by

$$Y = Zq(K, N) \quad (2)$$

where Z is an exogenous constant and $q_K > 0$, $q_{KK} < 0$, $q_N > 0$, $q_{NN} < 0$. Part of the firm's output is used to augment the physical capital stock. The quantity of investment is determined optimally by investing up until the point where the real return exactly equals the cost of investing, derived in equation **Hata! Başvuru kaynağı bulunamadı.** of the appendix and repeated here for convenience

$$Zq_K = R^d \quad (3)$$

This is simply the condition that the marginal product of capital is equal to the real return on capital.

We also assume a perfectly competitive labour market, which, given the production function above combined with equation (1) gives us the equilibrium condition that the marginal product of labour is equal to the real wage

$$Zq_N = W \quad (4)$$

Finally, the objective of the firm is to maximise a profit function of the form

$$\pi = \int_0^{\infty} e^{-\beta t} [Y(K, N) - I[1 + k]] dt \quad (5)$$

where k are the installation costs of investment. Solving this maximisation problem yields the optimal capital investment level $I = \dot{K}$ as shown in equation **Hata! Başvuru kaynağı bulunamadı.** in the appendix.

2.2 Households

Representative domestic agents maximise time separable utility functions of the form

$$\max_{\{A(t)+G(t)\}} U = \int_0^{\infty} U[A(t) + G(t)] e^{-\beta t} dt \quad (6)$$

where A represents domestic aggregate consumption index (as in Rogoff and Obstfeld, 1996), and G net government spending, subject to the following three constraints². We deal with each in turn.

Firstly, the government budget constraint

$$\dot{B} \equiv \dot{B}^d + \dot{B}^f / C = R^d B^d + R^d B^f / C + G \quad (7)$$

² A full symbols list is provided in Appendix F.

The issue of domestic bills, \dot{B} , to domestic residents, \dot{B}^d , and foreign residents, \dot{B}^f , finances government expenditure net of tax revenue, G , and the cost of debt servicing, $R^d B^d + R^f B^f / C$, where C is the real exchange rate defined as the price of one unit of foreign currency. Thus, the issue of bills involves an intertemporal transfer from the future to the present. We stress that we do not in this paper seek to model *how* the government comes to this decision of bill finance *vis-à-vis* alternative financing methods; rather our focus is on the *effects* of this method once it has been chosen, and the mechanism by which it affects the real exchange rate and net international debt. As in Obstfeld and Rogoff (1995), Ricardian Equivalence is a feature of this model. However, Ricardian Equivalence does not imply the same thing for domestic and foreign residents, since domestic residents know that bond financing increases their future tax liabilities, but the same thing cannot be said for foreign residents. This opens up a channel for fiscal policy to be effective.

Secondly, the stock market constraint, following Obstfeld and Rogoff (1995),

$$V^d \dot{X}^d \equiv X^d \dot{V}^d + X^d D^d \quad (8)$$

states³ that a change in the proportion (X^d) of the value of domestic firms⁴ that domestic individuals own (in other words, shares: the value of domestic claims to the entire future profits of domestic firms, V^d), $V^d \dot{X}^d$, is equal to the domestic proportion of the change in the stock market valuation of these shares, $X^d \dot{V}^d$, plus their proportion of dividends, $X^d D^d$.

Thirdly and lastly, the balance of payments constraint

$$\dot{H} = R^d B^f / C - R^f F^d + \Pi - T + H \left(1 + \frac{\dot{C}}{C} \right) (1 + R^f) \quad (9)$$

states that the net accumulation of foreign assets⁵ by domestic agents, $-\dot{H}$, can only be accumulated by running a trade surplus, T , adjusted accordingly to take account of the net

³ See Obstfeld and Rogoff (1996, p.100) for a discrete time formulation.

⁴ We assume that there are a large number of homogenous perfectly competitive domestic firms producing goods for both domestic and foreign consumption.

⁵ Net international debt, H , is defined in Appendix A.

income from holding these net asset stocks, $R^f F^d - R^d B^f / C$ and real net profit repatriation⁶, Π , defined as the foreign owned share of domestic dividends minus the domestic owned share of foreign dividends⁷, plus any capital gain from holding foreign money in terms of foreign goods. Hence, consumption is affected by the income from foreign asset holdings and the repayments on debt. Thus, the existence of the balance of payments constraint stresses the intertemporal choice being made by consumers to transfer their consumption intertemporally, by both domestic and international mechanisms.

Combining equations (8) through (9) yields the aggregate constraint

$$\begin{aligned}
 & -\dot{B}^d + \dot{X} - C\dot{H} && (10) \\
 & = Y + R^f F^d - A - G - I - R^d B^d - R^d B^f / C \\
 & \quad + X^d (\dot{V}^d / V^d + D^d / V^d) + X^f (\dot{V}^f / V^f + D^f / V^f) \\
 & \quad + H \left(1 + \frac{\dot{C}}{C} \right) (1 + R^f)
 \end{aligned}$$

In essence, therefore, the right hand side of the constraint represents net domestic ‘income’ (factor earnings, net interest from asset holdings, return on shares) minus ‘consumption’ (private, government and investment), reflected by the ‘saving’ (net wealth accumulation) on the left hand side.

⁶ $\Pi \equiv (1 - X^d) (D^d / C) - X^f D^f$

⁷ It is important to note that both countries are ‘large’ - agents in both countries hold each others’ assets.

2.3 Equilibrium Conditions

Maximising⁸ (6) subject to (10) yields the familiar Euler equations, which can be combined to yield the extended Blanchard (1981) arbitrage condition

$$R^d = \dot{V}^d/V^d + D^d/V^d \quad (11)$$

Finally, we have the standard uncovered interest parity condition⁹

$$1 + \frac{\dot{C}^e}{C} = 1 + \frac{\dot{C}}{C} = \frac{1 + R^d}{1 + R^f} \quad (12)$$

From equation (9), since $T_C > 0$ (the Marshall-Lerner condition: a depreciation improves the trade balance), $T_H > 0$ (wealth effects: a rise in international debt is a rise in foreign wealth which improves exports and a fall in domestic wealth which reduces imports, hence improving the trade balance), $T_V > 0$ (since the value of domestic firms reflects their relative productivity, rises in the stock market valuation of domestic firms tends to be associated with an improving trade balance) and $T_G < 0$ (increases in government spending worsen the trade balance), and assuming profit repatriation effects are small relative to trade effects, therefore, from equation (9), $\dot{H}_C < 0$, $\dot{H}_H < 0$ ¹⁰, $\dot{H}_V < 0$ and $\dot{H}_G > 0$

From equation (11), $\dot{V}^d = V^d R^d - D^d(C, H, V, G)$, since $D_C > 0$ (rise in domestic the real exchange rate improves domestic profits and hence dividends), $D_H < 0$ (increases in net international debt, through wealth effects, adversely affect domestic profitability¹¹), $D_V > 0$

⁸ The working is shown in Appendix B, and follows Obstfeld and Rogoff (1996) and Barro and Sala-i-Martin (1995).

⁹ We do not go into the derivation of this, since it is well known that it may also be derived from optimising behaviour: indeed UIP *is* an arbitrage condition equating the forward price of foreign assets (bills and shares) with the spot exchange rate minus the discounted value of the interest foregone by holding foreign assets. We assume the perfect foresight approximation to rational expectations. Thus, c is the only ‘jump variable’ in the model. Instantaneous adjustment of the real exchange rate captures the effect of price adjustment.

¹⁰ See Roberts and McCausland (1998) for further comments regarding the sign of \dot{H}_H .

¹¹ We make the standard assumption that the greatest proportion of domestic production is for the domestic market.

(the stock market valuation of firms is in effect the valuation of the entire future profit stream and is hence positively related to dividends)¹² and $D_G < 0$ (increased government spending decreases dividends because the issue of government bills implies depressed *future* consumption, which firms respond to by reducing current dividends), therefore $\dot{V}_C < 0$, $\dot{V}_H > 0$, $\dot{V}_V < 0$ ¹³ and $\dot{V}_G > 0$.

Finally, from equation (11)¹⁴, $R_C > 0$, $R_H < 0$, $R_V > 0$ and $R_G < 0$. Hence, from equation (12)¹⁵, $\dot{C}_C > 0$, $\dot{C}_H < 0$, $\dot{C}_V > 0$ and $\dot{C}_G < 0$.

Equations (9), (11) and (12) therefore capture the dynamics of the whole system, and, given the discussion above, may be summarised¹⁶ in matrix form by

$$\begin{bmatrix} \dot{C} \\ \dot{H} \\ \dot{V} \end{bmatrix} = \begin{bmatrix} \dot{C}_C & \dot{C}_H & \dot{C}_V \\ \dot{H}_C & \dot{H}_H & \dot{H}_V \\ \dot{V}_C & \dot{V}_H & \dot{V}_V \end{bmatrix} \begin{bmatrix} C \\ H \\ V \end{bmatrix} + \begin{bmatrix} \dot{C}_G \\ \dot{H}_G \\ \dot{V}_G \end{bmatrix} [G] \quad (13)$$

where the signs of the elements of the matrix are, from the discussion above: $\dot{C}_C > 0$, $\dot{C}_H < 0$, $\dot{C}_V > 0$ and $\dot{C}_G < 0$; $\dot{H}_C < 0$, $\dot{H}_H < 0$ ¹⁷, $\dot{H}_V < 0$ and $\dot{H}_G > 0$; $\dot{V}_C < 0$, $\dot{V}_H > 0$, $\dot{V}_V < 0$ and $\dot{V}_G > 0$. We now have all the tools in place necessary to proceed to the policy analysis conducted in the next section.

¹² A change in this stock market valuation is assumed not to affect internally generated physical capital augmentation.

¹³ From equation (11), $\dot{V}_V = R^d - D_V$. We assume initially that $R^d < D_V$, implying the stock market return dominates the return on net government borrowing, therefore $\dot{V}_V < 0$.

¹⁴ We have obtained above the signs of the partial derivatives of D^d , hence, $R^d = \dot{V}^d/V^d + D^d \left(\overset{+}{C}, \overset{-}{H}, \overset{+}{V}, \overset{-}{G} \right)$ from equation (11).

¹⁵ The partial derivatives for R^f will, of course, have the opposite signs to those given for R^d .

¹⁶ Although portfolio shares dynamically adjust to flow disequilibrium, they are, of course, constant in long run equilibrium, hence $\dot{X} = 0$. Furthermore, following Obstfeld and Rogoff (1996), in order to concentrate on the dynamics of domestic net international debt, the real exchange rate and the stock market, we assume $\dot{B}^d = 0$ and $\dot{V}^f = 0$.

¹⁷ See Roberts and McCausland (1998) for further comments regarding the sign of \dot{H}_H .

3 Policy Analysis

Following Kawai (1985), we can decompose the dynamic system represented by equation (13) into three dynamic sub-systems represented in equations (14) through (16), where variables are redefined in terms of deviations about long run equilibrium. The precise definitions of the matrix coefficients are given in Appendix C. Note that C is the ‘jump’ variable in this model in accordance with the perfect foresight approximation to rational expectations, which is the standard assumption employed in the literature in these models.

$$\begin{bmatrix} \dot{c} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} w_{11}^+ & w_{12}^- \\ w_{21}^- & w_{22}^- \end{bmatrix} \begin{bmatrix} c \\ h \end{bmatrix} + \begin{bmatrix} \omega_{11}^- \\ \omega_{21}^+ \end{bmatrix} [g] \quad (14)$$

$$\begin{bmatrix} \dot{v} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} x_{11}^- & x_{12}^+ \\ x_{21}^- & x_{22}^- \end{bmatrix} \begin{bmatrix} v \\ h \end{bmatrix} + \begin{bmatrix} \chi_{11}^+ \\ \chi_{21}^+ \end{bmatrix} [g] \quad (15)$$

$$\begin{bmatrix} \dot{c} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} z_{11}^+ & z_{12}^+ \\ z_{21}^- & z_{22}^- \end{bmatrix} \begin{bmatrix} c \\ v \end{bmatrix} + \begin{bmatrix} \zeta_{11}^- \\ \zeta_{21}^+ \end{bmatrix} [g] \quad (16)$$

The dynamics of these three sub-systems are illustrated in Figure 1 below, which shows the effect of a rise in bill financed government expenditure (g). Appendix D gives the derivations of their dynamic properties and Appendix E the slopes and shifts of the respective stationary loci.

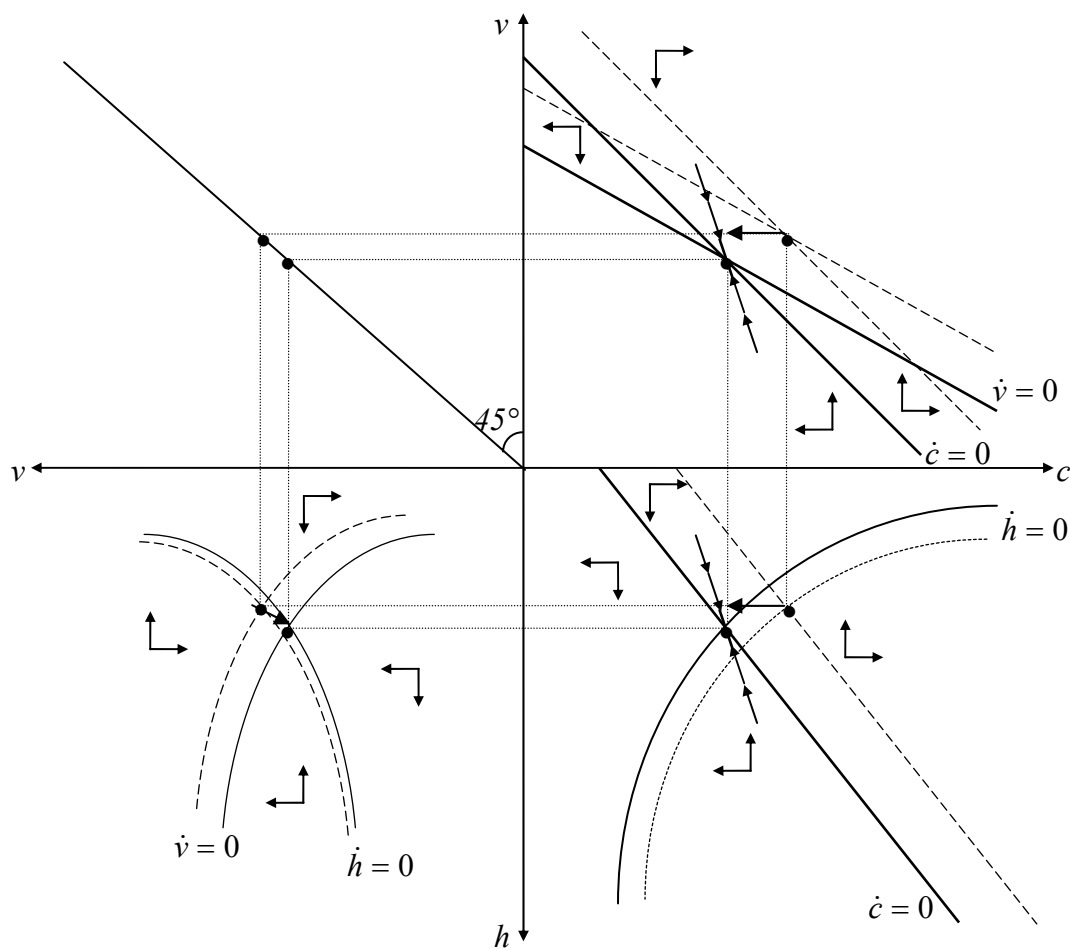


Figure 1

Effect of rise in government deficit
(the loci corresponding to the initial equilibria are represented by dashed lines)

Long Run Effects

We now consider the long run effects of an unanticipated rise in bill-financed government expenditure, represented by a rise in the parameter g . In the bottom right quadrant we illustrate the international debt and the real exchange rate ($c-h$) dynamics. A rise in g shifts both stationary loci leftwards (the original loci are denoted by dashed lines) resulting in a long run rise in net international debt (h) and fall in the domestic real exchange rate (c). The intuition behind these results is clear. Firstly there is the direct effect that an issue of government debt will partly be held by foreigners and therefore constitutes by definition part of net international debt. In addition, there is also an indirect effect. The issue of domestic

bills generates a rise in the return on domestic bills (R^d) and fall in the return on foreign bills (R^f). This excess of the domestic over the foreign return generates a capital inflow (foreign residents purchasing domestic bills). This leads to a balance of payments surplus. Balance is restored by a fall in the real exchange rate (c), which reduces the trade balance, and in turn, results in a rise in net international debt (h). In summary, therefore, the domestic real exchange rate falls and net international debt rises.

In the bottom left quadrant we illustrate the international debt and stock market ($h-v$) dynamics. A rise in g shifts both stationary loci rightwards resulting in a long run rise in net international debt (h) and a long run fall in the stock market value (v). We have given above some intuition as to the route through which net international debt rises, so all that remains is to explain the effect on stock market value. A fall in the domestic real exchange rate and a fall in domestic wealth (rise in net international debt) reduce the stock market value of future domestic profits.

In the top right quadrant we illustrate the stock market and the real exchange rate ($c-v$) dynamics. A rise in g shifts both stationary loci leftwards, resulting in a long run fall in the stock market value (v) and fall in domestic the real exchange rate (c). We have already provided some intuition behind these results above. The top left quadrant is a merely a pictorial device.

Short Run Adjustment

Finally, during the adjustment of the system, we noted that $R^d > R^f$. From UIP this implies the expectation of a rise in the domestic real exchange rate. The only way these expectations can be consistent with the long run equilibrium fall in domestic the real exchange rate is through the domestic real exchange rate initially falling by more than the required long run fall and then adjusting upwards. In other words, there is the real exchange rate overshooting. An alternative way of looking at this overshooting phenomena is in terms of the short run divergence between the returns on the different assets.

We show in Appendix D that the adjustment in the top right and bottom right quadrants is characterised by saddle-path behaviour towards long run equilibrium. This reflects the fact that both quadrants contain the 'jump variable' c (as already noted in footnote 9, c is the

only ‘jump variable’ in the model). On the other hand, adjustment in the bottom left quadrant is characterised by stable cyclical behaviour towards long run equilibrium (as shown in Appendix D).

Thus we now have the basic model in place, which assumes away the existence of non-linearities. We now proceed to relax these assumptions and analyse the policy consequences of the inherent non-linearities in the model. We address this issue in the next section.

4 The Irreversible Effects of Deficit Spending

In this section we relax some of the coefficient sign assumptions made in the previous section, and, in doing so, admit the possibility of multiple equilibria. It is well known in the hysteresis literature¹⁸ that it is ‘large’ changes that may lead to policy irreversibility in non-linear models such as this one, where ‘large’ is defined to be a change beyond the critical value that triggers the loss of an equilibrium point. A ‘large’ unanticipated rise in the (bill-financed) government deficit may lead to a loss of the net international credit/high stock market value equilibrium, and a structural shift to the net international debt/low stock market equilibrium. This is represented by a shift from E_0 to E_1 on Figure 2.

¹⁸ There are a large number of references on hysteresis and irreversibility, indeed the issues are covered today by most good advanced textbooks. Hysteresis is defined here in the mathematical sense of true remanence (rather than mere persistence). Hysteresis thus refers to a situation where an effect *remains*, even after its original cause has been removed. Varian (1979) is an early example of the application of this technique of analysis. Cross (1993) gives an overview of the more methodological foundations of hysteresis together with a list of more recent applications.

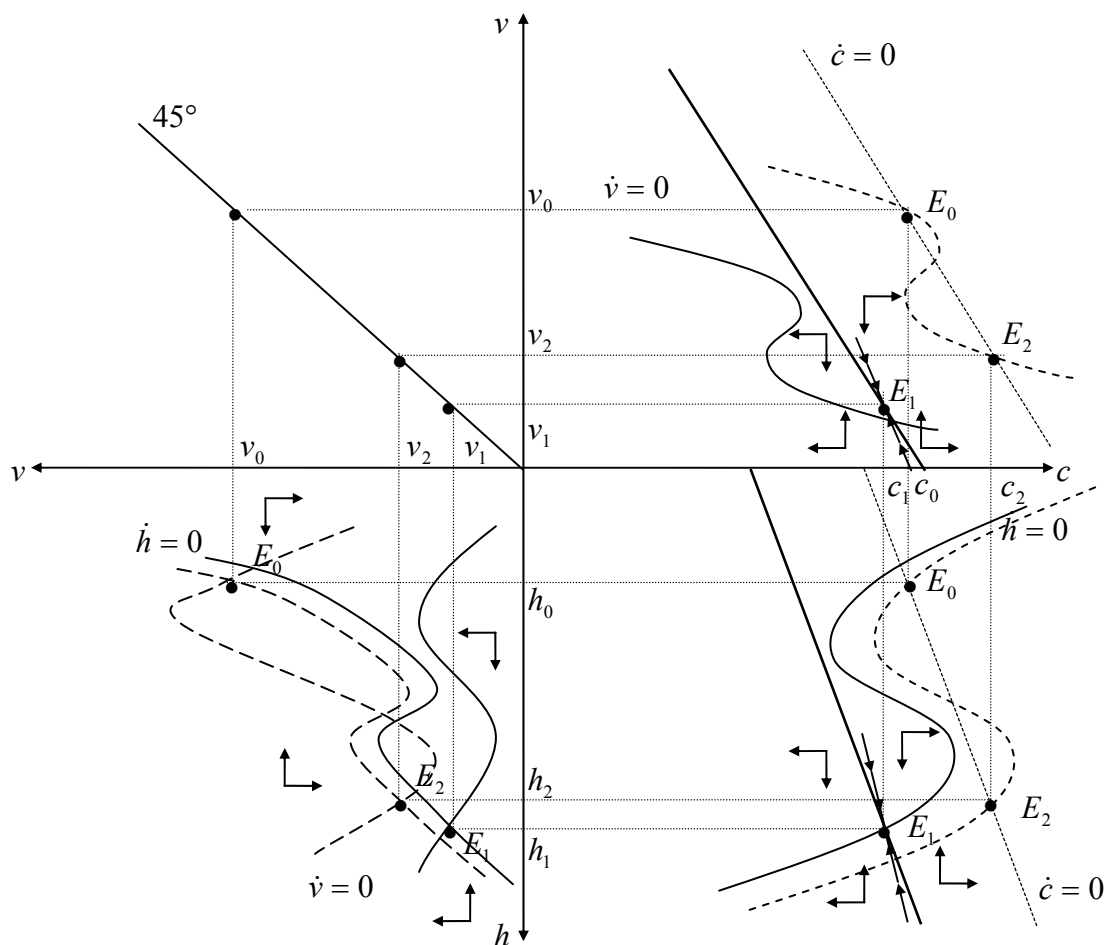


Figure 2

Effect of rise in government deficit: hysteresis

(the loci corresponding to the initial equilibria are represented by dashed lines)

(h, v, c show deviations around the long run equilibrium values of H, V, C , respectively, as shown in the appendix)

The shapes of the loci in Figure 2 are easily verified by appealing to the Intermediate Value Theorem as shown below.

Proposition 1

We have established in the text that

- $T_H > 0$
- $\dot{H}_H = R^d - T_H > 0$ or $\dot{H}_H = R^d - T_H < 0$

These imply that

\dot{H}_H is non-linear (see Lemma 1 below) ■

Lemma 1

Under Proposition 1, there exists multiple locally stable steady state equilibria if the following conditions are satisfied

- $\lim H = -F \Rightarrow T_H \rightarrow \infty \Rightarrow \dot{H}_H \rightarrow -\infty$
- $\lim H = B/C \Rightarrow T_H \rightarrow \infty \Rightarrow \dot{H}_H \rightarrow -\infty$
- $\exists H \ni \dot{H}_H > 0$

where $-F, B/C$ are the upper and lower bounds to H . See Roberts and McCausland (1999) for simulations using Cobb-Douglas functional forms that satisfy these conditions. The first two conditions state that the slope of \dot{H}_H in the \dot{H}, H space tends to minus infinity at both the upper and lower bounds. The third condition states that if there exists a region between these bounds in which $\dot{H}_H > 0$, then the Intermediate Value Theorem implies that the shape of the locus shown in Figure 2. Roberts and McCausland (1999) use the Cobb-Douglas form which satisfies these three conditions $\dot{H} = -\{A(H + F)^\theta C^{\alpha-1} - B(D - CH)^\theta C^{-\beta}\} + rH$, where the term in curly brackets is the trade balance (exports minus imports, where exports and imports are determined by wealth and the real exchange rate and the parameters A and B) and the term outside the brackets is the debt repayment term. The lower and upper bounds are found by setting exports and imports to zero respectively. The fact that $T_{HHH} > 0$ confirms that there is only a single minimum, and hence only a single region over which $\dot{H}_H > 0$.

Proof: Since \dot{H}_H is continuous in H , Lemma 1 follows from the Intermediate Value Theorem, noting Figure 2. ■

Proposition 2

We have established in the text that

- $D_V > 0, D_N > 0$
- $\dot{V}_V = R^d - D_V < 0$ or $\dot{V}_V = R^d - D_V > 0, \dot{V}_N = -D_N < 0$

These imply that

1. \dot{V}_V is non-linear (see Lemma 1 below) ■

Lemma 2

Under *Proposition 2*, there exists multiple locally stable steady state equilibria if the following conditions are satisfied

- $\lim V = -J \Rightarrow D_V \rightarrow \infty \Rightarrow \dot{V}_V \rightarrow -\infty$
- $\lim V = P \Rightarrow D_V \rightarrow \infty \Rightarrow \dot{V}_V \rightarrow -\infty$
- $\exists V \ni \dot{V}_V > 0$

Proof: Since $\dot{V}_V(V, N, R)$ is continuous in V , Lemma 1 follows from the Intermediate Value Theorem, noting Figure 1 and Figure 2 and Figure 3 below. ■

The first two conditions state that the slope of \dot{V}_V in the \dot{V}, V space tends to minus infinity at both the upper and lower bounds. The third condition states that if there exists a region between these bounds in which $\dot{V}_V > 0$, then the Intermediate Value Theorem implies that the shape of the locus shown in Figures 1-3.

Below we provide a simple plausible example of a dividend function that satisfies the sufficient conditions for the existence of multiple steady state equilibria listed in Lemma 2. We conduct simulations using Cobb-Douglas forms that satisfy the conditions given in *Lemma 2*. Suppose $D(C, H, V, G)$ takes the form

$$D = L(V + J)^\phi C^\alpha H^\gamma - M(P - V)^\eta C^{-\mu} H^\varepsilon \quad (17)$$

where the first term represents revenue and the second term represents costs, under the simplifying assumption that all profits are redistributed to shareholders in the form of dividends. L is a portmanteau coefficient representing all other factors determining revenue (and here specifically representing the net revenue effect of government spending), M is a

portmanteau coefficient representing all other factors determining costs, P and J are upper and lower bounds to stock market value (which are found by setting revenues and costs to zero respectively), and $0 < (\phi, \eta) < 1$ reflect the properties of diminishing marginal revenues and marginal costs. We assume the plausible parameter value of $R = 0.04$, $(\phi, \eta, \alpha, \mu, \gamma, \varepsilon) = 0.9$, $J = P = 1$ and equilibrium values of C and H of unity. The results are robust with respect to changes around these main values. A rise in G is represented by a fall in the exogenous portmanteau coefficient L (since we established earlier that $D_G < 0$ through a Ricardian Equivalence effect decreasing current consumption and hence firm revenue). A fall in L from 0.0219 to 0.0215 results in the three equilibria $(-0.785, 0, 0.785)$ being reduced to a single equilibrium (0.991) as shown on Figure 3.

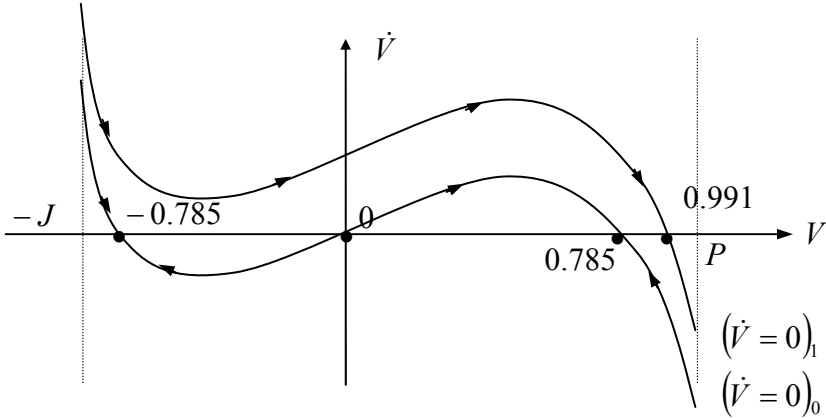


Figure 3

Now consider an unanticipated cut in bill financed government expenditure which reduces the deficit to its former level. This does not lead to the restoration of the initial equilibrium, but rather to the locally proximate equilibrium E_2 in figure 2, also characterised by net international debt/low stock market value.¹⁹ This may mimic to some degree the experience of developing countries, who, in trying to grow too quickly by deficit spending, end up in a far worse position, when the hysteresis and irreversibility costs of such spending are finally revealed.

¹⁹ Note that equilibrium E_2 has a higher level of ‘the real exchange rate’ and lower commodity prices than equilibrium E_0 . Higher the real exchange rate may, when considered in isolation, be ‘desirable’; however, in combination with net international debt and low stock market value, it is somewhat less desirable.

The European policy implications of this analysis are that, although in the short term there may be an incentive for the individual member country to over-borrow in advance of the imposition of binding criteria, there are damaging long run consequences. The country may indeed find itself unable to escape from a position of net international debt and low growth (low stock market value) despite cutting its deficit (reversing its policy). This has long run consequences for the economic performance of the eventual Union as a whole.

5 Conclusions

This paper looks at the effects of bill financed government deficit expenditure, where bills are held by both domestic and foreign residents. The issue of bills affects the dynamics of the stock market. In the short run the real returns on these different assets diverges: there is overshooting of the domestic real exchange rate. In the long run, we show that an increase in bill financed government deficit expenditure leads to a fall in the domestic real exchange rate, a rise in net international debt, a deterioration of the domestic trade balance and a fall in the stock market value.

Furthermore, due to non-linearities in the dynamic equations for international debt and stock market value, hysteresis effects are present in the model. This implies that a reversal of a deficit financing policy does not lead to a restoration of the initial state: in other words, the country gets stuck in a low stock market value and international debt trap. This has been shown both analytically and simulated using plausible parameter values. This result has obvious relevance to developing countries and additionally should provide a note of caution to countries running loose fiscal policies.