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Predicting the VIX and the Volatility Risk Premium: The Role of Short-run Funding Spreads Volatility Factors*

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Abstract

This paper presents an innovative approach to extract Volatility Factors which predict the VIX, the S&P500 Realized Volatility (RV) and the Variance Risk Premium (VRP). The approach is innovative along two different dimensions, namely: (1) we extract Volatility Factors from panels of filtered volatilities - in particular large panels of univariate ARCH-type models and propose methods to estimate common Volatility Factors in the presence of estimation error and (2) we price equity volatility risk using factors which go beyond the equity class namely Volatility Factors extracted from panels of volatilities of short-run funding spreads. The role of these Volatility Factors is compared with the corresponding factors extracted from the panels of the above spreads as well as related factors proposed in the literature. Our monthly short-run funding spreads Volatility Factors provide both in- and out-of-sample predictive gains for forecasting the monthly VIX, RV as well as the equity premium, while the corresponding daily volatility factors via Mixed Data Sampling (MIDAS) models provide further improvements.

Keywords: Factor asset pricing models, Volatility Factors, ARCH filters
JEL Codes: C2, C5, G1

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1 Introduction

This paper presents an innovative approach to extract volatility factors which are shown to predict the VIX, the S&P500 Realized Volatility (RV) and the Variance Risk Premium (henceforth VRP).[1] The approach is innovative along two different dimensions, namely: (1) we extract factors from panels of filtered volatilities - in particular large panels of univariate financial asset ARCH-type models and propose methods to estimate these factors in the presence of estimation/measurement error and (2) we price equity volatility risk using factors which go beyond the class of equity assets. More specifically we find that the most successful models feature volatility factors extracted from panels of volatilities of short-run funding spreads, which have both in- and out-of-sample predictive ability. This is especially useful given that the VIX is widely viewed by investors as the market gauge of fear (Whaley, 2000) and we provide evidence that our volatility factors can drive the VIX. In addition, our factors explain the VRP which is the difference between the implied and expected volatilities and is considered as an indicator of the representative agent's risk aversion (e.g. Bekaert and Hoerova, 2014) that is related to long-run risk models (e.g. Drechsler and Yaron, 2011).

Our analysis presents a new approach to extract common volatility factors based on panels of simple ARCH-type models which function as filters (the ARCH-type model parameter estimates are not of any direct interest), and Principal Component (PC) methods are applied to such panels of financial assets volatilities. In particular, even though ARCH-type models can be potentially misspecified, they can still be viewed as *filters* and deliver reliable estimates of volatility (see e.g. Nelson and Foster, 1994). Furthermore, we present methods that address the estimation error of filtered volatilities in extracting volatility factors. One method orthogonalizes the unsystematic component before extracting the factors, while the other method is based on the Instrumental Variables (IV) approach of estimating PCs. Moreover, our analysis studies the impact of the sampling frequency in extracting volatility factors and the role of high- versus low-frequency volatility factors as predictors in the context of Mixed Data Sampling (MIDAS) predictive regressions.

A number of papers extract factors from panels of option-based implied volatilities - see e.g. Carr and Wu (2009), Egloff, Leippold and Wu (2010), Zhou (2018), or from a cross-section of realized volatility measures, first considered by Anderson and Vahid (2007) who estimate the PC from the daily RVs of frequently traded stocks, and more recently by Christoffersen, Lunde and Olesen (2019) who also extract the common factor from the filtered ARMA logRVs of daily commodities, among others. On the other hand, Connor, Korajczyk and Linton (2006) develop a dynamic approximate factor model for a large panel of stock returns and estimate the heteroskedasticity in factor returns using a non-parametric local trend model. Barigozzi and Hallin (2017) develop a two-step general dynamic factor approach which accounts for the joint factor structure of stock returns and volatilities. On the contrary, some papers base their analysis on small cross-sections of assets such as Diebold and Nerlove (1989) and Engle, Ng and Rothschild (1990). Our approach is related, but different from the existing literature. Because we use simple ARCH-type models our

[1] Whenever we refer to the VIX we use the VIX^2 which is the component directly related to VRP.

analysis is neither confined to asset classes with traded options (since we do not use implied volatilities) nor restricted to assets with high-frequency intraday data and realized volatilities, given that for many financial assets high-frequency data may not be available, especially for a long time span. Nevertheless, our analysis on dealing with the estimation error for volatility factor extraction also applies to panels of RVs.

Traditionally, the extraction of risk factors is typically confined to a particular asset class. Fama-French factors are extracted from cross-sections of stock returns and are meant to price equity risk (but not say bonds or commodities returns). Level, slope and curvature factors are extracted from fixed income securities and are meant to price the term structure. A number of attempts have been made to extract factors jointly from stocks and bonds using the class of affine asset pricing models, see Bekaert, Engstrom and Grenadier (2010), Koijen, Lustig and Van Nieuwerburgh (2017), among others. Our approach also crosses asset classes and can be cast in an underlying affine asset pricing model. In particular, to predict equity volatility we do not exclusively rely on factors driven by stock returns. One can consider homogeneous panels stratified by asset class and compute principal components of the ARCH-type model filtered volatilities.

The approach of extracting factors from different subpanels of assets or types of economic indicators is also pursued in other studies. For instance, Ludvigson and Ng (2007) extract factors from two separate panels, the financial and macroeconomic indicators panels in an attempt to label factors and provide further understanding as to the economic driving source of each factor in a different setting from ours. Motivated by Ludvigson & Ng (2007) as well as the economic arguments that short-run funding spreads and long-run corporate and government spreads are good predictors of economic activity (e.g. Gilchrist and Zakrajšek, 2012), and indicators of illiquidity and bank credit risk (e.g. the TED and LOIS), especially during the recent US crisis (e.g. Adrian and Shin, 2010, Bekaert, Ehrmann, Fratzscher and Mehl, 2014 and Taylor and Williams, 2009), we estimate factors from these two panels. Extracting factors from these two classes/panels of assets allows us to label the factor and provide a better interpretation of the results as well as crystallize the driving source of the VIX, S&P500 RV and VRP. Moreover, our results show that by considering the factors from these two asset classes we can extract different types of volatility factors with different information content especially during the more volatile periods.[2]

Using our novel volatility factor approach we revisit the prediction of the VIX, the S&P500 RV and the VRP at different investment horizons. We find that the short-run funding spreads volatility factors are strongly significant for forecasting the VIX and the RV (at horizons 3 to 9 months). Furthermore we find that the VRP is driven by the volatility of the volatility of consumption growth empirical proxy mostly for longer horizons, which is consistent with the long-run risk models, while our short-run funding spreads volatility factor also turns out to be a driving factor for the VRP mostly at long horizons (6 and 9 months). In contrast, the long-run corporate and government bond spreads volatility factors turn out to be mostly insignificant in-sample predictors for the VIX, RV and VRP. Furthermore, we show using a comprehensive and robust

[2] An alternative approach is to consider group volatility factor models (e.g. in the spirit of Andreou, Gagliardini, Ghysels and Rubin, 2019) which is an avenue of future research.

empirical evidence that our proposed short-run funding spreads volatility factors have additional predictive ability for the VIX, RV and VRP over the traditional single factor model that focuses on consumption risk, as well as a number of other factors/indicators in the literature.[3] Our findings indicate the existence of an additional factor beyond the consumption growth volatility uncertainty proposed in Drechsler and Yaron (2011). Finally, our volatility factors can also predict the excess equity returns beyond some of the traditional predictors of the equity premium such as the VRP, the log price-dividend ratio, the log earnings-price ratio, among others. Last but not least, we find that the high-frequency (daily) volatility factors via MIDAS models provide prediction gains over the corresponding low-frequency (monthly) volatility factors, in traditional linear LS regression models.

The paper is organized as follows: Section 2 presents the factor analysis of panels of filtered volatilities. Section 3 provides the Monte Carlo simulations of the propositions in Section 2. Section 4 discusses the volatility factor estimation results. Section 5 presents the empirical analysis on the driving forces of the VIX, the S&P500 RV, the VRP and the equity premium and evaluates the role of our proposed factors. Section 6 discusses a number of robustness checks and the last section concludes the paper.

2 Factor Analysis with Panels of ARCH Filters

We start with the widely used class of continuous time affine diffusion (henceforth AD) asset pricing models. To fix notation, we follow the presentation of Duffie, Pan and Singleton (2000) and consider a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ where the filtration satisfies the usual conditions (see e.g. Protter, 2004) and $\mathbb{P}$ refers to the physical or historical probably measure.[4] Moreover, we suppose that the r-dimensional $\mathcal{F}$-adapted process $\mathcal{X}^f$ of state variables or factors is Markov in some state space $D \subset \mathbb{R}^r$, solving the stochastic differential equation:

$$d\mathcal{X}_t^f = \mu(\mathcal{X}_t^f)dt + \sigma(\mathcal{X}_t^f)dW_t \tag{2.1}$$

where W_t is an $\mathcal{F}_t$-adapted Brownian motion under $\mathbb{P}$ in $\mathbb{R}^r$, $\mu : D \to \mathbb{R}^r$, and $\sigma : D \to \mathbb{R}^{r\times r}$. Furthermore:

Assumption 2.1. *The distribution of $\mathcal{X}^f$, given an initial known $\mathcal{X}_0^f$ at t = 0, is completely characterized by a pair (K, H) of parameters determining the affine functions:*

$$\begin{array}{rcll} \mu(x) & = & K_0 + K_1 x, & K \equiv (K_0, K_1) \in \mathbb{R}^r \times \mathbb{R}^{r\times r} \\ (\sigma(x)\sigma(x)')_{ij} & = & (H_0)_{ij} + (H_1)'_{ij} x & H \equiv (H_0, H_1) \in \mathbb{R}^{r\times r} \times \mathbb{R}^{r\times r\times r}. \end{array} \tag{2.2}$$

[3] It is important to note that all our empirical analysis controls for lagged VIX and/or lagged S&P500 RV and therefore controls for information embedded in equity market risk measures. The above results are valid for the period from 1999m01-2016m09, with or without the period associated with Lehman Brothers bankruptcy and its aftermath.

[4] Note that our analysis requires continuous path processes, i.e. we exclude jumps. The continuous record asymptotics analysis for filtered volatilities requires absence of jumps. In principle the realized volatility measures can accommodate jumps, but a host of issues emerge when we tackle the panel data asymptotics, issues we leave for future research.

It will be convenient to focus exclusively on orthogonal factor cases. There is a well-known rotational indeterminacy in latent factor models, see e.g. in the context of affine diffusions the discussion in Dai and Singleton (2000). The panel data estimation procedures - introduced later - will impose a normalization of the factors. We assume here without loss of generality that the factors are orthogonal in population.

Assumption 2.2. *The r factors $\mathcal{X}^f$ are assumed orthogonal, and therefore the affine functions satisfy:*

$$\begin{array}{llll} K_1 = diag(k_1^{ii}) & & H_0 = diag(h_0^{ii}) & \\ (H_1)'_{ij} = 0 & \forall i \neq j & (H_1)'_{ii} = h_1^{ii}\mathbf{1}_i & \forall i \end{array} \tag{2.3}$$

where $\mathbf{1}_i$ is a $1 \times r$ vector of zeros except for the unit i^{th} element and h_1^{ii} is a scalar.

In a generic AD no-arbitrage asset price setting, Duffie et al. (2000) show that the bond, equity and variance premia at different investment horizons are linear functions of the same risk factors - i.e. state variables $\mathcal{X}_t^f$. In the term structure literature it is common to rotate the factors such that they correspond to the commonly used level, slope and curvature factors. The equity premia literature has instead focused on factors driven by macroeconomic fundamentals, in particular consumption uncertainty in the context of long-run risk economies studied by Bansal and Yaron (2004), where agents have a preference for early resolution of uncertainty and therefore dislike increases in economic uncertainty.[5] Suppose any asset i with (log) price denoted by p_t^i which has exposure to (some of) the risk factors, i.e. for the scalar δ_0^i and the $1 \times r$ vector δ^i :

$$dp_t^i \equiv \delta_0^i + \delta^i d\mathcal{X}_t^f \qquad \delta^i \neq 0 \tag{2.4}$$

such as for example the log price of an equity claim, log price of a zero-coupon bond, a risk spread, etc.

Following Duffie et al. (2000) and Carr & Wu (2009), we define the variance risk premium (VRP) as the difference between the time t expected equity return variance under the historical ($\mathbb{P}$) and under the risk-neutral ($\mathbb{Q}$) probability measures. Therefore, in an affine setting over horizon τ, the VRP can be written as:

$$\begin{array}{rcl} VRP(t,\tau) & = & E_t^{\mathbb{P}}[V_{t,t+\tau}^r] - E_t^{\mathbb{Q}}[V_{t,t+\tau}^r] = \delta_{vrp}(\tau) + \gamma_{vrp}(\tau)\mathcal{X}_t^f \\ \mathbb{E}_t^J[V_{t,t+\tau}^r] & = & \mu_{rv}^J(\tau) + \gamma_{rv}^J(\tau)\mathcal{X}_t^f \qquad J = \mathbb{P}, \mathbb{Q} \end{array} \tag{2.5}$$

where the parameters relate to the data generating processes for both $\mathbb{Q}$ and $\mathbb{P}$ measures (see, for example, Bollerslev, Tauchen and Zhou, 2009).

For some of the asset classes we need to distinguish spreads factors from volatility factors. Take for example a short-run credit spread. In such a case spreads factors pertain to credit spreads, whereas the volatility factors pertain to their volatility. Hence, we denote by c_t a credit spread and V_t^c its spot volatility. In a linear

[5]See in particular Eraker and Shaliastovich (2008) for the linear pricing characterization of long-run risk equilibrium models.

affine setting these can be written as:

$$c_t = \mu_c + \gamma_c \mathcal{X}_t^f, \qquad V_t^c = \mu_{cv} + \gamma_{cv} \mathcal{X}_t^f \ . \tag{2.6}$$

Obviously, the loadings γ_c and γ_{cv} may contain zeros such that the subset of factors which affect credit spreads may be different from the subset driving credit volatility. We consider a cross-section $i = 1, \ldots, N$, of asset spreads (and returns) spot volatilities written in a generic way, simplifying the notation, as:

$$\sigma^i(\mathcal{X}_t^f)^2 = \mu_{iv} + \gamma_{iv} \mathcal{X}_t^f. \tag{2.7}$$

Given volatility is latent we need to think about extracting its sample paths from observable data. More precisely, one can use proxies, i.e. filtered volatilities, to replace the left hand side latent spot volatility in the above equation. Finally, we assume that the cross-section of volatilities spans the factor space - namely:

Assumption 2.3. *Let* $X_t \equiv (\sigma^1(\mathcal{X}_t^f)^2, \ldots, \sigma^N(\mathcal{X}_t^f)^2)'$ *and* γ_v *be an* $N \times r$ *matrix and* μ_v *an* $N \times 1$ *vector. Then:*

$$X_t = \mu_v + \gamma_v \mathcal{X}_t^f$$

where γ_v *is of rank* r*.*

The above assumption guarantees that we can infer the volatility factor process from the cross-section of volatilities. Finally, it should also be noted that higher conditional moments of affine diffusions, if they exist, are also affine functions of the state process, see e.g. Duffie et al. (2000). In our analysis we will require that $\mathcal{X}_t^f$ has finite fourth unconditional moment, namely:

Assumption 2.4. *The process* $\mathcal{X}_t^f$ *has finite fourth moments, i.e.* $\mathbb{E} \parallel f_t \parallel^4 < M$*.*

This implies that the processes c_t and V_t^c in (2.6) have finite unconditional fourth moments as well and that uniformly across t all conditional (given the σ-field of $\mathcal{X}_\tau^f$, $\tau \leq t$) fourth moments of $\mathcal{X}_t^f$ are finite.

2.1 Panels of Volatility Proxies

There is a long history of modeling co-movements of volatilities via factor models. Early work involving small cross-sections of assets includes various ARCH factor models, see e.g. Engle et al. (1990), among others. Diebold & Nerlove (1989) suggest a closely related latent factor model. A number of recent papers entertain the idea to extract principal components from panels of realized volatilities, see e.g. Aït-Sahalia and Xiu (2017), Pelger (2019), among others. We do not have the luxury to work with realized volatilities as the assets we deal with are not traded at ultra-high frequencies such as liquid equities that are typically chosen (in particular S&P100 stocks). This applies to many financial assets. Hence, our volatility proxies are ARCH-type discrete time models. Therefore, our analysis is related to Connor et al. (2006) who develop a dynamic approximate factor model and rely on large panel data asymptotics to estimate a common volatility component. Also relevant to our work are Egloff et al. (2010) and Aït-Sahalia, Karaman and Mancini (2012), among others, who apply principal component analysis (PCA) to panels of variance swap rates. We do not directly observe volatility (nor $\mathcal{X}_t^f$) but have at our disposable, for a large set of assets, some estimates of volatility.[6] We lack continuous time observations p_t^i for asset i but have observations, denoted by $p^i_{[t:t+h]}$ over some discrete time intervals. Formally, we have:

Assumption 2.5. *Log price data are sampled discretely at some equidistant frequency h across all i denoted $p^i_{[t+kh:t+(k+1)h]}$, $\forall\, k \in \mathbb{N}$.*

We estimate (univariate) ARCH-type models viewed as filters through which one produces an estimate of the conditional variance. Hence, we construct a panel of univariate filtered volatilities across assets, denoted by $\widehat{V}^i_{[t:t+h]}$ for asset i and sampling interval h. The time series of cross-sections are sampled at the fixed time interval h. This may leave the impression that the underlying affine diffusion setting is detached from the large panel framework, and therefore irrelevant, for the purpose of our analysis, since in general there is no straightforward mapping from the continuous time process to a finite time grid discretization, except in a few special cases. However, the continuous time process is relevant because it provides: (a) the foundations for the volatility filters and their relationship to $\mathcal{X}_t^f$, and (b) the stochastic properties of the idiosyncratic (i.e. measurement) errors in the panel data model, such that they satisfy the so called approximate (using the terminology of Chamberlain and Rothschild, 1983) panel structure. Our analysis is inspired by Nelson and Foster (1994) who use continuous record asymptotics, i.e. involving log asset price data at arbitrary small time intervals, to characterize the distribution of the measurement error of discrete time volatility filters vis-à-vis continuous time diffusions. We do not entertain $h \downarrow 0$ asymptotics. Instead, we view the Nelson-Foster asymptotics as a guidance to the distribution of filtering errors, while keeping h fixed. More specifically:

Assumption 2.6. *The class of parametric filters that are considered feature the following updating scheme:*

$$
\begin{aligned}
\widehat{V}^i_{[t:t+h]} &= \widehat{V}^i_{[t-h:t]} + h\widehat{\kappa}^i_{t-h} + h^{1/2} g^i_t \\
\widehat{\kappa}^i_{t-h} &\triangleq \widehat{\kappa}^i\left(p^i_{[t-h:t]}, \widehat{V}^i_{[t-h:t]}, t, h\right) \\
g^i_t &\triangleq g^i\left(\widehat{\upsilon}^i_{x,t}, p^i_{[t-h:t]}, \widehat{V}^i_{[t-h:t]}, t, h\right) \\
\widehat{\mu}^i_{t-h} &\triangleq \widehat{\mu}^i(p^i_{[t-h:t]}, \widehat{V}^i_{[t-h:t]}, t, h) \\
\widehat{\upsilon}^i_{x,t} &\triangleq h^{-1/2}[p^i_{[t:t+h]} - p^i_{[t-h:t]} - h\widehat{\mu}^i_{t-h}]
\end{aligned}
\tag{2.8}
$$

where $\widehat{\kappa}^i$, $\widehat{\upsilon}^i_{x,t}$, and g^i_t satisfy regularity conditions appearing in Appendix A.

In particular, $\widehat{\kappa}^i$, $\widehat{\upsilon}^i_{x,t}$, and g^i_t are functions selected by the econometrician. The $\widehat{\kappa}^i$ and $\widehat{\mu}^i$ must be continuous

[6] We are working under the assumption that we can collect volatility data which span the space of all the risk factors, formally defined later. Alternatively, we can think of estimating a sub-block of factors pertaining to volatility, which we will still denote by $\mathcal{X}_t^f$ to avoid further complicating notation.

in all arguments and g_t^i must be differentiable in $\widehat{V}^i$, $\widehat{v}^i$ and h almost everywhere and must possess one-sided derivatives everywhere (for further details see Appendix A as well as Nelson & Foster, 1994). For instance, standard (E)GARCH models can be employed to filter the conditional volatility (e.g. Nelson, 1992). In practice, either class of models yields the correct filtered conditional volatilities despite the fact that the models are potentially misspecified. It is for this reason that in our empirical work we choose to use (E)GARCH models. Next, we need to analyze the properties of $u^i_{[t:t+h]}$ in equation (2.9) appearing below. We expect that there are commonalities across assets, i.e. univariate estimation errors are neither cross-sectionally nor temporally uncorrelated.

Proposition 2.1. *Let Assumptions 2.1 - 2.6 hold and as $h \downarrow 0$, the volatility filter defined in (2.8) satisfies:*

$$\begin{aligned} \widehat{V}^i_{[t:t+h]} &= \sigma^i(\mathcal{X}_t^f)^2 + u^i_{[t:t+h]} \\ &= \delta^i(\sigma(\mathcal{X}_t^f)\sigma(\mathcal{X}_t^f)')\delta^{i\prime} + u^i_{[t:t+h]} \\ &= (\sum_{j=1}^r (\delta_j^i)^2 h_0^{jj}) + \sum_{j=1}^r (\delta_j^i)^2 h_1^{jj} \mathcal{X}_t^j + u^i_{[t:t+h]}. \end{aligned} \tag{2.9}$$

Proof: see Appendix B

The above result involves $h \downarrow 0$. For any finite h we expect $u^i_{[t:t+h]}$ to feature discretization errors.[7] In addition, we may also expect that pervasive factors may appear in those errors.

2.2 Digression on Factor Model Assumptions

In the context of standard large scale factor models, some basic assumptions are made about the factors and the time series and cross-sectional dependence of the idiosyncratic errors. In particular, the aforementioned notion of an approximate factor model due to Chamberlain and Rothschild (1983) imposes regularity conditions such that the idiosyncratic errors are allowed to be mildly temporally and cross-sectionally correlated. To elaborate on this and to keep our analysis as close as possible to the standard large scale factor models in the literature, we adopt the commonly used notation with some modification and then discuss the mapping with the framework discussed so far. Moreover, given the nature of the volatility proxies, involving either spot or integrated volatilities, we adopt the following notation:

Assumption 2.7. *Consider the vector form model representation for $X_{[t:t+h]} = (\widehat{V}^i_{[t:t+h]}, i = 1, \ldots, N)'$. We also let F_t^0 be the $r \times 1$ vector of true population factors $F_t^0 = \sigma(\mathcal{X}_t^f)^2$. Moreover, for factor loadings $\tilde{\Lambda}^0 = (\lambda_1^0, \ldots, \lambda_N^0)'$, we have:*

$$X_{[t:t+h]} = \tilde{\Lambda}^0 F_t^0 + u_{[t:t+h]}. \tag{2.10}$$

[7] For convenience and to avoid extra notation we do not distinguish between $u^i_{[t:t+h]}$ for finite h versus the limiting process.

Note that we suppress the intercepts in the above equation, although they appear implicitly because $\sigma(\mathcal{X}_t^f)^2$ is an affine function of $\mathcal{X}_t^f$ according to Assumption 2.1. By the same token, the factor loadings λ_i^0 appearing in equation (2.10) depend on δ^i appearing in equation (2.4) and the linear affine slope coefficients appearing in Assumption 2.2. Finally, $u_{[t:t+h]} = (u^1_{[t:t+h]}, \ldots, u^N_{[t:t+h]})'$. To address the cross-sectional and temporal dependence across the elements of $u_{[t:t+h]}$ due to discretization and approximation errors we make the following assumption:

Assumption 2.8. *The vector $u_{[t:t+h]}$ for any $h > 0$ has the following factor structure, with F_t^0 defined in equation (2.10):*

$$u_{[t:t+h]} = \Upsilon^0 F_t^0 + \tilde{u}_{[t:t+h]} \tag{2.11}$$

Combining equations (2.10) and (2.11) yields:

$$X_{[t:t+h]} = (\tilde{\Lambda}^0 + \Upsilon^0) F_t^0 + \tilde{u}_{[t:t+h]} \equiv \Lambda^0 F_t^0 + \tilde{u}_{[t:t+h]} \tag{2.12}$$

and the matrix representation of the factor model in (2.12) is:

$$X = F^0 \Lambda^{0\prime} + \tilde{u} \tag{2.13}$$

where X is a $T \times N$ matrix of observations on (standardized) volatilities, $\tilde{u}$ is a $T \times N$ matrix of idiosyncratic errors, the true factor matrix $F^0 = (F_1^0, \ldots, F_T^0)'$ is $T \times r$ and the loading matrix Λ^0 is $N \times r$.

Several issues emerge when we examine the factor structure appearing in equations (2.12) and (2.13). First, we may not necessarily expect to find unbiased estimates for the loadings, i.e. estimates of $\tilde{\Lambda}^0$. Indeed, because we use filtered volatilities, the loadings $(\tilde{\Lambda}^0 + \Upsilon^0) \equiv \Lambda^0$ no longer represent the true underlying loadings, $\tilde{\Lambda}^0 = (\lambda_1^0, \ldots, \lambda_N^0)'$, but instead are contaminated by the estimation noise. If we were interested in the point estimates of the loadings this would be problematic. This is not our goal, however, as we only seek to estimate the factors which drive the cross-section of estimated volatilities. Second, we need to be careful about situations where $\tilde{\Lambda}^0$ and Υ^0 (nearly) off-set each other, resulting in a so-called weak factor situation (Onatski, 2018). There are a number of reasons to think this will not be the case in our particular setting, but to err on the cautious side we do entertain this possibility.

A general solution is to consider estimators which undo the bias. One such type of estimators we consider involves instruments assumed to be orthogonal to the volatility proxy errors $u^i_{[t:t+h]}$ appearing in equation (2.9). The idea to use instrumental variables (IV) in the context of PCA has been entertained in a number of papers, including Rao (1964), Wang and Qin (2002) and more recently Fan, Liao and Wang (2016). The issue of course is to find valid instruments, which will be discussed in the next subsection. The use of IV relies on a projection matrix $P_Z \equiv Z(Z'Z)^{-1}Z'$ where Z is a set of instruments with the property:

Assumption 2.9. *There is a set of instruments $Z \equiv [Z_1, \ldots, Z_T]'$ such that $E[Z_t^i u^i_{[t:t+h]}] = 0 \; \forall \; i$ and t.*

The procedure is simple, with data X being replaced by the projections $P_Z X,$ for which standard PCA is applied. The projected data are void of proxy errors, so that the bias issue is resolved, provided some additional assumptions apply, to which we turn later. While IV is a possible solution to the aforementioned bias problem, there are several reasons to believe that the estimation of factors, even with biased loadings, can be done with standard PCA in our particular application involving panels of volatility proxies. We discuss next a number of reasons why this is the case.

First, the consistency and asymptotic normality of the principal components estimator when both N and T go to infinity involves a number of assumptions stated as Bai (2003, Assumptions C-G). They notably require that the ratio between the i^{th} largest and the $(i+1)^{th}$ largest eigenvalues of the population covariance matrix of the data, is rising proportionally to N so that the cumulative effects of the normalized factors on the cross-sectional units strongly dominate the idiosyncratic influences asymptotically. The motivation for the potential of weak factors, as discussed by Onatski (2018), occurs in practice when the ratio of the adjacent eigenvalues of the finite sample analog of the population covariance matrix is small, which implies that in particular Assumption B in Bai (2003) would be violated. The existing literature suggests that this is not the case with panels of volatility proxies. The panels of realized volatilities, considered by Aït-Sahalia and Xiu (2017) and Pelger (2019), correspond to a setting similar to ours as the realized measures also feature errors whose properties are characterized by infill asymptotics. The findings in those papers are that there are only a few (namely typically one) factors and they explain more than 90 % of the cross-sectional variation. In a different setting, closer to ours Connor et al. (2006) find a single dominant factor.

Second, we verify some of the assumptions which rule out the presence of weak factors in a simulation setting discussed in the next section. We simulate discretizations of affine diffusions and study the bias properties of the loadings as well as the potential for the presence of weak factors in panels of volatility proxies. We find no simulation evidence for the presence of weak factors.

Third, looking at the simulations and panels used in the empirical section, it appears that none of the symptoms that might suggest weak factors are present. The ratios of the first-to-second and second-to-third largest eigenvalues turn out to be much larger than one, whereas all others are close to one. These results hold not only for the simulation design in Section 3 but also for the empirical analysis, details of which are discussed in Section 4. These results confirm the previous findings in the literature and are dissimilar to the maximal ratio of 1.75 across twenty eigenvalues cited by Onatski (2018) as motivation for weak factors of the Stock and Watson macro panel data.

Finally, in a setting not involving measurement errors, Egloff et al. (2010) and Aït-Sahalia et al. (2012), find that two factors explain close to 100 % of the variation in panels variance swap rates for the S&P500. This is important, as it could be argued that the measurement errors lead to a weak third factor, undetected due to the biased loadings. Another remedy, of course, is the IV-based procedure which will be considered as well.

In the remainder of this subsection we discuss the assumptions in order to obtain desirable estimates of the underlying factors. These assumptions pertain to standard PCA when applied to X or IV-based PCA when applied to $P_Z X$. For the sake of simplicity, we focus on X with the understanding that, ceteris paribus, the same conditions apply to the instrumented data.

The above equation (2.11) makes the statement that $u_{[t:t+h]}$ are related to the true factors - as expected from discretization and approximation errors. More important, however, is the fact that there are no factors other than F_t^0 which explain the co-movements among the elements of $u_{[t:t+h]}$.

Assumption 2.10. *The factors F_t^0 and errors $\tilde{u}_{[t:t+h]}$ in equations (2.11) and (2.12) satisfy Bai (2003, Assumptions C-G). More specifically, the factors F_t^0 and the errors are assumed to be weakly dependent (Assumption D, Bai, 2003) and $\tilde{u}_{[t:t+h]}$ are weakly serially and cross-sectionally dependent (Assumption C, Bai, 2003).*

2.3 Volatility Factor: PCA of panels of volatility proxies

With the assumptions discussed so far, we can proceed with the standard method of asymptotic principal components, initially considered by Connor and Korajczyk (1986) and refined by Bai and Ng (2002), Bai (2003), as an estimator of the factors in a large N and T setup. As noted before, the subsequent analysis applies either to X or else $P_Z X$. Again for convenience we focus only on X in the subsequent analysis.

We determine the number of factors using consistent selection methods such as those of Bai & Ng (2002) as well as Ahn and Horenstein (2013). Let r denote the estimated number of factors. Then the principal components method constructs a $T \times r$ matrix of estimated factors and a corresponding $N \times r$ matrix of estimated loadings by solving the following optimization problem:

$$\min_{\Lambda, F} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (\widehat{V}^i_{[t:t+h]} - \lambda_i F_t)^2 \tag{2.14}$$

subject to the normalization that $(F'F)/T = I_r$. The estimated factor matrix $\hat{F}$ is $\sqrt{T}$ times the eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix XX'. Moreover, $\tilde{\Lambda}' = (\hat{F}'\hat{F})^{-1}\hat{F}'X = \hat{F}'X/T$ are the corresponding factor loadings.

Finally, it is worth recalling that we do have biased estimates of the loadings, due to the fact that we have filtered volatilities (unless instrumental variables are being used). It is therefore also worth noting that we do not impose no-arbitrage conditions across pricing equations.[8] Then, Bai (2003, Theorem 1) implies that:

[8] The literature in general, and our paper in particular, avoids the often tedious cross-equation restrictions during the estimation of empirical asset pricing models. Imposing such conditions, which are assumed to hold in the data generating process, is a much debated topic in the term structure of interest literature (see e.g. Joslin, Le and Singleton, 2013, among others) and may be prone to misspecification issues. What really matters for our analysis is the extraction of factors.

Proposition 2.2. *Under Assumptions 2.1 – 2.10, as* N *and* $T \to \infty$ *and* $\sqrt{N}/T \to 0$*, there exists an* $r \times r$ *rotation matrix* $\mathcal{H}_r$ *with rank(*$\mathcal{H}_r$*) =* r*, such that for each* t *:*

$$\sqrt{N}(\hat{F}_t - \mathcal{H}_r F_t^0) = V_{NT}^{-1}\left(\frac{\hat{F}'F^0}{T}\right)\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\lambda_i^0\tilde{u}_{it} + o_p(1) \xrightarrow{d} N(0, V^{-1}Q\Gamma_t Q'V^{-1})$$

where V_{NT} *is a diagonal matrix with the* r *largest eigenvalues of the matrix* $(1/NT)XX'$*, and* Q *=* $plim_{N,T\to\infty}(\hat{F}'F^0)/T$*,* V *is a diagonal matrix with the* r *largest eigenvalues of the matrix* $\Sigma_\Lambda^{1/2}\Sigma_F\Sigma_\Lambda^{1/2}$ *with* $N^{-1}\Lambda'\Lambda \xrightarrow{p} \Sigma_\Lambda > 0$*, and* Σ_F *is the variance of the* F_t^0 *process. Finally,* Γ_t *=* $\lim_{N\to\infty}\sum_{i=1}^{N}\sum_{j=1}^{N}\lambda_i\lambda_j'\mathbb{E}(\tilde{u}_{it}\tilde{u}_{jt})$

Proof: see Appendix B

Note that the limiting distribution of factors involves the loadings. Here we need to distinguish the case of PCA with the raw data matrix X versus PCA applied to $P_Z X$. The former features loadings $(\tilde{\Lambda}^0 + \Upsilon^0)$ while the latter involves the unbiased $\tilde{\Lambda}^0$. From the perspective of the true loadings, the former can be viewed as non-pivotal since it involves unknown nuisance parameters pertaining to the contaminated loadings.

Finally, it should also be noted that the analysis in this section is also relevant when panels of realized volatilities are used to extract factors, as such measures are also affected by measurement noise. The existing literature has not recognized the potential biases in estimation of loadings and its implications.

2.4 Volatility Factor: PCA of panels of volatility proxies of idiosyncratic errors

Assumption 2.3 tells us that we can recover all the factors using the panel of volatility proxies. By the same token, we could also consider the factors related to the mean and extract them from a panel of spreads/returns. Analogous to equation (2.13), we can consider:

$$Y = F^0\Phi^{0\prime} + e \tag{2.15}$$

where Y is a $T \times N$ matrix of observations on (demeaned) spreads/returns, e is a $T \times N$ matrix of idiosyncratic errors, the true factor matrix $F^0 = (F_1^0, \ldots, F_T^0)'$ is $T \times r$ and the loading matrix Φ^0 is $N \times r$.[9] Ludvigson & Ng (2007) approximate the volatility of the factors by applying PCA to the above panel and computing the squares of the extracted factors, i.e. $\hat{F}_{kt}^2$, $k = 1, \ldots, r$.[10] We pursue an alternative approach. Consider, for example, the following square root processes in the context of a one- and two-factor

[9]There is again a discretization error of the affine diffusion model appearing in equations (2.1) through (2.4), which is assumed to be negligible.

[10]We avoid further complicating the notation by using the same notation $\hat{F}$ for estimators based on either (2.13) or (2.15), although it should be clear that these are different estimators.

setting, respectively:

$$d\mathcal{X}_{1t}^{f} = a_1(b_1 - \mathcal{X}_{1t}^{f})dt + \sigma_1\sqrt{\mathcal{X}_{1t}^{f}}dW_{1t} \tag{2.16}$$

and

$$\begin{aligned} d\mathcal{X}_{1t}^{f} &= a_1(b_1 - \mathcal{X}_{1t}^{f})dt + \sigma_1\sqrt{\mathcal{X}_{1t}^{f}}dW_{1t} \\ d\mathcal{X}_{2t}^{f} &= a_2(b_2 - \mathcal{X}_{2t}^{f})dt + \sigma_2\sqrt{\mathcal{X}_{1t}^{f} + \mathcal{X}_{2t}^{f}}dW_{2t} \end{aligned} \tag{2.17}$$

Our approach builds on the idea of volatility factors extracted from panels of volatilities and is related to the fact that the panel of spreads/returns, Y, in (2.15) involves not just the common mean returns/spreads component/factor, but it also involves the idiosyncratic errors, for which one can again think about how to define and estimate the volatilities of the panel of e in (2.15). In this context a two-step approach can be applied to extract first the common returns factor $\hat{F}$ and the corresponding estimated idiosyncratic components of returns or residuals, $\hat{e}$. In the second step the Volatility Factor (VF) can be extracted from the panel of univariate volatilities of $\hat{e}$.

In addition, we employ two alternative IV approaches to address the estimation/measurement error issue. One method extracts the common factors from the panel of fitted values obtained from the regressions of the variable of interest e.g. VIX or RV or VRP or excess returns, on each univariate AR-(E)GARCH filtered volatility pertaining to each financial asset in the panel. It therefore attempts to extract the volatility factor from the systematic component of the model with the filtered volatilities and orthogonalize the unsystematic error component, and hence it is denoted by VFe. The other approach is based on the IV projection discussed in Assumption 2.9. The Volatility Factor Instrumental Variable ($VFIV$) approach uses lags of the cross-sectional average of filtered volatilities as IVs. In particular, we extract the $VFIV$ as the principal component/common factor from the panel of fitted values of each univariate AR-(E)GARCH filtered volatility regressed on their cross-sectional average using lags of the cross-sectional average as instruments. This method is inspired by Fan et al. (2016) and is a simple version of Projected-PCA approach which attempts to remove the estimation/filtering error. The choice of lag length is based on the moment selection criteria of Andrews (1999) and Hall, Inoue, Jana and Shin (2007). Based on the Cragg and Donald (1993) statistic and the associated Stock and Yogo (2005) critical values, we find that 98% of our empirical panel series reject the null hypothesis of weak instruments, whilst the instruments orthogonality condition gains empirical evidence for 84% of these panel series, evaluated by using the Eichenbaum, Hansen and Singleton (1988) instrument orthogonality test. Further details on the empirical analysis are discussed in Sections 4.

3 Monte Carlo simulations

We evaluate the results of Propositions 2.1 and 2.2 via Monte Carlo simulations to study the effects of different sample sizes (N, T), sampling frequencies (or aggregation) and ARCH-type filters in extracting the Volatility Factors (VF) as well as the alternative volatility type factors that deal with the estimation error (VFe and $VFIV$), as described in the previous section. Moreover, we evaluate the performance of Bai & Ng (2002) information criteria (IC) as well as the Ahn & Horenstein (2013) estimators, the Eigenvalue Ratio (ER) and the Growth Ratio (GR), to estimate the number of volatility type factors.

3.1 Simulation Design

We start from the following single factor affine diffusion DGP for a cross-section of N asset returns:

$$\begin{aligned} dp_t^i &= \sum_{j=1}^{r} \delta_j^i d\mathcal{X}_{jt}^f + \left[\sum_{j=1}^{r} \gamma_j^i \mathcal{X}_{jt}^f\right]^{1/2} dW_t^i \qquad (3.1) \\ d\mathcal{X}_{1t}^f &= a_1(b_1 - \mathcal{X}_{1t}^f)dt + \sigma_1 \sqrt{\mathcal{X}_{1t}^f} dW_{1t}^f \\ &\vdots \qquad\qquad \vdots \\ d\mathcal{X}_{rt}^f &= a_r(b_r - \mathcal{X}_{rt}^f)dt + \sigma_r \sqrt{\mathcal{X}_{rt}^f} dW_{rt}^f \end{aligned}$$

where W_t^i and W_{jt}^f for $i = 1, \ldots, N$, $j = 1, \ldots, r$, are mutually uncorrelated Brownian motions and p_t^i are log prices of asset i. Simple Euler discretization of the above diffusions may pose problems with regards to the positivity constraint of the volatility factor process (see e.g. Deelstra and Delbaen, 1998). To that end we simulate for a time-step of size $1/n$, the approximating processes for $k \in \mathbb{N}$:

$$\begin{aligned} p_{k+1}^{(n)i} &= p_k^{(n)i} + \sum_{j=1}^{r} \delta_j^i (\mathcal{X}_{j,k}^{(n)f} - \mathcal{X}_{j,k-1}^{(n)f}) + \left[\sum_{j=1}^{r} \gamma_j^i (\mathcal{X}_{j,k}^{(n)f})^+\right]^{1/2} \varepsilon_{k+1}^i \qquad (3.2) \\ \mathcal{X}_{1,k+1}^{(n)f} &= \mathcal{X}_{1,k}^{(n)f} + \frac{a_1}{n}(b_1 - \mathcal{X}_{1,k}^{(n)f}) + \frac{\sigma_1 \varepsilon_{1,k}^{(n)f}}{\sqrt{n}} \sqrt{(\mathcal{X}_{1,k}^{(n)f})^+} \\ &\vdots \qquad\qquad \vdots \\ \mathcal{X}_{r,k+1}^{(n)f} &= \mathcal{X}_{r,k}^{(n)f} + \frac{a_r}{n}(b_r - \mathcal{X}_{r,k}^{(n)f}) + \frac{\sigma_r \varepsilon_{r,k}^{(n)f}}{\sqrt{n}} \sqrt{(\mathcal{X}_{r,k}^{(n)f})^+} \end{aligned}$$

where $(x)^+ = \max(0, x)$. Deelstra and Delbaen (1998) establish the strong convergence of the discretization scheme in (3.2) to the diffusion process in (3.1) as the step size decreases to zero. Following the estimates of the $AFF1V$ model in Chernov, Gallant, Ghysels and Tauchen (2003), we pick the following parameters for the simulation design. For the single factor model, r = 1, we select δ_1^i and γ_1^i as $NIID(1,1), \forall\, i$, and a_1 = 4.5, b_1 = 0.02, σ_1 = 1.[11] As the parameters pertain to annual sampling, we set n = 250 for daily returns. Lower-frequency realizations are obtained by aggregating the above high- or daily-frequency process to a low-frequency one such as weekly and monthly. All the errors are $N(0, 1/n)$. First, we simulate a cross-section of returns with $N = 50, 100$ and $T = 3780$ where T is adjusted according to the aggregation frequency, $n^{(f)} = 1, 5, 21$ referring to the daily, weekly and monthly frequencies, respectively. Hence we have $T/n^{(f)}$ sample sizes. Some of the choices of N and T are based on the empirical sample sizes considered in the empirical application. Then we create a panel of demeaned and standardized returns and construct a panel with the estimated idiosyncratic components of returns (or residuals) from the regression of each return on the principal component extracted from the panel of returns. We estimate univariate AR-(E)GARCH models with errors that follow the Normal or Generalized Error Distribution (GED) (Nelson, 1991) and create the corresponding panels of volatilities (AR_(E)GARCH_Norm and AR_(E)GARCH_GED, respectively) from which we extract the volatility type factors (VF, VFe and $VFIV$).[12] The Monte Carlo experiment is performed using 10000 simulations.

3.2 Simulation Results

The true/simulated factor from the discretized DGP at high frequency generated by (3.2) is obtained from a single factor affine diffusion for the cross section of N assets over a time-series sample, T. For the single factor model the DGP parameters described in the previous section represent the high-frequency (daily) process. The simulated daily process is then aggregated to represent weekly and monthly frequencies using $n^{(f)} = 5, 21$, respectively. For each of the simulated processes, at different frequencies, we follow the three approaches described in subsection 2.4 to obtain VF, VFe and $VFIV$. Note that as a proxy for the variable of interest in the second approach (for the extraction of VFe) we used the VF which also matches some of the properties of VIX and RV.

First, we evaluate the ability of Bai & Ng (2002) IC_{p2} criterion and the Ahn & Horenstein (2013) ER and GR estimators to determine the number of factors. We find that the ER and GR estimators perform well by estimating a single factor as generated by the single factor affine diffusion model in (3.1). In contrast, while the IC_{p2} criterion performs well for the PC which is estimated from the panel of the true/generated

[11] We checked if a truncation comes into play and if it influences the results for which we have not found any evidence. Nevertheless, we also used the CIR specification to simulate our model which turned out to give very similar results, albeit more computationally burdensome. We also simulated the two factors model and the simulation results are qualitatively the same and hence not reported for conciseness.

[12] The Skewed GED (SGED) (Theodossiou, 1998), Student's *t* (Bollerslev, 1987) and Skewed Student's *t* (Hansen, 1994) are also used as error distributions and the results are robust and similar to those of the GED (Nelson, 1991). The volatility type factors extracted from all these filters are highly correlated, ranging from $0.74 - 0.99$. In addition, the unconditional mean and unconditional variance of the simulated data are taken as the initial values for our AR-(E)GARCH models. Our results are also robust to different initial values based on estimating an AR-(E)GARCH model for a subset of the simulations as well as choosing as initial values the unconditional moments (mean and variance) based on the estimated parameters of the AR-(E)GARCH models.

volatilities, it seems to overestimate the number of factors, hitting always the maximum bound of factors, r_{max}, for the panels with the estimated AR-(E)GARCH volatilities that correspond to VF and VFe factors, except in the $VFIV$ case. This suggests that the IC_{p2} may be more sensitive to the estimation error which can be solved by adopting the $VFIV$ approach. These simulation results (found in Table A.1.1 in the Online Appendix (OA)) provide additional evidence that the Ahn & Horenstein (2013) estimators are not sensitive to the choice parameter, r_{max}.

Second, we evaluate the correlation of the simulated/true factors with the estimated volatility type factors in order to examine how the latter are affected by (a) the alternative approaches described above for extracting volatility type factors and (b) the effects of the sampling frequency in extracting the volatility type factors, and evaluate Propositions 2.1 and 2.2. The correlations of the simulated and estimated volatility type factors (VF, VFe and $VFIV$) are higher for the high-frequency (daily) DGP vis-à-vis the aggregated processes and the corresponding factors estimated from low-frequency data (shown by the correlations in Table A.1.1 in the OA). As the aggregation horizon increases (and the frequency of the true process becomes lower), the correlations of the simulated and estimated types of volatility factors are decreased especially for the AR-(E)GARCH_Norm models. This result provides support for Proposition 2.1 according to which volatility type factors are more accurately estimated with higher-frequency data. Moreover, we find that the volatility type factors extracted from AR-(E)GARCH_GED models yield higher correlations with the true volatility factor and are less sensitive to the sampling frequency. This is explained by the fact that the GED is a more flexible distribution to capture the heavy tail behavior (Nelson, 1991) found in different sampling frequencies.

Third, we evaluate the effect of the estimation error, denoted by ee_{it}, which is the difference between the true/simulated and estimated volatilities, using the auxiliary regression $ee_{it} = a + bVF_t^T + u_{it}$, where VF^T denotes the true volatility factor, for the different panels of volatility filters and find that as the sampling frequency increases, the $\Sigma b^2/N \to 0$, as the theory suggests. Moreover, we evaluate the relative Mean Squared Errors (MSE) of the estimated factors which account for the estimation error, i.e. VFe and $VFIV$, vis-à-vis the VF and find that both VFe and $VFIV$ are relatively more efficient than VF in MSE terms. In addition, the $VFIV$ is relatively more efficient than VFe especially for the AR-(E)GARCH_GED models. Regarding the MSEs of loadings, there are no substantial differences between the three volatility type factors. Last but not least, we investigate the variance signal of the first factor as a percentage of the first five factors. We find that VF and VFe factors yield variance signal ranging from 70%-100% while the variance signal of $VFIV$ appears to be always 100%, providing evidence of the strong factor case. These results are also found in Table A.1.1 in the OA.

Within our simulation design for the single factor affine diffusion model we also evaluate the properties of the residuals from volatility factor models. Following Bai (2003) Assumptions C, E and H we evaluate the time and cross-sectional dependence and heteroskedasticity. Using auxiliary regressions we find that the panel residuals from the volatility factor models have weak and insignificant temporal dependence for the

VF and VFe factors, while there is no evidence of temporal dependence for the $VFIV$ factors. Related is the assumption of weak cross-sectional dependence in the errors of factor models for which we find weak evidence. We also examine the dynamic heteroskedasticity using auxiliary regressions for the squared residuals and find that their corresponding coefficients range from 0.3 to 0.4 for VF and VFe and are negligible for $VFIV$. Similarly, there is weak evidence of static heteroskedasticity in the residuals of the VF and VFe factor models, while there is no evidence of static heteroskedasticity for the $VFIV$ factor model. Finally, regarding the assumption of weak dependence between factors and idiosyncratic errors (Assumption D, Bai, 2003) we find that the regression coefficient of the panel model residuals and the estimated volatility type factors is very close to zero in almost all cases. Hence, all the above residuals tests provide simulation evidence which suggests that the error assumptions satisfy the approximate factor structure assumptions in Bai (2003).

Last but not least, we consider the size of the ratios of the largest eigenvalues for the various types of estimated volatility factors following Onatski (2018) in order to check if there is an indication of weak factor structure in the simulations. We find that the largest ratios are as high as 6.5 and 2.8 (found in Table A.1.1 of the OA) suggesting that there is no evidence of weak factor structure. Moreover, Lettau and Pelger (2018) state that for the loadings in a strong factor model $\Lambda'\Lambda/N \rightarrow \Sigma$, where Σ is a full-rank, diagonal matrix. Evaluating this assumption we find supportive evidence for both simulation designs. We also get zero mean and low variance values for the difference between the true VF and the estimated types of volatility factors indicating "strong factor" case.

4 Empirical analysis of extracting volatility factors

We extract factors from panels of asset spreads and volatilities beyond equity and show how these help price equity volatility risk, the VIX, the RV of S&P500 stock market returns and the VRP. In our empirical analysis we control for the volatility of the S&P500 returns directly by including the lagged VIX and lagged RV, and therefore we can focus on the role of volatilities beyond equities. Motivated from the work of Bekaert, Ehrmann, Fratzscher and Mehl (2014), Gilchrist and Zakrajšek (2012) and Taylor and Williams (2009), among others, we extract common factors from financial assets risk panels of: (1) Short-run Funding (denoted by $SRFUN$) spreads and volatilities and (2) Long-run corporate and government Bond (denoted by $LRBON$) spreads and volatilities. It is worth mentioning at the outset that the $SRFUN$ factors turn out to be relatively more significant predictors than the $LRBON$ factors. The acronyms and the names of our factors and other variables employed in the empirical analysis are summarized in Table 1. While the analysis reported here focuses on $SRFUN$ and $LRBON$ spreads, it should be noted that we did examine other financial asset classes, including commodities returns and their spot and futures prices spreads and their volatilities, as well as the foreign exchange returns and volatilities, but did not find much evidence of predictability.

Traditionally factors are extracted from the returns and spreads of financial assets. Hence, the common (mean) factors of the short-run funding and the long-run corporate and government bond spreads, $SRFUN\_SF$ and $LRBON\_SF$, respectively, are extracted from the corresponding panels of spreads/returns. We also estimate monthly volatilities using AR-(E)GARCH models for the idiosyncratic components (residuals) obtained from the regression of each spread on the corresponding common spreads factor from the aforementioned panels.[13] We consider three different distributions for the AR-(E)GARCH models, the Normal, the GED and SGED, and report the volatility type factor results for all three distributions in order to examine the robustness of our empirical results.[14] We consider the GED being a more flexible distribution and SGED capturing skewness and the effects of large extreme events related to crises. The choice of the AR-(E)GARCH models is based on the fact that they are often considered as spot volatility benchmarks in many financial asset volatility empirical studies. Given that our results are robust to the GARCH type filters in this section we report results mainly for the AR-EGARCH filter (while Section 6 summarizes a comprehensive robustness analysis). Last but not least, we also estimate the corresponding daily volatility type factors from the daily AR-(E)GARCH models for a subset of the cross-section of the indicators for $SRFUN$ and $LRBON$ (listed in Tables A.1.2 and A.1.3) given the unavailability of a small number of series at daily frequency.[15] Our sample period refers to 1999m01 - 2016m09 (T = 213 monthly observations) and 06/01/1999 - 20/09/2016 (T = 3497 daily observations). We consider 1999 as the starting period due to the fact that the number of series in the cross-section, N, increases significantly in the aforementioned panels as opposed to having a starting period in 1990 where we have roughly half of the cross-section in these two classes of assets.

We extract both spreads/mean factors (SF) and volatility type factors for each panel. We consider the three different volatility type factors. The Volatility Factors (VF) are based on the estimated volatilities (AR-EGARCH fitted values) of the idiosyncratic component/residual of each spread regressed on the spreads factor (SF). The other two volatility type factors, namely the VFe and the $VFIV$, deal with the estimation error of filtered volatilities. We extract VFe from the panel of fitted values obtained from the regressions of the variable of interest (e.g. VIX, RV, VRP or returns) on each univariate AR-(E)GARCH filtered volatility for each idiosyncratic component in the panel. The $VFIV$ is based on the IV approach and uses lags of the cross-sectional average of the panel of the N filtered volatilities as instruments. In particular, we extract

[13]The following transformations are applied to the series before extracting the factors. First we define the spreads of interest rates and bond rates (found in Tables A.1.2 and A.1.3). To ensure stationarity we take the first difference of the above spreads, following e.g. Ludvigson & Ng (2007), which are thereby demeaned and standardized. Second, we estimate the univariate AR-(E)GARCH models for the panel of idiosyncratic components of spreads (residuals) from the regression of each spread on the common spreads factor and create the panel of the corresponding estimated volatilities which are then standardized before extracting the principal components or volatility type factors. We do not demean the fitted volatilities in order to ensure positivity of the extracted volatility factor.

[14]We also consider Student's t distribution for the volatility model panels which is highly correlated (0.97) with the corresponding factors using the GED and SGED. In addition, the factors are almost identical (0.99 correlation) whether using Student's t or Skewed Student's t GARCH type filters.

[15]Although our methods apply to RVs we do not pursue this approach in the empirical analysis due to unavailability of intraday data for most of the financial series in our panels. Resorting to the monthly RVs yields imprecise estimates because of the small number of observations/days aggregated within a month.

the common factors or principal components, $VFIV$, from the panel of the fitted values of each univariate AR-(E)GARCH filtered volatility regressed on their cross-sectional average using lags of the cross-sectional average as simple instruments.

We turn to evaluating the number of principal components for each of the two panels, the $SRFUN$ and $LRBON$, using the Bai & Ng (2002) information criteria (IC_{p1}, IC_{p2}, IC_{p3}, BIC_3) and the Ahn & Horenstein (2013) estimators (ER and GR). We find that the IC_p criteria hit the boundary of the maximum number of factors, $r_{max} = 5$, whereas the Ahn & Horenstein (2013) estimators choose one factor in most cases. These results are reported in the OA, Table A.1.4, for conciseness. While the BIC_3 performs better than IC_p criteria, in the sense that it does not hit the boundary of r_{max}, it still estimates a large number of factors being four in most cases. Interestingly, while the IC_p and BIC_3 estimate a large number of factors for all types of factors (SF, VF, VFe), this is not the case for the $VFIV$ approach, according to which all Bai & Ng (2002) criteria as well as Ahn & Horenstein (2013) estimators choose one factor (Table A.1.4). This evidence suggests that the IV estimation of volatility factors alleviates the problem of the IC_p criteria that hit the boundary of r_{max}. While alternative criteria can often yield different number of factors empirically, given the difference in their penalty terms, it is worth mentioning that the choice of factors based on this approach follows an unconditional setup (i.e. does not condition on the explanatory or predictive power of each factor for the VIX or RV or VRP). Hence we also consider the complementary method which selects factors in a predictive regression using a hard thresholding approach (e.g. Bai and Ng, 2008). We find that for the VIX using the $SRFUN$ factors, only one factor, namely the first volatility factor, is selected among alternative volatility and spreads factors and their lags using the Bai and Ng (2008) approach. Thus the results based on the Bai & Ng (2008) hard thresholding targeted predictor approach, the Ahn & Horenstein (2013) as well as our predictive regressions in Tables 2-4 (which include additional variables too) provide empirical support for a single factor for each panel and especially the $SRFUN$ factors. In addition, the first common volatility type factor explains the largest variation in each panel and also provides a framework to label each of the factors, referring to the subpanel being used to construct it. For instance, the first volatility factor, $SRFUN\_VF$, explains 81% of the variation of this panel, while the $LRBON\_VF$ explains 87% of the cross-sectional variation of the panel. Similar results apply to the rest of the volatility type factors (VFe and $VFIV$).

Next we empirically evaluate the properties of the residuals extracted from volatility factor models. Following Bai (2003) we evaluate the existence of weak temporal and cross-sectional dependence and heteroskedasticity (Assumptions C, E and H). Using auxiliary regressions we find that the panel residuals from the volatility type factor models have weak and insignificant temporal dependence. Similarly, using auxiliary regressions for the squared residuals we find weak evidence of heteroskedasticity. Finally, regarding the assumption of weak dependence between factors and idiosyncratic errors (Assumption D in Bai, 2003) we find that the regression coefficient of the panel model residuals and each of the corresponding alternative estimated volatility factors (VF, VFe and $VFIV$) is insignificant and close to zero.

Following Lettau & Pelger (2018) a strong factor affects a very large number of underlying assets in the panel. Hence, we examine if the first volatility type factor (VF, VFe, $VFIV$) affects each filtered volatility in the panel and find that all spreads volatilities in the panel are significantly explained by the first factor. These results are robust using both parametric and non-parametric (the Kendall's tau and Spearman's rank) correlation tests. We also investigate the variance signal of the first factor as a percentage of the first five factors and find that VF and VFe yield variance signal close to 50% while the variance signal of $VFIV$ appears to be 100%, providing evidence of a strong factor. Last but not least, we also examine the estimated loadings regarding the strong factor case. According to Lettau & Pelger (2018) under the strong factor assumption, $\Lambda'\Lambda/N \rightarrow \Sigma$, where Σ is a full-rank, diagonal matrix. We evaluate this assumption and find supportive evidence for the volatility type factors for both the $SRFUN$ and $LRBON$.

Turning to the volatility type factors we examine more closely the cross-section of each panel and which series drive each factor. The short-run funding spreads volatility panel ($N = 45$) comprises volatilities of short-run funding spreads indicators such as the TED, the LIBOR, the Eurodollar, the (Non) Commercial (Non) Financial papers of different short-run maturities (of 7 Days,1,3,6,12 months) spreads with respect to the Federal Funds rate (FF). The list of short-run funding variables and spreads definitions for this panel as well as their data source is found in the OA Table A.1.2. The spreads considered in this panel have a shorter horizon of less than one year relative to those from the long-run corporate and government bond spreads panel below. The R^2 of each series with the first volatility factor, $SRFUN\_VF$ (which explains 81% of the variation of this panel) is also reported. The $SRFUN\_VF$ loads heavily on the volatility of (a) the spread of the 7- and 15-day A2/P2/F2 nonfinancial commercial paper with respect to FF with $R^2 \approx 0.70$, (b) the 3-month/1-week AA financial commercial paper spread with $R^2 \approx 0.65$ and (c) the 3-month financial commercial paper spreads with respect to Treasury bill spread as well as the LIBOR and Eurodollar spreads with $R^2 \approx 0.60$. Similar results can be obtained for $SRFUN\_VFe$ and $SRFUN\_VFIV$.

The long-run bond spreads panel ($N = 55$) involves volatilities of relatively long-run corporate and government bond spreads (longer than one year) from different industries, indices, maturities, rating categories vis-à-vis the corresponding government bond maturity (e.g. 1,5,7,10 years). The list of long-run bond spreads series definitions of this panel and their data source is found in the OA Table A.1.3. The first $LRBON\_VF$ (which explains 87% of the variation of this panel) loads heavily on the following types of volatility spreads vis-à-vis the corresponding maturity government bond: the Merrill Lynch (ML) US high yield (BB) option-adjusted as well as the ML US high yield semi-annual yield to worst, the ML high yield corporate master II: effective yield, the ML US corporate master (A) option-adjusted. These individual volatilities have an $R^2 \approx 0.80$ with the $LRBON\_VF$. The volatility of one of the benchmark corporate spreads, namely the Moody's bond spread Baa-Aaa, also yields a high $R^2 \approx 0.65$ with the factor, $LRBON\_VF$. Similar results apply to $LRBON\_VFe$ and $LRBON\_VFIV$.

In Figure 1 we plot the two monthly volatility factors, namely the short-run funding spreads volatility factor ($SRFUN\_VF$) and the long-run corporate and government bond spreads volatility factor ($LRBON\_VF$)

during the monthly period 1999m01 - 2016m09. We note the spike during the global financial crisis in our factors - a spike which also appears in the VIX, RV and the VRP. This will prompt us to also look at samples with and without the financial crisis as well, to examine the sensitivity of our results to the crisis period. In Figure 1 we observe the relatively different behavior of $SRFUN\_VF$ from the other two volatility factors during the crisis. The $SRFUN\_VF$ starts increasing in September 2007 well before the long-run corporate and government bond spreads volatility factor. In fact the first peak of the $SRFUN\_VF$ is much higher relative to the other volatility factor in Figure 1. This first peak of $SRFUN\_VF$ in February 2008 is associated with diminished liquidity in the interbank funding rates and the announcement of the Fed to reduce its target for the federal funds rate as well as the primary credit rate. The second peak in $SRFUN\_VF$ is of the same size and coincides with the peak in the $LRBON\_VF$ in September 2008 due to the bankruptcy of Lehman Brothers. Another interesting feature from Figure 1 is that $SRFUN\_VF$ reverts to its low historical mean level shortly after the Lehman crisis whereas the level of the $LRBON\_VF$ remains at a relatively higher level in the post-Lehman period and up to the end of 2016. The correlation, ρ, between $LRBON\_VF$ and $SRFUN\_VF$ is 0.40 since they refer to different classes of risky assets. Interestingly some of the well known US broader economic conditions factors/indices based on spreads (as opposed to volatilities), such as the Chicago Fed National Conditions Index (NFCI) also correlates highly with $LRBON\_SF$ ($\rho = 0.90$), $LRBON$ volatility type factors ($\rho \approx 0.70$) and with $SRFUN\_SF$ ($\rho = 0.80$).

In addition, we compare our spreads factors with other related factors such as the Gilchrist & Zakrajšek (2012) spread, GZ_SPR, the NFCI and its subcomponents (credit, leverage and risk) as well as the St Louis Financial Stress Index (FSI). Figure 2 shows the time series behavior of our long-run corporate and government bond spreads factor ($LRBON\_SF$) with the aforementioned factors, GZ_SPR, FSI and NFCI. It is evident how highly correlated these factors are. The $LRBON\_SF$ is highly correlated with the GZ_SPR ($\rho = 0.95$), the FSI ($\rho = 0.85$) and NFCI ($\rho = 0.90$), due to the large number of common long-run corporate and government bond spreads series. In contrast, $SRFUN\_SF$ is relatively less but still highly correlated with these well-known factors given its correlation with GZ_SPR is $\rho = 0.67$ and with NFCI is $\rho = 0.80$. Similarly our corresponding volatility type factors e.g. $SRFUN$ factors also correlate with some of the traditional spreads factors such as FSI ($\rho \approx 0.80$) and NFCI ($\rho \approx 0.60$). It is worth mentioning at the outset that our volatility type factors perform relatively better empirically than the corresponding spreads factors in explaining the VIX, the RV and the VRP. Hence we focus our discussion on the volatility type factors, but also provide some results for comparison purposes with the corresponding spreads factors at the end of subsection 5.1 and in Section A.2 in the OA. The correlation matrix of our factors with the aforementioned established factors in the literature can be found in Table A.1.5 in the OA. Finally, we mention that our factors provide improved in- and out-of-sample results vis-à-vis the aforementioned factors in the literature for the VIX, RV and VRP, as summarized in the empirical Section 5 and in the robustness Section 6 (and detailed in the OA).

Furthermore, we repeat our analysis extracting factors from the returns, spreads and volatilities of the series at daily frequency. The daily cross-section is smaller due to the fact that some variables have a shorter time-series sample. Hence for the daily analysis the long-run corporate and government bond spreads panel includes a cross-section of $N = 54$ and the short-run funding spreads panel includes $N = 39$ (and the unavailable daily series are marked with a ($^+$) in Tables A.1.2-A.1.3 of the OA). The daily vis-à-vis the monthly frequency plays an important role for volatility factor estimation as implied by Proposition 2.1 and the simulation evidence in Section 3, which we further investigate empirically using linear - Least Squares (LS) and MIDAS - Nonlinear Least Squares (NLS) predictive regressions both in- and out-of-sample in Section 5.

5 What drives the VIX, RV and VRP?

The objective of this section is threefold: First, to examine the ability of our spreads and volatility type factors extracted from the short-run funding spreads and the long-run corporate and government bond spreads panels, to explain the monthly VIX, the S&P500 RV and the VRP at different horizons, and to compare our factors with existing factors in the literature, such as the GZ_SPR, the FSI, the NFCI, among others. We evaluate the alternative specifications of extracting volatility factors based on the panels of (i) the volatilities of the idiosyncratic components (residuals) of the spreads factors (denoted by VF) and (ii) the alternative volatility type factors which deal with the estimation error (denoted by VFe and $VFIV$, defined in the previous section) for explaining and predicting the VIX, RV and VRP. Second, we assess whether the higher-frequency (daily) factors provide additional information both in- and out-of-sample vis-à-vis the corresponding low-frequency (monthly) factors in explaining the VIX, RV and VRP. The higher sampling frequency can be quite relevant when extracting volatility type factors and when using these as predictors of other volatility benchmark indicators such as the VIX or the RV of S&P500. Third, given our factors can explain the VRP we examine whether they are also relevant predictors for the equity risk premium.

Our monthly RV of the S&P500 is the summation of the 78 within day five-minute squared returns covering the normal trading hours from 9:30am to 4:00pm plus the close-to-open overnight squared returns. For a typical month with 22 trading days, this leaves us with a total of T = 22 $\times$ 78 = 1716 five-minute returns augmented with 22 overnight squared returns. The variance risk premium, VRP, is not directly observable and therefore an empirical proxy can be constructed. Following Bollerslev, Tauchen and Zhou (2009), Zhou (2018) and Drechsler & Yaron (2011) we assume that $E_t(\text{RV}_{t+1}) = \text{RV}_t$, i.e. RV_t follows a random walk, such that the $\text{VRP}_t = \text{VIX}_t - \text{RV}_t$ becomes directly observable at time t. For comparison purposes we follow their definition of VRP_t in this section. Different methods and models can also be adopted to approximate the conditional expectation of RV_t such that, for instance, the conditional expectation is replaced by the forecasts of different reduced-form model specifications for RV. Indeed, Bekaert and Hoerova (2014) emphasize this point and show that alternative RV forecasts not only affect the VRP but also have implications on the role of the VRP in predicting stock returns, economic activity and financial instability. The robustness Section

6 also revisits our empirical analysis using one of the VRP measures in Bekaert & Hoerova (2014) for their best performing RV model. Our robustness results (in Section 6 and in the OA subsection A.2.1 and Table A.2.1) show that our $SRFUN$ volatility type factors are robust and significant predictors for the Bekaert & Hoerova (2014) VRP measure.

5.1 Do the volatility factors explain the VIX, RV and VRP?

In-sample predictive regressions results are reported for the monthly VIX, RV and VRP using our volatility type factors as predictors at monthly and daily frequencies. It is worth mentioning from the outset that over this period there is no empirical evidence of reverse Granger causality of the volatility type factors related to the short-run funding spreads (discussed in detail in the OA Section A.2 and summarized in Section 6).

Table 2 presents the estimated monthly linear predictive regression models for the VIX during the period 1999m01 - 2016m09 for forecasting horizons of $H = 3$, 6 and 9 months, which aim to evaluate if the monthly $SRFUN$ volatility type factors are significant predictors of the VIX. The choice of H is motivated by the CBOE VIX futures contracts. The following models are specified: In the first model the VIX depends on its own lag, VIX$(-H)$. In the second model the VIX is driven by the volatility of consumption growth following Drechsler & Yaron (2011) and by VIX$(-H)$. Following Drechsler & Yaron (2011) and Bollerslev et al. (2009), we proxy the volatility of consumption using the AR-GARCH fitted values of the monthly per capita consumption on non-durables and services which is denoted by DLC_V. The corresponding single factor model specification for the VIX which is closer to the consumption-based asset pricing theory of, e.g. Bansal & Yaron (2004), Drechsler & Yaron (2011), and includes only the DLC_V$(-H)$ is presented only when this factor turns out to be significant in models for the VRP in Table 4. The remaining models considered in Table 2 incorporate the $SRFUN$ volatility type factors which are based on the panels of the volatilities of the residuals from the regression of spreads on the common factor of spreads. We consider AR-(E)GARCH panels of the residuals to extract these common volatility factors. For the monthly VF, VFe and $VFIV$ predictors we report the LS estimates of linear regression models and the corresponding Newey West (NW) Heteroskedastic and Autocorrelation Consistent (HAC) errors. Our predictive models are also estimated using the IV method, instead of LS, to address the generated regressor issue. Longer lags of $(H+1)$ up to $(H+3)$ of these factors are used as instruments and the choice of lag length is based on the moment selection criteria of Andrews (1999) and Hall et al. (2007). Overall, we find that the IV approach of estimating the predictive regressions yields robust empirical results compared to the LS approach.

In order to investigate further Proposition 2.1 and the role of high-frequency data in extracting volatility factors we also examine the role of daily (as opposed to monthly) factors in predictive regressions for the monthly VIX, RV and VRP (in Tables 2, 3 and 4, respectively). Hence we present the empirical results of MIxed-DAta Sampling (MIDAS) predictive regression models, introduced by Ghysels, Santa-Clara and Valkanov (2006), according to which the dependent variable is either the low-frequency (monthly) VIX or

RV or VRP and the predictors are either low-frequency variables such as the volatility of consumption or high-frequency (daily) volatility type factors. The MIDAS model estimated by NLS is given by: $Y_t^M = \beta_0 + \beta_M X_{t-H}^M + \beta_H \sum_{i=1}^{N_D} w_i(\theta^D) F_{t-H,i}^D + \epsilon_t$, where Y_t^M refers to VIX or RV or VRP, X_t^M represents the monthly volatility of consumption, F_t^D refers to the daily-frequency factors and $w_i(\theta^D)$ is a parsimonious and data-driven aggregation scheme of the high-frequency data using, e.g. polynomial functions such as the exponential Almon or simpler step functions. The actual step size is chosen based on minimizing the AIC. In Tables 2-4 the adjusted R^2 (adjR^2) for the MIDAS regressions are reported in the last column for comparison with the linear LS adjR^2 that use the common (monthly) frequency predictors.

Three broad empirical results can be inferred from Table 2: First, all types of volatility factors for the panel of short-run funding spreads are statistically significant (at least at the 5% level) for all forecast horizons, $H = 3, 6, 9$ months, and model specifications considered for the VIX (except of $SRFUN\_VFe$ with Normal distribution which appears significant only for $H = 6$ and 9 months) over the period 1999m01-2016m09. $SRFUN$ volatility type factors predict a positive impact on the VIX which increases with H, for a given model. While most $SRFUN$ volatility type factors are significant predictors of the VIX at all horizons, it appears that $SRFUN\_VF$ is relatively a more noisy predictor and in particular $SRFUN\_VFIV$ (given the relatively higher standard errors). These empirical results are consistent with the simulation evidence related to the ratios of MSE of VFes and $VFIV$s versus VFs (found in Table A.1.1 and discussed in subsection 3.2). The effect of lagged VIX appears significant mostly in shorter horizons of $H = 3$ and 6 while the volatility of consumption turns out to be insignificant for all H. Second, the adjR^2 of the monthly LS predictive models are relatively similar across the different volatility type factors, except the monthly $SRFUN\_VFe$ with the Normal volatility filter distribution, which yields lower adjR^2 in LS models relative to the other distributions (GED and SGED). This evidence is also consistent with the simulation results. What is worth noting is that the monthly $SRFUN$ volatility type factors yield an improvement of almost 60% in terms of adjR^2 for longer horizons of $H = 6$ and 9 months vis-à-vis the benchmark models which include the lagged VIX (and the volatility of the consumption growth). In addition, the adjR^2 gains from the MIDAS regression models which use the daily information of the volatility type factors are higher vis-à-vis the corresponding linear LS monthly models for $H = 6$ and 9 months. Interestingly, for longer forecasting horizons (e.g. $H = 9$) the adjR^2 of the MIDAS regression models yield almost 50% gains vis-à-vis the corresponding adjR^2 of the LS models for explaining the VIX. This result provides empirical support for the MIDAS regression analysis according to which the relatively higher-frequency (daily) sampling scheme of regressors combined with a data-driven aggregation scheme can yield improved in-sample fit. These results are consistent with Proposition 2.1 according to which higher sampling frequency yields more accurate PCs. The corresponding MIDAS models in Table 2 with the daily factors are estimated using either the step function or the exponential Almon lag polynomial which yield similar adjR^2's and the choice of step size is based on the AIC. The daily $SRFUN\_VFe$ provides the highest in-sample adjR^2 gains and statistical significance for long horizon ($H = 9$) vis-à-vis the other factors and the benchmark models for the VIX, while the daily $SRFUN\_VF$ and $SRFUN\_VFIV$ outperform

$SRFUN\_VFe$ in shorter horizons.

Table 3 turns to the role of our volatility type factors in forecasting S&P500 RV at different horizons H for the same sample period. It has a similar structure to that of Table 2. The difference in Table 3 is that the models control for the lags of both the RV and the VIX following e.g. Bekaert, Hoerova and Duca (2013). Two broad results follow from Table 3: First, all $SRFUN$ volatility type factors are statistically significant at all forecasting horizons ($H = 3, 6, 9$, except of $SRFUN\_VFe$ with Normal distribution). As expected all volatility type factors have a positive impact on the RV. The lagged RV is significant only for the short horizon of $H = 3$ whereas the VIX and DLC_V are insignificant at all horizons considered. These results are robust to the IV estimation of the linear predictive regressions due to generated regressors. Second, the adjR^2 gains in models with our monthly factors are similar across all H and the volatility type factors considered. On the other hand, the adjR^2 of the MIDAS regression models with the daily $SRFUN$ volatility type factors show substantial gains vis-à-vis those of the monthly LS regression models especially as the horizon increases, given by $H = 9$ months. These results provide further empirical support of Proposition 2.1 which extend the relative gains from higher-frequency volatility type factors in the context of MIDAS predictive regressions.

Table 4 turns to the corresponding predictive regression models for the VRP for the period 1999m01 - 2016m09. For the VRP we consider the benchmark model which includes the lagged VIX, the second model which is related to the single factor long-run risk model of e.g. Bansal & Yaron (2004) and Drechsler & Yaron (2011) including only the consumption growth volatility, and the third model which includes both variables. The reported results in Table 4 yield the following interesting conclusions: First, in accordance with the theory the VRP can be explained by the single factor model namely the volatility of the volatility of consumption which is approximated by the DLC_V that turns out to be significant in all horizons ($H = 3, 6, 9$ months). For the longer horizon $H = 9$ the DLC_V is also significant in the presence of the lagged VIX and our volatility type factors. Second, our $SRFUN$ volatility type factors are significant and have a positive impact for longer horizons of $H = 6, 9$ months while they are insignificant for shorter horizons (except of $SRFUN\_VFIV$ with Normal distribution which appears to be significant for $H = 3$). These results are robust to the IV estimation approach. Third, the VRP MIDAS regression models show that the daily frequency of all the $SRFUN$ volatility type factors provide better in-sample fit, given the relatively higher adjR^2, vis-à-vis the monthly frequency of factors, for longer horizons of $H = 6$ and 9 months. The adjR^2 of the MIDAS models redoubles vis-à-vis the corresponding linear models. Overall the $SRFUN$ volatility type factors with GED and SGED distributions at $H = 6$ and 9 months yield stronger in-sample results in terms of the statistical significance of these factors (relative to the volatility type factors with Normal distribution) and the $SRFUN\_VFe$ factors yield the highest adjR^2 in MIDAS models.

The results in Tables 2-4 refer to the sample period that ends in 2016m09 and include a binary/dummy variable in the constant of the models to capture the mean shift effect of the bankruptcy of Lehman Brothers and its aftermath during September and October 2008. We therefore re-examine our results, not controlling for the unusual nature of the global financial crisis in the OA Table A.2.4. We also consider an alternative approach in dealing with the Lehman Brothers period by trimming the sample and we obtain similar results.[16] Additional robustness checks for the results in Tables 2-4 are reported in Section 6.

So far, we focused exclusively on discussing the effects of volatility type factors and in particular those extracted from the volatilities of short-run funding spreads. Our empirical analysis does not deal exclusively with panels of volatility filters but it also extracts factors from various panels of the spreads and returns. Hence, we also consider the spreads factors $LRBON\_SF$ and $SRFUN\_SF$ from the long-run corporate and government bond spreads and the short-run funding spreads panels, respectively. Established factors in the literature that have some common information in terms of the actual spreads series with our factors are the FSI, the NFCI, among others. Hence, we consider the aforementioned factors and their corresponding volatilities obtained from the estimated AR-EGARCH models denoted as FSI_V and NFCI_V, and NFCI sub-indices which refer to series classified in three categories: Risk, Credit and Leverage. The correlations of these factors are also reported in Table A.1.5. The FSI and NFCI are highly correlated with the $LRBON\_SF$ (with $\rho \approx 0.90$).[17] The volatilities of FSI and NFCI, FSI_V and NFCI_V, are relatively less correlated with all our $SRFUN$ and $LRBON$ volatility type factors with correlation coefficients ranging from $0.30 - 0.65$. This is expected given that our volatility type factors are extracted from panels of volatilities. Interestingly, the $SRFUN\_SF$ as well as all our volatility type factors are different from existing factors in the literature, given their relatively low correlation of $0.53 - 0.80$. Last but not least, it is worth mentioning that both the FSI and NFCI incorporate equity market indices and most importantly the VIX, which is not the case for our factors.

Turning to the predictive models with the aforementioned alternative factors, (including the Lehman Brothers dummy variable), one can evaluate the significance of these established factors to explain the VIX, the RV and the VRP. The overall result from these predictive models (reported in Table A.1.6 of the OA) is that for $H = 6, 9$ the $SRFUN\_VF$, $SRFUN\_VFe$ and $SRFUN\_VFIV$ provide the highest adjR^2 and the strongest significance (at 1%-5% level) relative to the other factors, in explaining the VIX, the RV and the VRP. The corresponding volatility type factors from the $LRBON$ panel turn out to be insignificant in most models of the VIX and RV. Our $SRFUN$ volatility type factors still perform better than the alternative volatility factors, however, the financial indices, FSI and NFCI, yield similar adjR^2 for the VIX and RV models for $H = 3$ and VRP models for $H = 6, 9$. Concluding, the $SRFUN\_VF$, $SRFUN\_VFe$ and $SRFUN\_VFIV$ turn out to be the relatively most significant factors across different model specifications followed by the FSI and NFCI.

[16] Using a trimming of 2% and 2.5% removes many of the outliers including the Lehman Brothers bankruptcy period. In particular 2008m10, 2008m11 and 2009m02 are excluded using a trimming of 2% while the 2.5% additionally trims 2008m09. Our results remain robust when trimming the sample.

[17] The FSI and our $LRBON\_SF$ have 9 series in common which are related to all corporate and government bond spreads, as opposed to the $SRFUN\_SF$ which has only 3 series in common, namely the TED, the LOIS and the 3-month financial commercial paper minus the 3-month Treasury bill.

5.2 Out-of-sample predictability for the VIX and RV

This section examines the out-of-sample (OOS) performance of our volatility type factors in predicting the VIX and the S&P500 RV, given that the in-sample results showed that these factors are relatively more significant for these variables as opposed to VRP.[18] For our out-of-sample exercise we estimate the factors recursively by the principal component method prior to forecasting (i.e. at each forecast origin) following e.g. Goncalves, McCracken and Perron (2017). It is also important to mention that our approach also re-estimates the volatility models updating their estimation accordingly before the recursive estimation of the factors, so that we consider a fully recursive scheme. Hence this OOS approach tries to avoid the look-ahead bias criticism often posed in financial applications. In addition, we also consider two OOS prediction periods, before and after the Lehman Brothers crisis starting in 2007m01 and 2009m01, respectively. Since we are dealing with evaluating the predictive ability of nested models with estimated factors we apply the recent techniques in Goncalves et al. (2017). To examine the relative out-of-sample forecasting performance of the factors we evaluate (i) the relative MSE of the models with our alternative monthly factors vis-à-vis the benchmark models (which exclude all factors) reported in Table 5 as well as (ii) the relative MSE of the daily factors using the MIDAS-NLS models vis-à-vis the monthly factors via the traditional linear LS models reported in Table 6.

In Table 5 we evaluate the out-of-sample forecasting performance of the monthly volatility type factors (VF, VFe and $VFIV$) extracted from the panels of $LRBON$ and $SRFUN$, for $H = 3, 6, 9$ months for the two OOS periods. The top Panel A of Table 5 reports the MSE ratios for the VIX using the AR benchmark model with and without the DLC_V, while the second Panel B reports the MSE ratios for the RV using the Distributed Lag benchmark model with the lags of RV and VIX with and without DLC_V. All MSE ratios are reported with the competing factor model in the numerator. Reported MSE ratios being less than one imply that the models with our factors yield statistically significant OOS MSE gains vis-à-vis the aforementioned benchmark models. The (*) denotes the rejection of the null hypothesis of equal predictive ability of the corresponding benchmark model and the competing model which also includes the alternative factors (at 5% significance level) using the MSE-F test statistic and the McCracken (2007) critical values.[19] The overall results in Table 5 suggest that the factors extracted from the panel of the volatilities of short-run funding spreads yield MSE gains vis-à-vis the benchmark models for both OOS forecasting periods and each forecasting horizon, $H = 3, 6, 9$ months. In contrast, the factors extracted from the panel of the volatilities of long-run corporate and government bond spreads yield MSE gains mainly for the pre-Lehman Brothers crisis period while the results become weaker for post-Lehman Brothers crisis period especially for predicting VIX. The MSE ratios for $SRFUN$ volatility type factors perform better than $LRBON$ volatility type factors for forecasting VIX while the results for RV using $LRBON$ volatility type factors appear

[18] The OOS results for the VRP are also relatively more weak than those of the VIX and RV and therefore are not reported.

[19] We treat the MSE-F test of equal predictive ability as a one-sided to the right test, following Clark and McCracken (2015), because under the forecasting principle a more parsimonious model is preferred.

to be unstable across forecasting horizons.[20] Moreover the $SRFUN$ volatility type factors with SGED distribution in most cases provide greater gains for the OOS forecasts relative to other distributions for both out-of-sample periods. Interestingly, in the post-Lehman forecasting period the $SRFUN$ volatility type factors perform relatively better. The MSEs of the models with the $SRFUN\_VF$ and $SRFUN\_VFe$ show gains up to 40% vis-à-vis the benchmark AR model of the VIX for both forecasting periods, for $H = 9$. Similarly, for forecasting the RV, the $SRFUN\_VFe$, but especially the $SRFUN\_VF$, can yield MSE gains up to 30% vis-à-vis the benchmark model, which is more pronounced in the post-Lehman forecasting period (as shown in Panel B of Table 5). Overall, the OOS results show that our $SRFUN$ volatility type factors are robust and significant predictors of the VIX and RV (vis-à-vis those from $LRBON$).

In Table 6 we turn to the evaluation of the out-of-sample forecasts for the VIX and the RV comparing the performance of the daily vis-à-vis the monthly volatility type factors via the MSE ratio of the MIDAS-NLS versus the traditional linear LS regression models, for the post-Lehman period. For each predictive regression model the predictor used (one at a time) refers to the volatility type factors, $SRFUN\_VF$ and $SRFUN\_VFe$, using the AR-EGARCH model with Normal, GED and SGED distributions.[21] The forecasts of the models are evaluated using the MSE from the MIDAS model with the daily factors vis-à-vis the LS model with the corresponding monthly factors via the MSE ratio of the two models, MSE(MIDAS)/MSE(LS). We report results for both the recursive and fixed sample forecasting schemes. The overall picture from the results reported in Table 6 suggests that in most cases both the $SRFUN$ daily factors (VF and VFe) improve upon the corresponding monthly factors in forecasting the VIX and RV, for most forecasting horizons using the recursive method. These results provide additional empirical evidence related to the fact that the higher sampling frequency in financial assets can yield volatility type factors which along with MIDAS model can improve the OOS predictive ability of traditional linear regression models with lower frequency factors.

5.3 Equity premium predictability

So far we have focused on pricing volatility risk with volatility factors. An affine asset pricing model implies that risk pricing for each asset class is linear in $\mathcal{X}^f$, including expected excess log returns (on the market portfolio). In particular, the equity risk premium - in analogy with the variance risk premium in equation (2.5), can be written as:

$$E_t^{\mathbb{P}}[r_{t,t+\tau}] - E_t^{\mathbb{Q}}[r_{t,t+\tau}] = \gamma_{er}(\tau)\mathcal{X}_t^f. \tag{5.1}$$

Bollerslev et al. (2009) find that the VRP is a significant predictor of the expected stock returns. Given that the empirical evidence above, shows that some of our factors can predict the VIX, the RV and the VRP, we can therefore examine if our factors can also predict excess returns. Specifically within the traditional

[20]For $H = 6$ there are no gains using $LRBON$ volatility type factors while for $H = 3$ and $H = 9$ there are substantial improvements in terms of MSE.

[21]The $VFIV$ results are similar with VF and VFe and not reported for conciseness.

equity predictability literature we evaluate both the in- and out-of-sample forecasting ability of our spreads and volatility factors and examine if they have any additional predictive ability beyond that of the VRP as well as some of the most popular predictors of returns (e.g. the log Price-Dividend ratio, log(P/D), the Moody's bonds default spread Baa-Aaa, the log Price-Earnings ratio, log(P/E) and the term spread, TMSP).

Table 7 presents the in-sample monthly S&P500 excess returns LS predictive regression results for $H = 1$ and 3 months with robust standard errors based on the Newey West HAC estimator for the period 1999m01-2016m09, excluding the Lehman Brothers period. The corresponding results without the Lehman Brothers dummy variable are reported in the OA Table A.1.7. The VRP is taken as the benchmark predictor in the first model (first column) as well as the rest of the models in Table 7 given the results in Bollerslev et al. (2009). Hence, all models in the first panel in Table 7 control for the VRP and include the following factors one at a time in order to address the relative role of each factor as a predictor: our short-run funding spreads factors ($SRFUN\_SF$) as well as the volatility type factors, $SRFUN\_VF$ and $SRFUN\_VFe$. By analogy, and because it is related to some of the traditional predictors (e.g. Baa-Aaa, TMSP), we also consider the long-run corporate and government bond spreads factor ($LRBON\_SF$) and the corresponding volatility type factors, $LRBON\_VF$ and $LRBON\_VFe$.[22] We also consider the GZ credit spread (GZ_SPR) given as an alternative corporate risk spread and its volatility GZ_SPR_V. The in-sample results for $H = 1$ in the first panel of Table 7 show that VRP is significant in all models which is consistent with the evidence in Bollerslev et al. (2009). Interestingly our risk factors and in particular the $SRFUN\_VF$ and $SRFUN\_VFe$ and the spreads factor $SRFUN\_SF$ are always statistically significant even in the presence of VRP. In addition, the spreads factors, $LRBON\_SF$ and GZ_SPR, are also significant but their corresponding volatility predictors turn out to be insignificant. In addition, the $SRFUN$ volatility type factors provide higher adjR^2 compared to the corresponding models which include only the VRP as predictor. In particular, these gains can be as high as around 20% when adding the monthly $SRFUN\_VF$ or $SRFUN\_VFe$ as a predictor vis-à-vis the benchmark model with only the VRP predictor. The MIDAS models which employ the daily frequency of the factors (and the monthly VRP benchmark) provide additional gains in terms of adjR^2 which can reach up to 55% vis-à-vis the benchmark model with just the VRP(-H).

Turning to the second panel of Table 7 the models include both the VRP and the Baa-Aaa default spread as well as our factors. We find that the Moody's long-run spread turns out to be an insignificant predictor in the presence of the VRP and our short-run funding spreads volatility type factors ($SRFUN\_VF$ and $SRFUN\_VFe$) are always significant predictors with almost 20% adjR^2 gains in the common-frequency LS models vis-à-vis the benchmark model which includes both the VRP and Baa-Aaa. Using the corresponding MIDAS model with the daily $SRFUN$ volatility predictors the gains in terms of adjR^2 can go almost 45%. Similar results are obtained for the term spread, TMSP, which is another popular predictor of the excess market returns considered in the third panel of Table 7.

Last but not least, we show that all $SRFUN$ volatility type factors are still significant in the presence of

[22] The $VFIV$ factor results as well as the results for all the volatility type factors are similar with the reported ones.

both the VRP and log price-dividend ratio (log (P/D)), which is also one of the strong predictors of the equity premium. Similarly, the three spreads factors ($SRFUN\_SF$, $LRBON\_SF$ and GZ_SPR) are also strong predictors of excess returns. In the monthly linear LS models the $SRFUN\_SF$ and $LRBON\_SF$ yield the relatively higher adjR^2's. Interestingly, while the adjR^2's gains of spreads factors ($SRFUN\_SF$ and $LRBON\_SF$) are also supported by the daily MIDAS models with even higher corresponding adjR^2, it is the daily $SRFUN$ volatility type factors in MIDAS models that yield the highest relative in-sample adjR^2 gains. Performing the same analysis with log price-earnings ratio (log(P/E)) we find weaker results, only the $SRFUN\_VF$ with the SGED distribution is significant. The second part of Table 7 repeats the in-sample equity predictability analysis for $H = 3$ months and the $SRFUN$ volatility type factors appear to perform even better in some cases while the other factors' results become weaker.

The out-of-sample (OOS) predictive ability of our factors for forecasting the S&P500 excess returns for 1 and 3 months ahead using the recursive method is evaluated for the pre- and post-Lehman Brothers crisis OOS periods. Using the traditional constant or historical mean/average benchmark model for excess returns we report the corresponding MSE ratio of each model vis-à-vis this benchmark. The MSE ratios of our volatility type factors yield OOS relative forecasting improvements in both periods but especially in the longer OOS since 2007m01 for the VRP. These results are reported in the OA, Table A.1.8. In fact, it is mostly the MIDAS models with daily volatility type factors that yield out-of-sample significant MSE gains in both periods.

6 Robustness checks

In this section we summarize the main findings of an extensive robustness analysis. The detailed discussion and corresponding tables are found in the OA, Section A.2.

We examine the robustness of our results using an alternative measure of the VRP. Following Bekaert & Hoerova (2014) we use the VRP based on their best performing RV forecasting model, denoted by VP8, which is correlated with our VRP measure (in the previous sections) with simple (rank) correlation coefficient being 0.52 (0.76). Re-estimating the VRP models in Table 4 using the VP8, we find that the significance of $SRFUN$ volatility type factors is not only robust for the longer horizons $H = 6$ and 9, but also turns out to be significant for a shorter horizon of $H = 3$ (found in subsection A.2.1, Table A.2.1 in the OA). Similarly, the in-sample S&P500 excess returns results are also robust when using VP8, i.e. $SRFUN$ volatility type factors remain significant (Table A.2.2).

Turning to different AR-(E)GARCH filters, for extracting volatility type factors we find that our in-sample results are robust with and without the Lehman Brothers dummy (found in subsection A.2.2, Tables A.2.3-A.2.5 in the OA). Our results are also robust to volatility type factors extracted from the panels of log volatility filters as well as to the log specification of our predictive regressions, following the literature on logRV type models. In general, the log predictive regressions for the VIX and RV models are robust to the results in Tables 2 and 3, while the results for the VRP are stronger in terms of significance for the non-log transformation in Table 4 vis-à-vis the logVRP specifications (found in subsection A.2.3, Tables A.2.6 and A.2.7 in the OA). Moreover, evaluating the reverse Granger causality, we find that during 1999m01 - 2016m09, there is no empirical evidence that the VIX, the RV and the VRP Granger cause the $SRFUN$ volatility type factors. Additional results are reported in the OA, subsection A.2.4 and Table A.2.8.

Some studies acknowledge that the volatility of volatility of consumption is difficult to measure empirically and approximate consumption volatility by monthly industrial production (IP) volatility (DLIP_V) or the Chicago Fed National Activity Index volatility (CFNAI_V) instead of the DLC_V (e.g. Bollerslev et al., 2009, Zhou, 2018). Using these alternative proxies we find that, in general, the volatility per capita consumption DLC_V is more significant than that of the other two proxies and do not affect the significance of our proposed volatility factor e.g. $SRFUN\_VF$ (in subsection A.2.5 in the OA).

Turning to other financial indicators and factors we evaluate their predictive ability vis-à-vis our proposed factors. The CBOE Skewness index is an insignificant predictor in our models and does not affect the significance of the other volatility factors (subsection A.2.6, Table A.2.11 in the OA). Similar results apply using the Chang, Christoffersen and Jacobs (2013) Realized Skewness factor (for the sample ending in 2008m12). We also consider various alternative indicators of liquidity in order to establish whether these are also predictors of the VIX, RV and VRP and to what extend these are related to our $SRFUN$ volatility factor. Employing the three measures of liquidity proposed by Pastor and Stambaugh (2003) and a CDS factor in the spirit of Adrian and Shin (2010) we find that these are insignificant predictors of the VIX, RV and VRP and do not affect the significance of our $SRFUN$ volatility factors. Further details of these results can be found in subsection A.2.7 of the OA (and Tables A.2.12 and A.2.13).

Finally, some individual series in our three cross-sections of financial assets have been monitored as recent indicators of financial distress and as leading indicators of economic activity. Examples of these indicators are the TED and the Baa-Aaa as well as various energy and precious metals futures returns indicators. We address the predictive ability of many of these indicators and their volatilities (one at time) vis-à-vis the $SRFUN\_VF$. Two interesting results can be extracted from the short-run funding spreads indicators, shown in Table A.2.14, subsection A.2.8 in the OA. First we find that the volatilities of the 7-day A2/P2/F2 nonfinancial commercial paper minus the Fed funds rate and the TED spreads yield the relatively highest adjR^2 and statistical significance among the rest of the predictors for explaining the VIX and the RV but lower than our $SRFUN\_VF$, except of the case of $H = 9$ in explaining the RV. Second, the volatility of the spread of the 3-month financial commercial paper minus treasury bill yields similar adjR^2 with that of the $SRFUN\_VF$ in explaining the VRP and higher statistical significance only for $H = 6$.

7 Conclusions

We propose a procedure which consists of collecting a large panel of asset returns or spreads, monthly and daily in our case, or possibly any other (higher) frequency. For each series in the panel one fits a standard ARCH-type volatility model on the estimated idiosyncratic component of spreads which takes into account the common spreads factors. Therefore a panel of filtered volatilities is obtained from which principal components are extracted to represent common volatility factors. We study the theoretical properties of such a procedure, in particular how the combination of volatility filtering and principal component analysis relate to the class of affine diffusions often used in theoretical and empirical asset pricing models, the role of sampling frequency and the role of estimation error in filtering volatilities. Monte Carlo simulation evidence shows that our proposed procedures have the expected sampling properties that fall in line with the asymptotic theory.

The literature emphasizes almost exclusively on estimating parametric models for the S&P500 involving both cash and options data (e.g. Chernov and Ghysels (2000) and Pan (2002) and many subsequent papers) or some type of non-parametric procedure combining high-frequency cash market data and options on the market index (see Bollerslev et al. (2009) and many subsequent papers). Most studies find that either volatility risk or more specifically a disaster fear affecting consumption and the overall economy are the driving forces of the VIX and the VRP. Our analysis goes beyond the confines of the market index. The empirical analysis takes as given that volatility risk is pervasive and interconnected across different asset classes. There are at least two interpretations of our empirical findings. First, it is fair to say that the panel data setting allows us to estimate volatility risk more precisely. There is in fact a theoretical justification for this argument, although it is not explicitly exploited in the current paper. Hence, we can take for granted that the panel-based approach yields better estimates of volatility factors. Are these improved estimates telling us indirectly something about consumption volatility risk - which is hard to pin down using either conventional aggregate consumption or activity related series? Is this fundamental volatility risk also related to common volatility factors of short-run funding spreads volatilities or the volatility factor? Our empirical analysis provides supportive evidence for these. A second interpretation of our results is that we uncover drivers of the VIX and VRP that relate to a different asset pricing model, not necessarily driven by consumption volatility risk. Should we think of production-based asset pricing models? A default risk interpretation of our short-run funding spreads factor would surely not rule this out. Note that here too the argument of more efficient estimation of volatility risk factors through panel data methods, as advocated in this paper, equally applies. Should we think of our results as given more credence to models which rely on financial intermediation risk channels? It surely may, and perhaps we are also better at capturing this risk channel with our novel approach, compared to say looking at the aggregate balance sheets of intermediaries or the CDS spreads of their parent companies. Our robustness analysis suggests that in terms of capturing the dynamics of the VIX and VRP we do better than the direct measures of financial intermediaries balance sheet constraints. Hence, our empirical findings provide challenges and food for thought for future research.

Appendices

A Regularity Conditions for Volatility Proxies

The filter appearing in (2.8) satisfies:

Assumption A.1. *For all* $i = 1, \ldots, N$ *the functions* $\widehat{\kappa}^i$, $\widehat{\mu}^i$ *and* g_t^i *satisfy:*

- $\widehat{\kappa}^i$ *and* $\widehat{\mu}^i$ *are continuous in all arguments*
- g_t^i *is differentiable in* $\widehat{v}_{x,t}^i$, $\widehat{V}_{[h_i:t-h_i]}^i$ *and* h_i *almost everywhere and must possess one-sided derivatives everywhere.*

We proceed with a high-frequency data sampling uniform across all $i = 1, \ldots, N$, namely:

Assumption A.2. *Let* $h \triangleq (\sup_{1\leq i\leq N} h_i) \downarrow 0$, *where* N *is the number of assets in the cross-section. Moreover, let* $\forall$ $i : q_{[h_i:t]}^i \triangleq h^{-1/4}\, \widehat{V}_{[h:t]}^i$ - V_t^i, *which satisfies for all* $i = 1, \ldots, N$, *uniformly on every bounded* (x, q, t) *set:*

$$h^{-1/2}E[q_{[h:t+h]}^i - q_{[h:t]}^i | p_t^i = x, \mathcal{X}_t^f = \mathcal{X}, q_{[h:t]}^i = q] \to qB(x,y,t),$$

$$h^{-1/2}Var[q_{[h:t+h]}^i - q_{[h:t]}^i | p_t^i = x, \mathcal{X}_t^f = \mathcal{X}, q_{[h:t]}^i = q] \to C(x,y,t),$$

as $h \downarrow 0$ *where:*

$$\begin{aligned} B^i(x,y,t) &= \lim_{h\downarrow 0} E[\partial g^i\left(\widehat{v}_{x,t}^i, p_{[h:t-h]}^i, \widehat{V}_{[h:t-h]}^i, t, h\right)/\partial V^i | p_{[h:t-h]}^i = x, \widehat{V}_{[h:t-h]}^i = y] \\ C^i(x,y,t) &= \lim_{h\downarrow 0} E[(g^i\left(\widehat{v}_{x,t}^i, p_{[h:t-h]}^i, \widehat{V}_{[h:t-h]}^i, t, h\right))^2 | p_{[h:t-h]}^i = x, \widehat{V}_{[h:t-h]}^i = y] \end{aligned} \tag{A.1}$$

Further $B^i(x,y,t)$ *and* $C^i(x,y,t)$ *are twice continuously differentiable in* x *and* y.

Assumption A.3. *For some* $d > 0$ *and for all* $i = 1, \ldots, N$ *then both:*

$$\begin{gathered} E[h^{-1/2}|x_{t+h}^i - x_t^i|^{2+d} | x_t^i = x, V_t^i = y] \\ E[h^{-1/2}|V_{t+h}^i - V_t^i|^{2+d} | x_t^i = x, V_t^i = y] \end{gathered}$$

are bounded as $h \downarrow 0$, *uniformly on every bounded* (x,y,t) *set, and*

$$\limsup_{h\downarrow 0} E[g^i\left(\widehat{v}_{x,t}^i, p_{[h:t-h]}^i, \widehat{V}_{[h:t-h]}^i, t, h\right)^{2+d} | x_t^i = x, V_t^i = y, q_t = q]$$

is bounded uniformly on every bounded (x,y,q,t) *set.*

B Proof of Propositions 2.1 and 2.2

We start with the proof of Proposition 2.1. As stated in equation (2.4) suppose any asset i with (log) price p_t^i which has exposure to (some of) the risk factors, i.e.:

$$dp_t^i = \delta_0^i + \delta^i d\mathcal{X}_t^f \tag{B.1}$$

where $\mathcal{X}_t^f$ is an affine diffusion which satisfies Assumption 2.1. Combining equation equations (2.1) and (B.1) implies that p_t^i satisfies the diffusion:

$$dp_t^i = \mu^i(\mathcal{X}_t^f)dt + \sigma^i(\mathcal{X}_t^f)dW_t \tag{B.2}$$

where by Itô's lemma: $\mu^i(\mathcal{X}_t^f) \equiv \delta^i\mu(\mathcal{X}_t^f)$ and $\sigma^i(\mathcal{X}_t^f) \equiv (\delta^i\sigma(\mathcal{X}_t^f)$.

For each $h_i > 0$ (we allow the sampling scheme to differ across assets) the discrete time Markov process $p_{[h_i:t]}^i$ satisfies for each $\Delta > 0$ and integer l,

- $\mu_{\Delta,h_i}^i(x) = h_i^{-\Delta}E[p_{[h_i:lh_i+1]}^i - p_{[h_i:lh_i]}^i|p_{[h_i:lh_i]}^i = x]$
- $\sigma_{\Delta,h_i}^i(x) = h_i^{-\Delta}Var[p_{[h_i:lh_i+1]}^i - p_{[h_i:lh_i]}^i|p_{[h_i:lh_i]}^i = x]$

where: $\mu_{\Delta,h_i}^i(x) \to \mu^i(x)$, $\sigma_{\Delta,h_i}^i(x) \to \sigma^i(x)$ and for some $d > 0$, $h_i^{-\Delta}E[(p_{[h_i:kh_i+1]}^i - p_{[h_i:kh_i]}^i)^{2=d}|p_{[h_i:kh_i]}^i = x]$ $\to 0$, where $\to$ refers to convergence uniform on every bounded x set.

Then following Stroock and Varadhan (1979) and Nelson & Foster (1994), $p_{[h_i:t]}^i \Rightarrow p_t^i$, where $\Rightarrow$ denotes weak convergence. Moreover, let F^i be the cumulative distribution of the starting point x_0, and $F_{h_i}^i$ be the cumulative distribution of the starting point $p_{[h_i:0]}^i$. If $p_{[h_i:0]}^i$ sets p_0^i with probability one for all h_i, then the weak convergence is uniform on every bounded p_0^i set. The result in equation (2.9), in particular:

$$\widehat{V}_{[t:t+h]}^i = (\sum_{j=1}^{r}(\delta_j^i)^2 h_0^{jj}) + \sum_{j=1}^{r}(\delta_j^i)^2 h_1^{jj}\mathcal{X}_t^j + u_{[t:t+h]}^i$$

then follows from Theorem 3.1 in Nelson & Foster (1994) who establish that as $h \downarrow 0 \ : \ h^{-1/4}u_{[t:t+h]}^i \xrightarrow{d} N(0, C^i(x,y,t)/2B^i(x,y,t))$, uniformly (the so called unbiased large M case) with $B^i(x,y,t)$ and $C^i(x,y,t)$ defined in equation (A.1).

The proof of Proposition 2.2 follows largely along the lines of Bai (2003, Theorem 1). Bai (2003, Assumptions A-B) follows from Assumptions 2.3 and 2.4, implying bounded non-random loadings and finite fourth moments of the factors. Assumptions 2.8 and 2.10 guarantee that the remaining conditions hold such that the asymptotic result follows. □

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Figure 1: The short-run funding spreads volatility factor ($SRFUN\_VF$) and the long-run corporate and government bond spreads volatility factor ($LRBON\_VF$) during 1999m01 - 2016m09.

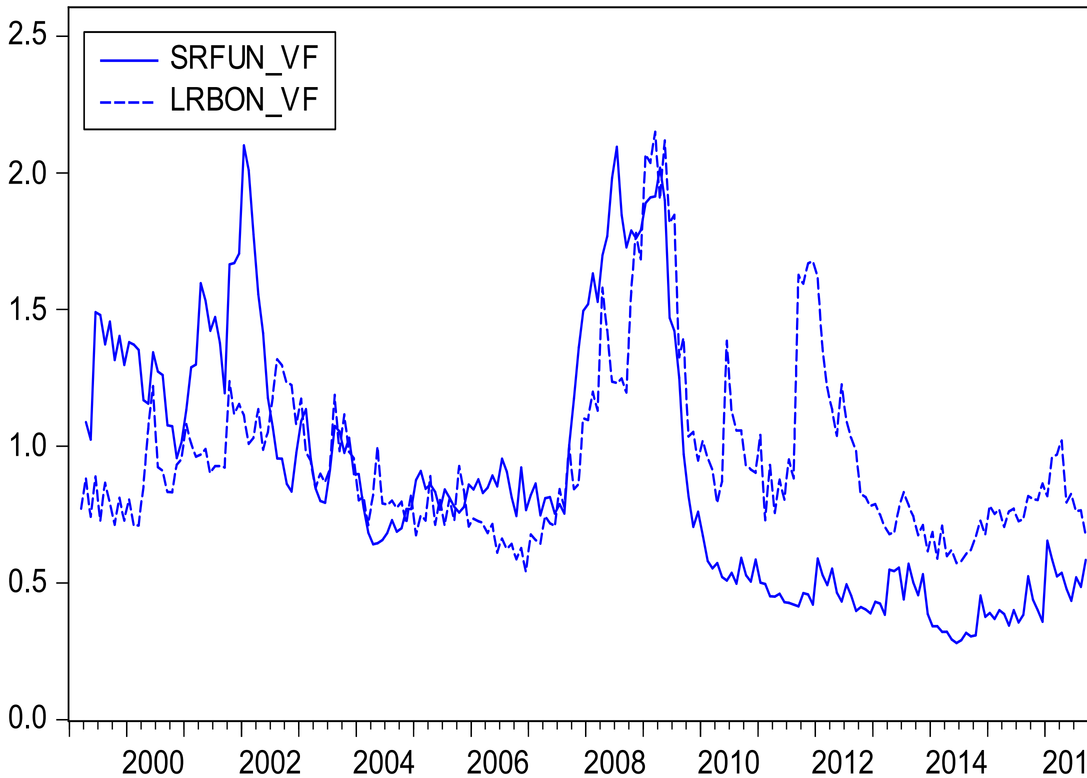

Figure 2: The long-run corporate and government bond spreads factor ($LRBON\_SF$), the Gildchrist and Zakrajšek spread factor (GZ_SPR), the Chicago Fed National Financial Index (NFCI) and the St Louis Fed Financial Stress Index (FSI) during 1999m01 - 2016m09.

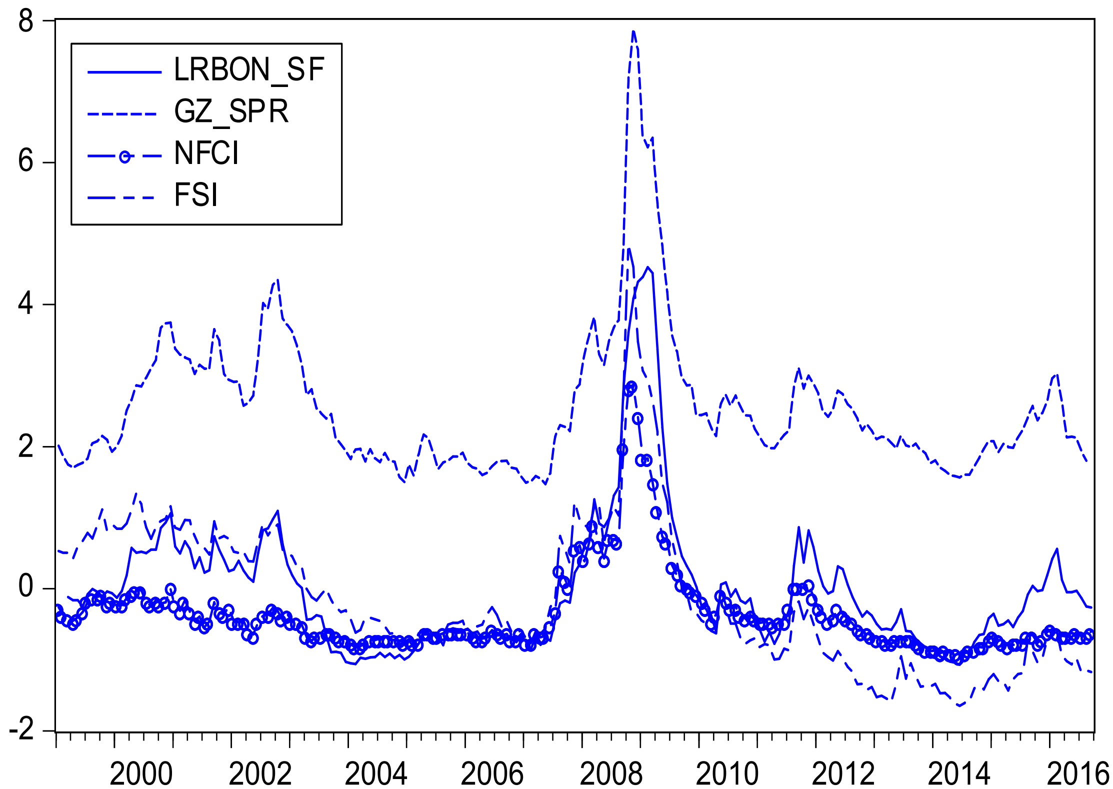

Table 1: Acronyms and names of variables and factors

The list of variables definitions for the short-run funding ($SRFUN$) and long-run corporate and government bond ($LRBON$) spreads panels used to extract the factors are found in the OA, Tables A.1.2 and A.1.3, respectively. This table lists the acronyms and names of factors and some series used in the empirical analysis.

Acronym	Long Name
Baa-Aaa	Moody's Baa- and Aaa-rated long-run industrial bond spread
CDS	First principal component of the credit default swaps extracted from 6 primary dealers listed in Bloomberg
CFNAI_V	Fitted volatility from an AR-EGARCH of Chicago Fed National Activity Index
DLC_V	Fitted volatility from an AR-EGARCH of consumption (Services and Non-durables) growth rate
DLIP_V	Fitted volatility from an AR-EGARCH of industrial production growth rate
FSI	St Louis Fed Financial Stress Index
FSI_V	Estimated volatility from an AR-EGARCH of St Louis Fed Financial Stress Index
GZ_SPR	Gilchrist & Zakrajšek (2012) Spreads factor
GZ_SPR_V	Estimated volatility from an AR-EGARCH of Gilchrist & Zakrajšek (2012) Spreads factor
log(P/D)	log Price-Dividend ratio
log(P/E)	log Price-Earnings ratio
$LRBON\_VF$	Common factor of the panel of AR-EGARCH estimates of the residuals obtained from the regression of each long-run bond spread with $LRBON\_SF$
$LRBON\_VFe$	Common factor of the panel of the fitted values of VIX/RV/VRP/returns when regressed on AR-EGARCH volatilities of residuals obtained from the regression of each long-run bond spread with $LRBON\_SF$
$LRBON\_VFIV$	Common factor of the panel of the fitted values of AR-EGARCH volatilities of the residuals obtained from the regression of each long-run bond spread with $LRBON\_SF$ when regressed on lags of the cross-sectional average of these AR-EGARCH volatilities
$LRBON\_SF$	long-run corporate and government bond spreads Factor
NFCI	Chicago Fed National Financial Conditions Index
NFCI_Credit	NFCI subindex which refers to the Credit series
NFCI_Leverage	NFCI subindex which refers to the Leverage series
NFCI_Risk	NFCI subindex which refers to the Risk series
NFCI_V	Estimated volatility from an AR-EGARCH of Chicago Fed National Financial Conditions Index
PrecMet	S&P GSCI Precious Metals Total Excess Return Index
$SRFUN\_VF$	Common factor of the panel of AR-EGARCH estimates of the residuals obtained from the regression of each short-run funding spread with $SRFUN\_SF$
$SRFUN\_VFe$	Common factor of the panel of the fitted values of VIX/RV/VRP/returns when regressed on AR-EGARCH volatilities of residuals obtained from the regression of each short-run funding spreads with $SRFUN\_SF$
$SRFUN\_VFIV$	Common factor of the panel of the fitted values of AR-EGARCH volatilities of the residuals obtained from the regression of each short-run funding spreads with $SRFUN\_SF$ when regressed on lags of the cross-sectional average of these AR-EGARCH volatilities
$SRFUN\_SF$	Short-run funding spreads factor
TED	3-Month London Interbank Offered Rate minus the 3-Month Treasury Bill
TMSP	Term spread
7DayNonFinCP-FF	7-Day A2/P2/F2 Nonfinancial Commercial Paper-FedFunds rate (FF)
3mFinCP-Tbill	3-Month Financial commercial paper-Treasury bill

Table 2: Predictive regression models for the VIX using the short-run funding ($SRFUN$) spreads Volatility Factors

The estimation results of the linear LS predictive regression models for different lag horizons, H, are reported and (***), (**), (*) refer to rejecting the null hypothesis of insignificant predictors at 1%, 5% and 10% respectively. Standard errors (SE) found in the parentheses refer to the NW HAC estimator with 12 lags. The Lehman Brothers bankruptcy dummy variable enters as constant and excludes the following observations: 2008m09 up to 2008m10; it is significant and unreported in this table for conciseness. The short-run funding spreads volatility factor ($SRFUN\_VF$) is extracted from the panels of the residuals of AR-EGARCH models with three alternative distributions (Normal, GED and SGED), obtained from the regression of each spread with the common spreads factor, $SRFUN\_SF$, reported in Panel A. The corresponding results for the factors for correcting the estimation error (VFe and $VFIV$) are also reported in Panels B and C. The MIDAS regression models are estimated using the corresponding daily volatility factors. The number of daily lags is estimated using either the step function or the exponential Almon lag polynomial and the results are robust. The choice of daily lags is based on the AIC. Reported are the adjR^2 of the monthly LS and MIDAS NLS regression models with the daily factors. The acronyms of the variables are defined in Table 1.

Horizon	$H=3$					$H=6$					$H=9$				
				R^2					R^2					R^2	
Factors	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS
	0.51			0.46		0.28			0.27		0.20			0.24	
	(0.05)***					(0.06)***					(0.07)***				
	0.49	5.54		0.46		0.25	4.96		0.27		0.15	9.99		0.24	
	(0.06)***	(4.51)				(0.07)***	(9.85)				(0.08)*	(10.31)			
VF								**Panel A**							
Norm	0.42		1.34	0.49	0.49	0.14		2.37	0.37	0.48	0.13		2.62	0.37	0.74
	(0.55)***		(0.55)**			(0.09)		(1.09)**			(0.07)		(1.07)**		
	0.41	-2.60	1.64	0.50	0.49	0.10	-11.33	3.41	0.44	0.47	-0.01	-6.56	3.51	0.42	0.73
	(0.05)***	(7.73)	(0.68)**			(0.07)	(14.96)	(1.43)**			(0.08)	(10.96)	(1.31)***		
GED	0.42		1.38	0.49	0.49	0.09		2.89	0.42	0.56	-0.01		3.23	0.42	0.72
	(0.05)***		(0.48)***			(0.09)		(1.08)***			(0.10)		(1.12)***		
	0.42	-0.45	1.39	0.49	0.49	0.11	-7.83	3.07	0.42	0.56	0.00	-3.56	3.31	0.42	0.72
	(0.05)***	(6.93)	(0.57)**			(0.06)*	(14.35)	(1.36)**			(0.08)	(10.61)	(1.30)**		
SGED	0.40		1.56	0.50	0.49	0.06		3.09	0.43	0.57	-0.03		3.33	0.42	0.70
	(0.05)***		(0.54)***			(0.09)		(1.11)***			(0.10)		(1.10)***		
	0.41	-2.60	1.64	0.50	0.49	0.10	-11.33	3.41	0.44	0.56	-0.01	-6.56	3.51	0.42	0.70
	(0.05)***	(7.73)	(0.68)**			(0.07)	(14.96)	(1.43)**			(0.08)	(10.96)	(1.31)***		
VFe								**Panel B**							
Norm	0.50		0.60	0.46	0.32	0.25		1.38	0.31	0.32	0.16		1.49	0.27	0.73
	(0.05)***		(0.43)			(0.06)***		(0.67)**			(0.07)		(0.56)***		
	0.48	3.78	0.52	0.46	0.32	0.24	0.35	1.37	0.30	0.31	0.14	5.10	1.39	0.27	0.73
	(0.06)***	(5.67)	(0.49)			(0.06)***	(10.93)	(0.81)*			(0.08)	(10.47)	(0.62)**		
SGED	0.44		1.16	0.48	0.34	0.09		2.83	0.43	0.36	-0.02		3.28	0.45	0.78
	(0.05)***		(0.41)***			(0.07)		(0.96)***			(0.08)		(1.03)***		
	0.44	-0.66	1.18	0.48	0.34	0.13	-11.22	3.11	0.44	0.36	0.01	-7.99	3.47	0.45	0.77
	(0.05)***	(6.95)	(0.50)**			(0.05)**	(14.37)	(1.22)**			(0.06)	(9.97)	(1.19)***		
GED	0.46		1.03	0.47	0.35	0.11		2.98	0.41	0.24	-0.01		3.81	0.46	0.71
	(0.05)***		(0.55)*			(0.10)		(1.24)**			(0.11)		(1.27)***		
	0.44	3.85	0.99	0.47	0.35	0.11	0.12	2.98	0.40	0.23	-0.03	3.66	3.77	0.46	0.71
	(0.05)***	(5.79)	(0.58)*			(0.07)	(12.58)	(1.34)**			(0.08)	(9.55)	(1.36)***		
$VFIV$								**Panel C**							
Norm	0.39		1.76	0.51	0.53	0.11		2.49	0.38	0.47	0.05		2.26	0.32	0.59
	(0.05)***		(0.50)***			(0.08)		(0.86)***			(0.08)		(0.81)***		
	0.40	-0.71	1.78	0.51	0.53	0.12	-3.92	2.56	0.38	0.48	0.03	3.04	2.21	0.32	0.59
	(0.05)***	(5.35)	(0.55)***			(0.06)*	(12.09)	(1.04)**			(0.07)	(11.90)	(0.95)**		
GED	0.41		1.58	0.50	0.51	0.08		3.02	0.43	0.47	0.01		2.96	0.39	0.57
	(0.05)***		(0.57)***			(0.08)		(1.12)***			(0.09)		(1.07)***		
	0.41	-1.33	1.62	0.50	0.51	0.11	-8.46	3.22	0.44	0.47	0.01	-2.11	3.01	0.39	0.57
	(0.05)***	(7.37)	(0.68)**			(0.06)*	(14.28)	(1.39)**			(0.07)	(11.00)	(1.26)**		
SGED	0.40		1.45	0.50	0.51	0.06		2.73	0.44	0.48	-0.02		2.75	0.41	0.58
	(0.05)***		(0.49)***			(0.08)		(0.97)***			(0.09)		(1.00)***		
	0.41	-2.73	1.51	0.50	0.51	0.10	-11.09	2.99	0.45	0.47	0.00	-4.88	2.86	0.41	0.57
	(0.05)***	(7.37)	(0.59)**			(0.06)	(14.25)	(1.20)**			(0.07)	(11.59)	(1.17)**		

Table 3: Predictive regression models for the RV using the short-run funding ($SRFUN$) spreads Volatility Factors

The estimation results of the linear LS predictive regression models for different lag horizons, H, are reported and (***), (**), (*) refer to rejecting the null hypothesis of insignificant predictors at 1%, 5% and 10% respectively. Standard errors (SE) found in the parentheses refer to the NW HAC estimator with 12 lags. The Lehman Brothers bankruptcy dummy variable enters as constant and excludes the following observations: 2008m09 up to 2008m10; it is significant and unreported in this table for conciseness. The short-run funding spreads volatility factor ($SRFUN\_VF$) is extracted from the panels of the residuals of AR-EGARCH models with three alternative distributions (Normal, GED and SGED), obtained from the regression of each spread with the common spreads factor, $SRFUN\_SF$, reported in Panel A. The corresponding results for the factors for correcting the estimation error (VFe and $VFIV$) are also reported in Panels B and C. The MIDAS regression models are estimated using the corresponding daily volatility factors. The number of daily lags is estimated using either the step function or the exponential Almon lag polynomial and the results are robust. The choice of daily lags is based on the AIC. Reported are the adjR^2 of the monthly LS and MIDAS NLS regression models with the daily factors. The acronyms of the variables are defined in Table 1.

Horizon	$H=3$						$H=6$						$H=9$					
					R^2						R^2						R^2	
Factors	RV(-H)	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	RV(-H)	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	RV(-H)	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS
	0.18	0.11			0.51		0.71	0.59			0.44		-0.5	0.15			0.44	
	(0.07)***	(0.09)					(0.46)	(0.82)					(0.85)	(0.12)				
	0.21	0.05	6.95		0.51		0.74	0.54	0.59		0.44		-0.51	0.16	-0.24		0.44	
	(0.07)***	(0.09)	(4.24)				(0.52)	(0.16)	(8.48)				(0.78)	(0.13)	(8.86)			
VF									**Panel A**									
Norm	0.21	0.00		1.27	0.52	0.51	0.07	-0.04		1.87	0.49	0.62	-0.06	0.12		1.95	0.49	0.82
	(0.06)***	(0.07)		(0.72)*			(0.05)	(0.09)		(1.01)*			(0.08)	(0.10)		(0.95)**		
	0.22	-0.03	-0.54	1.51	0.53	0.51	0.08	-0.08	-13.15	2.76	0.52	0.62	-0.04	0.03	-13.03	2.6	0.50	0.82
	(0.06)***	(0.08)	(6.88)	(0.82)*			(0.06)	(0.09)	(11.92)	(1.29)**			(0.08)	(0.12)	(8.80)	(1.05)**		
GED	0.22	-0.01		1.33	0.52	0.50	0.14	-0.16		2.32	0.51	0.64	0.02	-0.07		2.31	0.50	0.81
	(0.06)***	(0.07)		(0.64)**			(0.07)**	(0.14)		(1.02)**			(0.10)	(0.17)		(0.97)**		
	0.23	-0.02	1.50	1.29	0.52	0.50	0.1	-0.09	-10.16	2.53	0.51	0.64	-0.02	0.01	-10.77	2.52	0.51	0.81
	(0.06)***	(0.08)	(6.16)	(0.69)*			(0.06)*	(0.09)	(11.08)	(1.19)**			(0.08)	(0.12)	(8.35)	(1.05)**		
SGED	0.22	-0.03		1.49	0.53	0.50	0.13	-0.17		2.42	0.51	0.65	0.01	-0.06		2.27	0.50	0.79
	(0.06)***	(0.08)		(0.74)**			(0.06)**	(0.14)		(1.08)**			(0.10)	(0.17)		(0.96)**		
	0.22	-0.03	-0.54	1.51	0.53	0.50	0.08	-0.08	-13.15	2.76	0.52	0.65	-0.04	0.03	-13.03	2.60	0.50	0.80
	(0.06)***	(0.08)	(6.88)	(0.82)*			(0.06)	(0.09)	(11.92)	(1.29)**			(0.08)	(0.12)	(8.80)	(1.05)**		
VFe									**Panel B**									
Norm	0.18	0.11		0.13	0.50	0.36	0.07	0.06		0.12	0.44	0.48	-0.05	0.16		0.4	0.44	0.92
	(0.07)**	(0.10)		(0.48)			(0.05)	(0.08)		(0.28)			(0.08)	(0.12)		(0.43)		
	0.21	0.05	6.98	0.04	0.50	0.36	0.07	0.06	-0.01	0.12	0.44	0.48	-0.06	0.17	-1.38	0.42	0.44	0.92
	(0.07)***	(0.10)	(4.62)	(0.51)			(0.06)	(0.11)	(8.94)	(0.30)			(0.08)	(0.11)	(9.15)	(0.43)		
GED	0.22	0.03		0.9	0.51	0.34	0.15	-0.12		2.05	0.49	0.40	0.05	-0.06		2.39	0.50	0.91
	(0.06)***	(0.08)		(0.39)**			(0.06)***	(0.10)		(0.95)**			(0.11)	(0.15)		(1.07)**		
	0.23	0.01	3.87	0.82	0.51	0.34	0.12	-0.06	-8.07	2.22	0.49	0.40	0.01	0.02	-10.18	2.6	0.50	0.91
	(0.06)***	(0.08)	(5.33)	(0.41)**			(0.05)**	(0.08)	(10.07)	(1.11)**			(0.09)	(0.11)	(7.90)	(1.17)**		
SGED	0.22	0.01		0.94	0.52	0.37	0.15	-0.16		1.89	0.51	0.55	0.04	-0.07		1.91	0.50	0.86
	(0.06)***	(0.07)		(0.43)**			(0.06)**	(0.12)		(0.81)**			(0.10)	(0.16)		(0.79)**		
	0.23	.000	1.23	0.91	0.52	0.37	0.10	-0.07	-13.26	2.17	0.52	0.55	-0.01	0.03	-13.96	2.20	0.51	0.86
	(0.06)***	(0.08)	(6.32)	(0.47)*			(0.05)**	(0.08)	(11.26)	(0.96)**			(0.08)	(0.11)	(7.88)*	(0.85)***		
$VFIV$									**Panel C**									
Norm	0.20	0.01		1.29	0.52	0.56	0.10	-0.09		1.80	0.48	0.57	-0.04	0.05		1.35	0.46	0.73
	(0.06)***	(0.08)		(0.63)**			(0.05)*	(0.11)		(0.87)**			(0.09)	(0.14)		(0.74)*		
	0.21	-0.01	2.23	1.25	0.52	0.56	0.07	-0.03	-6.83	1.93	0.48	0.58	-0.06	0.1	-5.72	1.45	0.46	0.73
	(0.07)***	(0.09)	(5.02)	(0.66)*			(0.06)	(0.09)	(10.89)	(1.00)*			(0.07)	(0.10)	(10.63)	(0.85)*		
GED	0.22	-0.03		1.61	0.53	0.55	0.13	-0.15		2.32	0.51	0.57	0.00	-0.02		1.88	0.48	0.67
	(0.05)***	(0.08)		(0.79)**			(0.06)**	(0.13)		(1.02)**			(0.10)	(0.16)		(0.90)**		
	0.22	-0.03	-0.22	1.61	0.53	0.55	0.09	-0.07	-10.58	2.53	0.51	0.58	-0.04	0.05	-9.13	2.06	0.48	0.67
	(0.06)***	(0.08)	(6.77)	(0.84)*			(0.06)	(0.09)	(11.05)	(1.17)**			(0.08)	(0.11)	(9.09)	(0.99)**		
SGED	0.23	-0.04		1.44	0.54	0.55	0.14	-0.17		2.06	0.51	0.58	0.01	-0.04		1.75	0.49	0.68
	(0.05)***	(0.08)		(0.68)**			(0.07)**	(0.14)		(0.91)**			(0.10)	(0.18)		(0.92)*		
	0.22	-0.03	-1.32	1.47	0.53	0.55	0.09	-0.08	-12.3	2.32	0.52	0.58	-0.04	0.04	-11.10	1.98	0.49	0.68
	(0.06)***	(0.08)	(6.79)	(0.74)**			(0.06)	(0.09)	(11.18)	(1.05)**			(0.08)	(0.12)	(10.04)	(1.01)*		

Table 4: Predictive regression models for the VRP using the short-run funding ($SRFUN$) spreads Volatility Factors

The estimation results of the linear LS predictive regression models for different lag horizons, H, are reported and (***), (**), (*) refer to rejecting the null hypothesis of insignificant predictors at 1%, 5% and 10% respectively. Standard errors (SE) found in the parentheses refer to the NW HAC estimator with 12 lags. The Lehman Brothers bankruptcy dummy variable enters as constant and excludes the following observations: 2008m09 up to 2008m10; it is significant and unreported in this table for conciseness. The short-run funding spreads volatility factor ($SRFUN\_VF$) is extracted from the panels of the residuals of AR-EGARCH models with three alternative distributions (Normal, GED and SGED), obtained from the regression of each spread with the common spreads factor, $SRFUN\_SF$, reported in Panel A. The corresponding results for the factors for correcting the estimation error (VFe and $VFIV$) are also reported in Panels B and C. The MIDAS regression models are estimated using the corresponding daily volatility factors. The number of daily lags is estimated using either the step function or the exponential Almon lag polynomial and the results are robust. The choice of daily lags is based on the AIC. Reported are the adjR^2 of the monthly LS and MIDAS NLS regression models with the daily factors. The acronyms of the variables are defined in Table 1.

Horizon	$H=3$					$H=6$					$H=9$				
				R^2					R^2					R^2	
Factors	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS	VIX(-H)	DLC_V(-H)	Factor(-H)	LS	MIDAS
	0.21			0.43		0.14			0.36		0.09			0.33	
	(0.02)***					(0.03)***					(0.03)***				
		10.12		0.34			10.28		0.34			11.39		0.35	
		(2.14)***					(2.56)***					(1.90)***			
	0.20	2.14		0.42		0.12	5.59		0.37		0.05	9.41		0.35	
	(0.02)***	(2.39)				(0.03)***	(2.90)*				(0.03)*	(2.42)***			
VF								**Panel A**							
Norm(-H)	0.20		0.16	0.42	0.41	0.12		0.50	0.37	0.44	0.08		0.68	0.35	0.66
	(0.03)***		(0.27)			(0.04)***		(0.26)*			(0.03)***		(0.26)***		
	0.19	1.76	0.15	0.42	0.43	0.08	3.23	0.65	0.38	0.44	0.00	5.78	0.90	0.38	0.66
	(0.02)***	(2.80)	(0.28)			(0.04)**	(4.13)	(0.30)**			(0.03)	(3.01)*	(0.33)***		
GED(-H)	0.20		0.19	0.42	0.42	0.10		0.66	0.38	0.48	0.03		0.93	0.37	0.68
	(0.03)***		(0.23)			(0.04)**		(0.23)***			(0.03)		(0.28)***		
	0.19	1.89	0.15	0.42	0.44	0.09	3.99	0.57	0.38	0.49	0.01	6.84	0.78	0.38	0.69
	(0.02)***	(2.71)	(0.26)			(0.04)**	(3.88)	(0.26)**			(0.03)	(2.92)**	(0.32)**		
SGED(-H)	0.20		0.21	0.42	0.43	0.09		0.75	0.38	0.50	0.02		1.06	0.38	0.67
	(0.03)***		(0.25)			(0.04)**		(0.25)***			(0.03)		(0.27)***		
	0.19	1.76	0.15	0.42	0.44	0.08	3.23	0.65	0.38	0.52	0.00	5.78	0.90	0.38	0.67
	(0.02)***	(2.80)	(0.28)			(0.04)**	(4.13)	(0.30)**			(0.03)	(3.01)*	(0.33)***		
VFe								**Panel B**							
Norm(-H)	0.21		-0.10	0.42	0.31	0.14		0.36	0.36	0.66	0.09		0.46	0.34	0.80
	(0.02)***		(0.25)			(0.03)***		(0.24)			(0.03)***		(0.36)		
	0.20	2.57	-0.11	0.42	0.32	0.11	6.00	0.34	0.37	0.69	0.05	9.56	0.43	0.36	0.81
	(0.02)***	(2.43)	(0.25)			(0.03)***	(3.06)*	(0.27)			(0.03)*	(2.12)***	(0.40)		
SGED(-H)	0.19		0.47	0.43	0.32	0.11		0.87	0.37	0.65	0.05		1.13	0.35	0.80
	(0.02)***		(0.29)			(0.03)***		(0.33)***			(0.03)		(0.50)**		
	0.18	1.90	0.44	0.42	0.34	0.09	5.37	0.78	0.38	0.67	0.01	8.79	0.98	0.37	0.81
	(0.02)***	(2.43)	(0.30)			(0.03)***	(3.39)	(0.31)**			(0.03)	(2.41)***	(0.45)**		
GED(-H)	0.21		0.00	0.42	0.33	0.10		0.67	0.38	0.65	0.03		0.88	0.37	0.81
	(0.03)***		(0.24)			(0.04)**		(0.22)***			(0.03)		(0.26)***		
	0.20	2.87	-0.07	0.42	0.35	0.09	3.11	0.59	0.38	0.67	0.01	5.98	0.74	0.38	0.81
	(0.03)***	(2.69)	(0.25)			(0.04)**	(4.02)	(0.28)**			(0.03)	(2.77)**	(0.33)**		
$VFIV$								**Panel C**							
Norm(-H)	0.17		0.55	0.44	0.47	0.09		0.72	0.38	0.50	0.03		0.90	0.37	0.61
	(0.03)***		(0.21)**			(0.03)***		(0.22)***			(0.03)		(0.24)***		
	0.17	0.91	0.53	0.43	0.48	0.08	4.16	0.63	0.38	0.51	0.00	7.70	0.76	0.38	0.61
	(0.02)***	(2.21)	(0.22)**			(0.03)***	(3.22)	(0.25)**			(0.03)	(2.22)***	(0.26)***		
GED(-H)	0.20		0.11	0.42	0.47	0.09		0.78	0.39	0.52	0.02		1.08	0.39	0.59
	(0.03)***		(0.25)			(0.04)**		(0.26)***			(0.03)		(0.33)***		
	0.19	2.86	0.04	0.42	0.47	0.08	3.67	0.70	0.39	0.53	0.00	6.40	0.94	0.39	0.59
	(0.02)***	(2.45)	(0.26)			(0.03)**	(4.32)	(0.31)**			(0.03)	(3.16)**	(0.37)**		
SGED(-H)	0.20		0.13	0.42	0.47	0.08		0.74	0.39	0.52	0.01		1.00	0.39	0.58
	(0.03)***		(0.23)			(0.04)**		(0.21)***			(0.03)		(0.24)***		
	0.19	2.49	0.07	0.42	0.47	0.08	2.72	0.68	0.39	0.53	0.00	5.58	0.87	0.40	0.58
	(0.03)***	(2.47)	(0.24)			(0.03)**	(4.28)	(0.27)**			(0.03)	(2.71)**	(0.28)***		

Table 5: Out of Sample Recursive Forecasting relative MSE results for the VIX and S&P500 RV using alternative volatility type factors

The models of the VIX and RV are estimated using the predictors reported in each row for two periods (i) for the estimation period up to 2006m12, the OOS is 2007m01 - 2016m09 and (ii) for the estimation period up to 2008m12, the OOS is 2009m01 - 2016m09 for $H = 3, 6, 9$ months ahead using the recursive method. The notation FACTOR refers to the alternative Volatility type Factors (VF, VFe and $VFIV$) used as predictors in each column (one at a time), using the panels of AR-EGARCH fitted values with alternative error distributions (Normal, GED, SGED). The forecasts of the models are obtained using the recursive method and are evaluated using the relative Mean Squared Error (MSE) which is calculated as MSE = MSE(competing)/MSE(benchmark), where the benchmark model does not include the factor. The (*) denotes the rejection of the null hypothesis of equal predictive ability of the benchmark for either the VIX or RV and competing models with the additional factors, at the 5% significance level, using the MSE-F test statistic based on the McCracken (2007) critical values.

	OOS period: 2007-2016			OOS period: 2009-2016			OOS period: 2007-2016			OOS period: 2009-2016		
	H=3	H=6	H=9	H=3	H=6	H=9	H=3	H=6	H=9	H=3	H=6	H=9
Panel A: Regression models for VIX	$SRFUN_VF$_Norm						$LRBON_VF$_Norm					
VIX(-H),FACTOR(-H)	**0.91***	**0.82***	**0.83***	**0.94***	**0.82***	**0.80***	**0.95***	1.02	**0.97***	**0.99***	1.08	**0.97***
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.92***	**0.82***	**0.82***	**0.96***	**0.82***	**0.80***	**0.95***	1.01	**0.98***	0.99	1.06	**0.98***
	$SRFUN_VF$_GED						$LRBON_VF$_GED					
VIX(-H),FACTOR(-H)	**0.93***	**0.83***	**0.77***	**0.90***	**0.80***	**0.72***	**0.98***	**0.95***	**0.95***	1.18	1.37	1.04
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.93***	**0.83***	**0.77***	**0.91***	**0.80***	**0.72***	**0.97***	**0.93***	**0.96***	1.14	1.33	1.04
	$SRFUN_VF$_SGED						$LRBON_VF$_SGED					
VIX(-H),FACTOR(-H)	**0.93***	**0.82***	**0.77***	**0.93***	**0.80***	**0.72***	1.00	1.02	**0.99***	1.16	1.42	1.06
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.92***	**0.80***	**0.75***	**0.93***	**0.77***	**0.70***	0.99	1.00	0.99	1.14	1.37	1.07
	$SRFUN_VFe$_Norm						$LRBON_VFe$_Norm					
VIX(-H),FACTOR(-H)	**0.99***	**0.88***	**0.86***	1.01	**0.83***	**0.80***	**0.95***	1.02	**0.97***	**0.99***	1.08	**0.97***
VIX(-H),DLC_V(-H),FACTOR(-H)	1.00	**0.85***	**0.85***	1.04	**0.79***	**0.80***	**0.95***	1.01	**0.98***	0.99	1.06	**0.98***
	$SRFUN_VFe$_GED						$LRBON_VFe$_GED					
VIX(-H),FACTOR(-H)	**0.95***	**0.87***	**0.79***	**0.94***	**0.85***	**0.79***	**0.98***	**0.95***	**0.95***	1.18	1.37	1.04
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.95***	**0.87***	**0.80***	**0.94***	**0.86***	**0.79***	**0.97***	**0.93***	**0.96***	1.14	1.33	1.04
	$SRFUN_VFe$_SGED						$LRBON_VFe$_SGED					
VIX(-H),FACTOR(-H)	**0.96***	**0.83***	**0.71***	**0.97***	**0.78***	**0.63***	1.00	1.02	**0.99***	1.16	1.42	1.06
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.97***	**0.80***	**0.69***	0.99	**0.75***	**0.62***	0.99	1.00	0.99	1.14	1.37	1.07
Panel B: Regression models for RV	$SRFUN_VF$_Norm						$LRBON_VF$_Norm					
RV(-H),VIX(-H),FACTOR(-H)	**0.89***	**0.87***	**0.89***	**0.85***	**0.79***	**0.86***	**0.83***	**0.96***	**0.81***	**0.65***	1.08	**0.68***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.89***	**0.86***	**0.87***	**0.86***	**0.78***	**0.84***	**0.83***	1.01	**0.80***	**0.65***	1.22	**0.66***
	$SRFUN_VF$_GED						$LRBON_VF$_GED					
RV(-H),VIX(-H),FACTOR(-H)	**0.92***	**0.85***	**0.84***	**0.83***	**0.73***	**0.79***	**0.84***	1.03	**0.76***	**0.66***	1.26	**0.60***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.91***	**0.84***	**0.83***	**0.83***	**0.71***	**0.77***	**0.83***	1.07	**0.75***	**0.65***	1.36	**0.58***
	$SRFUN_VF$_SGED						$LRBON_VF$_SGED					
RV(-H),VIX(-H),FACTOR(-H)	**0.90***	**0.86***	**0.85***	**0.84***	**0.75***	**0.81***	**0.85***	1.01	**0.77***	**0.69***	1.19	**0.61***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.89***	**0.83***	**0.83***	**0.82***	**0.69***	**0.77***	**0.84***	1.05	**0.76***	**0.67***	1.27	**0.60***
	$SRFUN_VFe$_Norm						$LRBON_VFe$_Norm					
RV(-H),VIX(-H),FACTOR(-H)	1.00	**0.89***	**0.91***	1.10	**0.82***	**0.90***	**0.82***	**0.85***	**0.90***	**0.82***	**0.85***	**0.90***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	0.99	**0.88***	**0.90***	1.07	**0.79***	**0.88***	**0.82***	**0.85***	**0.90***	**0.82***	**0.85***	**0.90***
	$SRFUN_VFe$_GED						$LRBON_VFe$_GED					
RV(-H),VIX(-H),FACTOR(-H)	**0.95***	**0.88***	**0.95***	**0.90***	**0.82***	**0.97***	**0.81***	1.01	**0.74***	**0.62***	1.22	**0.56***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.95***	**0.87***	**0.94***	**0.91***	**0.81***	**0.96***	**0.81***	1.05	**0.73***	**0.62***	1.33	**0.54***
	$SRFUN_VFe$_SGED						$LRBON_VFe$_SGED					
RV(-H),VIX(-H),FACTOR(-H)	1.03	**0.93***	**0.95***	1.10	**0.89***	**0.97***	**0.83***	**0.97***	**0.75***	**0.65***	1.10	**0.58***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	1.04	**0.91***	**0.94***	1.11	**0.86***	**0.95***	**0.82***	1.00	**0.74***	**0.64***	1.19	**0.57***

Table 6: Out of Sample Forecasting relative MSE results for the VIX and S&P500 RV using daily vs monthly Volatility Factors in MIDAS vs LS predictive models

The models of the VIX and RV are estimated using the predictors reported in each row for the estimation period up to 2008m12 (and OOS period 2009m01 - 2016m09). For each predictive regression model the predictor used in each column (one at a time), refers to the Volatility type Factors (VF, VFe and $VFIV$) using the AR-EGARCH model (with the Normal, GED, SGED distributions). The forecasts of the models are evaluated using the Mean Squared Error (MSE) from the MIDAS model with the daily factors vis-à-vis the LS model with the corresponding monthly factors, which is calculated as the relative MSE = MSE(MIDAS)/MSE(LS). The MIDAS models are estimated using the exponential Almon lag polynomial. The (*) denotes the rejection of the null hypothesis of equal predictive ability of the benchmark and competing models at the 5% significance level using the MSE-F test statistic based on the McCracken (2007) critical values. Results are reported for both Fixed and Recursive OOS methods.

	Recursive OOS			Fixed OOS		
	MSE(MIDAS)/MSE(LS)			MSE(MIDAS)/MSE(LS)		
	H=3	H=6	H=9	H=3	H=6	H=9
Panel A: Regression models for VIX	$SRFUN_VF$_Norm			$SRFUN_VF$_Norm		
VIX(-H),FACTOR(-H)	**0.87***	**0.77***	**0.81***	**0.90***	**0.80***	**0.80***
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.85***	**0.76***	**0.85***	**0.88***	**0.80***	**0.89***
	$SRFUN_VF$_GED			$SRFUN_VF$_GED		
VIX(-H),FACTOR(-H)	**0.98***	**0.92***	1.04	**0.89***	**0.79***	**0.81***
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.90***	**0.91***	1.05	**0.89***	**0.79***	**0.88***
	$SRFUN_VF$_SGED			$SRFUN_VF$_SGED		
VIX(-H),FACTOR(-H)	**0.99***	**0.92***	0.99	**0.84***	**0.90***	**0.86***
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.92***	**0.93***	1.06	**0.89***	**0.90***	**0.88***
	$SRFUN_VFe$_Norm			$SRFUN_VFe$_Norm		
VIX(-H),FACTOR(-H)	**0.95***	**0.92***	**0.91***	**0.98***	1.01	1.07
VIX(-H),DLC_V(-H),FACTOR(-H)	**0.99***	**0.92***	**0.91***	1.08	1.01	1.08
	$SRFUN_VFe$_GED			$SRFUN_VFe$_GED		
VIX(-H),FACTOR(-H)	1.05	1.02	**0.97***	**0.97***	**0.97***	1.12
VIX(-H),DLC_V(-H),FACTOR(-H)	1.08	**0.99***	**0.84***	1.05	1.00	**0.97***
	$SRFUN_VFe$_SGED			$SRFUN_VFe$_SGED		
VIX(-H),FACTOR(-H)	1.13	**0.97***	**0.99***	1.29	1.00	1.04
VIX(-H),DLC_V(-H),FACTOR(-H)	1.16	**0.98***	**0.97***	1.35	1.09	1.05
Panel B: Regression models for RV	$SRFUN_VF$_Norm			$SRFUN_VF$_Norm		
RV(-H),VIX(-H),FACTOR(-H)	**0.74***	**0.61***	**0.61***	**0.93***	**0.74***	**0.85***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.75***	**0.62***	**0.66***	**0.96***	**0.75***	**0.99***
	$SRFUN_VF$_GED			$SRFUN_VF$_GED		
RV(-H),VIX(-H),FACTOR(-H)	1.06	**0.83***	**0.55***	**0.85***	**0.72***	**0.99***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.96***	**0.84***	**0.60***	**0.94***	**0.72***	1.10
	$SRFUN_VF$_SGED			$SRFUN_VF$_SGED		
RV(-H),VIX(-H),FACTOR(-H)	1.09	**0.84***	**0.59***	**0.75***	**0.90***	2.35
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	1.01	**0.87***	**0.64***	**0.93***	**0.89***	2.43
	$SRFUN_VFe$_Norm			$SRFUN_VFe$_Norm		
RV(-H),VIX(-H),FACTOR(-H)	**0.71***	**0.73***	**0.88***	**0.87***	**0.81***	1.41
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.78***	**0.72***	**0.91***	1.05	**0.86***	1.47
	$SRFUN_VFe$_GED			$SRFUN_VFe$_GED		
RV(-H),VIX(-H),FACTOR(-H)	**0.82***	**0.76***	**0.72***	1.01	**0.88***	**0.98***
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.85***	**0.75***	**0.75***	1.14	**0.90***	1.04
	$SRFUN_VFe$_SGED			$SRFUN_VFe$_SGED		
RV(-H),VIX(-H),FACTOR(-H)	**0.79***	**0.73***	**0.82***	**0.97***	**0.85***	1.22
RV(-H), VIX(-H),DLC_V(-H),FACTOR(-H)	**0.83***	**0.72***	**0.86***	1.10	**0.86***	1.30

Table 7: In sample results for the S&P500 excess returns predictability models using monthly and daily factors

The LS estimation results of the linear regression model are reported and (***), (**), (*) refer to rejecting the null hypothesis of insignificant predictors at 1%, 5% and 10% respectively. The variables are defined in Table 1. The predictive models refer to $H = 1$ and 3 for the monthly S&P500 excess returns during 1999m01 - 2016m09. Standard errors (SE) found in the parentheses refer to the NW HAC estimator with 12 lags. The Lehman Brothers bankruptcy dummy variable enters as constant and excludes the following observations: 2008m09 up to 2008m10. Reported are also the adjR^2 from the LS model with monthly factors as well as the adjR^2 from the MIDAS model which specifies daily instead of monthly factors. All the estimated models include a constant which is not reported in the tables below for conciseness. The reported results for the Volatility Factors refer to VF and VFe type factors. The results for $VFIV$ are relatively weaker and therefore unreported.

$H = 1$ month

		$SRFUN$				$LRBON$							
		VF		VFe		VF		VFe		$SRFUN\_SF$	$LRBON\_SF$	GZ_SPR	GZ_SPR_V
		SGED	GED	SGED	GED	SGED	GED	SGED	GED				
VRP(-H)	0.45	0.49	0.49	0.48	0.51	0.50	0.50	0.50	0.50	0.44	0.46	0.45	0.46
	(0.13)***	(0.16)***	(0.15)***	(0.14)***	(0.15)***	(0.14)***	(0.14)***	(0.14)***	(0.14)***	(0.14)***	(0.15)***	(0.15)***	(0.14)***
FACTOR(-H)		-1.59	-1.40	-3.92	-2.81	-1.03	-0.96	-4.19	-2.45	-0.48	-0.45	-4.64	-8.29
		(0.56)***	(0.45)***	(1.63)**	(1.02)***	(0.86)	-0.73	(2.93)	(1.61)	(0.23)**	(0.22)**	(2.66)*	(3.28)**
R^2 LS	0.11	0.13	0.13	0.12	0.12	0.11	0.11	0.12	0.12	0.12	0.12	0.11	0.11
R^2 MIDAS		0.17	0.17	0.17	0.17	0.12	0.12	0.18	0.14	0.15	0.14		
VRP(-H)	0.45	0.50	0.50	0.48	0.51	0.54	0.55	0.53	0.55	0.43	0.48	0.46	0.45
	(0.13)***	(0.15)***	(0.15)***	(0.14)***	(0.15)***	(0.16)***	(0.16)***	(0.14)***	(0.15)***	(0.14)***	(0.15)***	(0.16)***	(0.13)***
Baa-Aaa(-H)	-0.24	0.59	0.42	0.14	0.17	0.82	1.04	0.80	1.11	1.01	2.38	1.76	0.43
	(0.55)	-0.57	(0.55)	(0.54)	-0.54	(0.94)	-0.99	(0.83)	(0.90)	(0.80)	(0.89)***	(1.06)*	(0.71)
FACTOR(-H)		-1.84	-1.56	-4.04	-2.92	-1.80	-1.83	-6.76	-4.66	-0.81	-1.38	-12.25	-11.57
		(0.58)***	(0.49)***	(1.70)**	(1.02)***	(1.33)	-1.18	(4.24)	(2.54)*	(0.45)*	(0.36)***	(5.02)**	(6.25)*
R^2 LS	0.11	0.13	0.12	0.12	0.12	0.11	0.11	0.11	0.12	0.12	0.13	0.12	0.11
R^2 MIDAS		0.18	0.18	0.18	0.18	0.12	0.12	0.17	0.14	0.16	0.15		
VRP(-H)	0.46	0.50	0.50	0.49	0.51	0.50	0.50	0.50	0.50	0.43	0.46	0.45	0.46
	(0.13)***	(0.16)***	(0.16)***	(0.14)***	(0.15)***	(0.14)***	(0.14)***	(0.14)***	(0.14)***	(0.15)***	(0.15)***	(0.15)***	(0.14)***
TMSP(-H)	-2.23	-4.67	-5.46	-3.29	-1.97	3.56	3.90	-0.26	2.38	4.25	0.57	5.43	-1.39
	(23.88)	-22.44	(22.54)	(21.62)	-23.06	(24.52)	-24.38	(22.55)	(23.87)	(24.14)	(22.89)	(23.40)	(24.32)
FACTOR(-H)		-1.60	-1.41	-3.93	-2.81	-1.07	-0.99	-4.19	-2.48	-0.49	-0.45	-4.84	-8.23
		(0.57)***	(0.45)***	(1.64)**	(1.03)***	(0.92)	-0.79	(3.06)	(1.73)	(0.24)**	(0.23)*	(2.93)*	(3.38)**
R^2 LS	0.11	0.13	0.12	0.12	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
R^2 MIDAS		0.17	0.17	0.17	0.17	0.12	0.12	0.18	0.14	0.14	0.13		
VRP(-H)	0.50	0.52	0.52	0.51	0.52	0.58	0.58	0.56	0.57	0.47	0.52	0.51	0.49
	(0.14)***	(0.14)***	(0.14)***	(0.14)***	(0.13)***	(0.14)***	(0.15)***	(0.14)***	(0.14)***	(0.15)***	(0.16)***	(0.17)***	(0.15)***
log(P/D)(-H)	-3.51	-3.18	-3.22	-3.18	-3.17	-4.20	-4.31	-4.04	-4.21	-6.03	-5.34	-5.03	-4.55
	(1.68)**	(1.25)**	(1.38)**	(1.49)**	(1.54)**	(1.33)***	(1.31)***	(1.33)***	(1.27)***	(1.41)***	(1.42)***	(1.19)***	(1.44)***
FACTOR(-H)		-1.34	-1.13	-2.73	-1.90	-1.71	-1.60	-6.06	-3.77	-1.16	-0.96	-9.77	-17.94
		(0.51)***	(0.56)**	(1.71)	(1.07)*	(0.73)**	(0.62)**	(2.80)**	(1.50)**	(0.34)***	(0.22)***	(1.89)***	(5.18)***
R^2 LS	0.14	0.15	0.15	0.14	0.14	0.15	0.15	0.15	0.15	0.18	0.17	0.16	0.15
R^2 MIDAS		0.21	0.21	0.21	0.21	0.17	0.17	0.18	0.18	0.20	0.19		
VRP(-H)	-1.37	-0.26	-0.60	-0.84	-0.81	-1.38	-1.29	-1.18	-1.19	-1.09	-1.12	-1.25	-1.22
	(1.14)	-1.40	(1.43)	(1.25)	-1.45	(1.31)	-1.32	(1.33)	(1.40)	(1.37)	(1.45)	(1.34)	(1.25)
log(P/E)(-H)	0.53	0.50	0.51	0.52	0.53	0.53	0.53	0.53	0.53	0.50	0.52	0.52	0.53
	(0.15)***	(0.18)***	(0.17)***	(0.16)***	(0.16)***	(0.15)***	(0.15)***	(0.16)***	(0.16)***	(0.18)***	(0.18)***	(0.17)***	(0.17)***
FACTOR(-H)		-1.42	-1.05	-2.43	-1.69	0.03	-0.11	-1.25	-0.62	-0.31	-0.16	-0.97	-4.26
		(0.81)*	(0.73)	(1.92)	-1.55	(0.99)	-0.83	(3.43)	(2.02)	(0.32)	(0.30)	(2.69)	(5.01)
R^2 LS	0.12	0.13	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12
R^2 MIDAS		0.17	0.17	0.17	0.17	0.12	0.13	0.17	0.15	0.15	0.14		

Table 7 - Continued

$H = 3$ months

		$SRFUN$				$LRBON$							
		VF		VFe		VF		VFe		$SRFUN\_SF$	$LRBON\_SF$	GZ_SPR	GZ_SPR_V
		SGED	GED	SGED	GED	SGED	GED	SGED	GED				
VRP(-H)	0.34	0.37	0.38	0.37	0.40	0.38	0.38	0.37	0.37	0.33	0.35	0.34	0.33
	(0.12)***	(0.11)***	(0.12)***	(0.12)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.13)**	(0.12)***	(0.12)***	(0.12)***
FACTOR(-H)		-1.50	-1.38	-4.15	-3.49	-0.76	-0.67	-2.23	-1.29	-0.24	-0.26	-2.99	-5.03
		(0.72)**	(0.66)**	(1.33)***	(1.49)**	(0.91)	(0.81)	(3.37)	(2.01)	(0.38)	(0.37)	(3.43)	(4.25)
R^2 LS	0.10	0.11	0.11	0.11	0.12	0.10	0.10	0.09	0.09	0.10	0.10	0.09	0.09
R^2 MIDAS		0.18	0.18	0.16	0.13	0.13	0.13	0.12	0.14	0.13	0.15		
VRP(-H)	0.35	0.40	0.40	0.38	0.42	0.48	0.49	0.44	0.45	0.32	0.37	0.35	0.33
	(0.13)***	(0.11)***	(0.11)***	(0.12)***	(0.11)***	(0.12)***	(0.12)***	(0.12)***	(0.12)***	(0.13)**	(0.11)***	(0.10)***	(0.12)
Baa-Aaa(-H)	0.51	1.46	1.30	0.98	1.07	2.13	2.36	1.67	2.08	1.78	3.83	3.42	1.43
	(0.49)	(0.52)***	(0.56)**	(0.45)**	(0.49)**	(0.73)***	(0.78)***	(0.71)**	(0.81)**	(0.72)**	(0.99)***	(0.93)***	(0.69)***
FACTOR(-H)		-2.14	-1.88	-4.99	-4.19	-2.80	-2.68	-7.65	-5.47	-0.81	-1.73	-17.74	-16.04
		(0.59)***	(0.65)***	(1.49)***	(1.32)***	(1.05)	(0.96)	(3.56)	(2.35)	(0.39)**	(0.46)***	(4.45)***	(5.84)**
R^2 LS	0.09	0.13	0.12	0.11	0.12	0.11	0.11	0.10	0.11	0.11	0.13	0.12	0.10
R^2 MIDAS		0.17	0.17	0.17	0.14	0.13	0.13	0.13	0.15	0.13	0.16		
VRP(-H)	0.34	0.37	0.37	0.36	0.40	0.38	0.37	0.37	0.36	0.32	0.34	0.33	0.32
	(0.13)***	(0.12)***	(0.12)***	(0.12)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.14)**	(0.13)***	(0.12)***	(0.13)***
TMSP(-H)	9.06	8.49	7.64	9.78	10.97	15.03	14.95	10.91	12.54	13.80	11.79	14.66	12.36
	(24.18)	(22.48)	-22.53	-20.95	(22.71)	(23.88)	(23.90)	(23.12)	(23.67)	(24.83)	(23.69)	(24.31)	(24.62)**
FACTOR(-H)		-1.49	-1.36	-4.13	-3.49	-0.90	-0.79	-2.38	-1.47	-0.28	-0.28	-3.54	-5.54
		(0.72)**	(0.67)**	(1.38)***	(1.49)**	(0.94)	(0.83)	(3.32)	(2.02)	(0.39)	(0.37)	(3.67)	(4.44)
R^2 LS	0.09	0.11	0.11	0.11	0.11	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
R^2 MIDAS		0.17	0.17	0.15	0.13	0.13	0.13	0.12	0.14	0.13	0.15		
VRP(-H)	0.38	0.40	0.40	0.39	0.42	0.45	0.45	0.43	0.43	0.36	0.40	0.39	0.37
	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.12)***	(0.11)***	(0.11)***	(0.11)
log(P/D)(-H)	-3.19	-3.02	-3.03	-2.94	-2.78	-3.83	-3.90	-3.61	-3.72	-5.02	-4.63	-4.41	-4.24
	(1.18)***	(1.01)***	(1.02)***	(1.01)***	(1.08)**	(1.14)***	(1.15)***	(1.14)***	(1.15)***	(1.38)***	(1.33)***	(1.09)***	(1.16)***
FACTOR(-H)		-1.26	-1.12	-3.04	-2.65	-1.40	-1.27	-3.95	-2.50	-0.80	-0.70	-7.58	-14.06
		(0.51)**	(0.54)**	(1.57)*	(1.33)**	(0.62)**	(0.54)**	(2.12)*	(1.30)*	(0.35)**	(0.28)**	(2.13)***	(4.24)***
R^2 LS	0.12	0.13	0.13	0.12	0.13	0.12	0.12	0.12	0.12	0.14	0.13	0.13	0.12
R^2 MIDAS		0.18	0.18	0.18	0.15	0.17	0.17	0.14	0.17	0.16	0.18		
VRP(-H)	-0.62	1.12	0.71	0.48	1.09	-0.33	-0.33	-0.47	-0.47	-0.47	-0.37	-0.36	-0.55
	(0.95)	(0.93)	-0.96	-1.09	(0.94)	(1.08)	(1.04)	(0.97)	(0.94)	(0.96)	(0.99)	(1.06)	(1.05)
log(P/E)(-H)	0.38	0.33	0.35	0.35	0.37	0.39	0.39	0.38	0.38	0.36	0.37	0.36	0.36
	(0.11)***	(0.12)***	(0.12)***	(0.11)***	(0.12)***	(0.11)***	(0.11)***	(0.11)***	(0.11)***	(0.12)***	(0.12)***	(0.12)***	(0.11)***
FACTOR(-H)		-2.24	-1.81	-5.03	-5.08	-0.50	-0.45	-1.04	-0.56	-0.17	-0.17	-1.93	-3.21
		(0.75)***	(0.72)**	(2.18)**	(1.81)***	(0.91)	(0.77)	(3.24)	(1.84)	(0.38)	(0.35)	(3.27)	(4.90)***
R^2 LS	0.09	0.11	0.11	0.11	0.12	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
R^2 MIDAS		0.17	0.17	0.15	0.12	0.14	0.14	0.11	0.14	0.13	0.14		