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CONSUMERS' HETEROGENEITY, PUBLICNESS OF GOODS AND THE SIZE OF PUBLIC SECTOR

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Consumers' Heterogeneity, Publicness of Goods and the Size of Public Sector

Christos Bilanakos*

Abstract

This article studies the relationship between the level of consumers' inequality (or heterogeneity) and the size of government for the case of an impure public good. It is shown that the size of redistribution (represented by the level of subsidy provided to the firm) increases with the publicness of the good but may decrease with the level of consumers' inequality. Under the assumption of Nash bargaining between consumers and producers with respect to the level of subsidy, it is also shown that the actual size of government will be inefficiently small if the level of inequality is relatively low but can be inefficiently large (implying that the good will be overproduced in equilibrium) if the level of inequality is relatively low and the publicness of the good is high enough.

Keywords: Impure public good, perfect competition and monopoly, subsidy, Inequality, Nash Bargaining.

JEL Classifications: C71, H11, H21, H41.

1. Introduction

It is well known that the First Welfare Theorem fails – i.e. the competitive equilibrium is not Pareto efficient – in the presence of a public good. This kind of inefficiency is due to the free-rider problem associated with the presence of a positive consumption externality. If, at the same time, the market of the good under question is monopolistic, then the inefficiency problem is accentuated and the output quantity is further distorted below the optimum if the monopolist cannot implement perfect price discrimination. In such cases, a benevolent social planner who has complete information about consumers' demand and firms' cost functions can restore efficiency by implementing an appropriately designed quantity-based or price-based intervention mechanism (such as a subsidy scheme).[1]

Of course, a good may be neither purely public (nondepletable and nonexcludable) nor purely private. The concept of an impure public good was first examined in studies focusing on publicly provided goods that are subject to crowding effects (Bergstrom and Goodman, 1972; Borcherding and Deacon, 1972). These studies use a crowding function

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[1] See Mas-Colell et al (1995, Ch. 11.C) for more details on the private provision of public goods.

to take into account the fact that the amount of good (or the "service level") captured by each individual is negatively related to the number of consumers. For the case of a privately provided good (with which this paper is also concerned), the impure public good model has been based on the characteristics approach (Cornes and Sandler, 1984; 1994). In this framework, each consumer purchases some quantity of the good under question but derives utility from the good's characteristics rather than from the good itself. In particular, it is assumed that each unit of the good bought by the consumer jointly generates β units of a private characteristic and γ units of a public characteristic (where $\beta,\gamma>0$) over which the individual utility function is defined.

This paper studies the case of an impure public good by abstracting both from crowding effects and from the indirect characteristics approach. In order to take intermediate cases of publicness into account, we assume that each individual captures (consumes) the quantity bought by herself in the market plus a proportion of the total quantity bought by other consumers. This proportion represents the degree of publicness for the good under question and can vary between zero (implying that the good is purely private) and one (implying that the good is purely public).

There is clear evidence that the degree of government intervention in both perfectly and imperfectly competitive markets can often be either inefficiently high or inefficiently low. Therefore, some goods will be underproduced (relative to the first-best level of output) and other goods will be overproduced in equilibrium. This kind of government failure can be attributed to the biased objective function of a non-benevolent government serving the interests of some particular social group to the detriment of others. For instance, the high levels of subsidies provided to producers of agricultural products in USA or in EU countries have often been attributed to governmental decisions favoring large farmers' interests at the expense of consumers-taxpayers. However, the main branch of the literature has followed the median voter approach to account for the actual degree of government intervention in the market. According to this approach, the size of redistribution schemes (implemented through taxes and subsidies) simply reflects the median voter's preferences or income in political regimes where decisions are made by majority voting. In this context, economic models using the median voter approach (e.g. Romer, 1975; Roberts, 1977; Meltzer and Richard, 1981; Alesina and Rodrik, 1994; Persson and Tabellini, 1994) typically reach the conclusion that the size of redistribution (which can be interpreted as the size of public sector) increases with the level of income inequality between consumers.[2] This is due to the fact that any increase in inequality also increases the degree of government intervention preferred by the median voter. However, empirical evidence on the relationship between inequality and redistribution remains ambiguous and does not fully support this theoretical prediction, thus perhaps questioning the validity of the median voter approach itself.[3] In light of these controversies, more recent studies (e.g. Ackert et al, 2007; Durante and Putterman, 2009; Grosser and Reuben 2010; Hocht et al, 2012) remain within the median voter framework but assume that consumers-voters have other-regarding preferences (reflecting, for example, their inequality aversion or fairness considerations) in order to explain the actual size of redistribution and, in particular, the relationship between income inequality and the size of government.

This paper studies the relationship between the size of government, the level of consumers' heterogeneity or inequality and the publicness of the good under question by abstracting both from exogenous assumptions about the government's objective function and from the largely unrealistic median voter noncooperative approach. In particular, it is assumed here that the level of subsidy (representing the government size) is determined by an implicit arbitrator who divides the gains from trade according to the bargaining power of different social groups. More specifically, we adopt an axiomatic bargaining approach in the form of the Nash solution (Nash, 1950), according to which the final policy selection is the result of a reasonable social compromise between the interests of consumers and producers.[4]

The model studies the degree of government intervention in the market for a good of varying publicness under the assumption that consumers have heterogeneous preferences. To the extent that people with a higher income are more willing to pay and thus have a

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[2] For an exception to the rule, see Katsimi and Moutos (2006) who show that the relationship between inequality and redistribution can be non-positive even in a median voter context when the government uses tax revenues to finance the provision of a public good also produced by the private sector. For other studies based on the median voter approach, see Lee and Roemer (1999) and Benabou (2000).

[3] Some empirical studies (e.g. Meltzer and Richard (1983), Easterly and Rebelo (1993), Alesina and Rodrik (1994) and Milanovic (2000)) support the hypothesis of a positive relationship between inequality and redistribution. However, other studies (e.g. Clarke (1995), Lindert (1996), Perotti (1996) and Rodriguez (1999)) do not confirm this theoretical prediction.

[4] This approach seems to be a rather more realistic description of decision-making in modern institutionalized democracies than the directly democratic median voter approach.

more intense preference for the good under question, the degree of preference heterogeneity can also be interpreted as the level of income inequality between consumers[5]. The main findings of the paper can be summarized as follows: First, the optimal size of government is larger in a monopolistic market than in a competitive market. This is a hardly surprising result given the double source of inefficiency that distorts the monopolistic market and calls for a relatively heavier government intervention to implement the Pareto optimal allocation in equilibrium. Second, both the optimal and the actual size of government strictly increase with the publicness of the good and weakly decrease with the degree of consumers' heterogeneity (inequality). This means that a higher level of income inequality or preference heterogeneity may imply a smaller government size, contrasting the usual prediction of median voter models which find a positive relationship between inequality and redistribution. Third, the actual level of subsidy (i.e. the size of government) can be inefficiently high when the publicness of the good is high enough and the level of consumers' inequality is relatively low. That is, a low enough level of inequality may imply the overproduction of quasi-public goods due to the inefficiently high degree of government intervention in the market. This result contrasts the standard prediction of equilibrium underproduction for public goods (or, more generally, for goods subject to positive consumption externalities). On the other hand, if the publicness of the good is relatively low or the level of inequality is relatively high, then the level of subsidy will be inefficiently low (i.e. the government size will be too small) and the good will be underproduced in equilibrium.

The rest of the paper is organized as follows: Section 2 introduces the basic setup of the model. Section 3 characterizes the competitive and monopolistic equilibrium for a good of varying publicness as a function of the subsidy scheme implemented by the government. Section 4 computes the Pareto optimal allocation and uses it as a benchmark to determine the optimal corrective subsidy that restores efficiency both in a competitive and in a monopolistic market. Section 5 characterizes the actual level of subsidy in a monopolistic market under the assumption of Nash bargaining between consumers and producers and evaluates the actual size of government from a welfare point of view. Section 6 concludes the paper and discusses its possible extensions.

[5] The next section provides a justification of this interpretation in more detail.

2. The Model

Consider a setting with *n* consumers and one good (labeled as good *A*) of varying publicness in addition to *m* private goods. We assume a constant-returns-to-scale technology for the single profit-maximizing firm producing good *A*, implying the following cost function:

$c(q) = c \cdot q$, where $c > 0$ and *q* is the produced quantity of good *A*.

Define a benefit function $V(\cdot)$ over the level of good *A* captured by each consumer *i=1,…,n*, where: $V(0) = 0$, $V'(\cdot) > 0$ and $V''(\cdot) < 0$.

Imagine that a market exists for good *A* and each consumer chooses the quantity (A_i) of the good bought in the market by taking as given its price (*p*). In order to capture the varying publicness of the good (rather than assume that the good has a purely private property and a purely public property, as does the characteristics approach), we assume that each individual *i* consumes the quantity of the good bought by herself plus a proportion $\delta \in [0,1]$ of the total quantity bought by other consumers:

$$A_i + \delta \sum_{j \neq i} A_j$$

According to this formulation, the publicness of good *A* increases with the value of parameter *δ*, which represents the degree of nondepletability and/or nonexcludability of the good. The corner values *δ=0* and *δ=1* correspond to the extreme cases of a purely private and a purely public good, respectively.

We assume *n=2* for simplicity. The redistribution scheme is effectuated through a subsidy $s \in [0, c]$ provided to the firm per unit of produced output. The cost of the subsidy is paid by consumers in the form of lump-sum taxes T_1, T_2 (where T_i is the tax paid by consumer *i*) and the total tax burden is equally shared between consumers 1 and 2: $T_1 = T_2 = T$, where $T_1 + T_2 = 2T = sq$ or $T = sq/2$.

We also assume *m=1* and leisure (ℓ) is the purely private good used as the numeraire commodity. Each consumer *i=1,2* has a different income (M_i) and her preferences are represented by a utility function which is separable in ℓ and *A*:

$$\phi_i = u(\ell_i) + V(A_i + \delta A_j) \text{ , where } u' > 0 \text{ and } u'' < 0 \tag{1}$$

The budget constraint faced by consumer i will be binding at the solution of her utility maximization problem and has the following form:

$$\ell_i + pA_i = M_i - T \tag{2}$$

We can substitute (2) into (1) to get:

$$\phi_i = u(M_i - T - pA_i) + V(A_i + \delta A_j) \tag{3}$$

Then, we can use a first-order Taylor expansion to approximate $u(\cdot)$ as:[6]

$$u(M_i - T - pA_i) \approx u(M_i) - (pA_i + T)\cdot u'(M_i) \tag{4}$$

Rewrite (3) by use of (4) as:

$$\phi_i = u(M_i) - (pA_i + T)\cdot u'(M_i) + V(A_i + \delta A_j)$$

Of course, this utility function represents the same preferences as the following one:

$$U_i = \frac{1}{u'(M_i)}\cdot V(A_i + \delta A_j) - pA_i - T$$

We conclude that consumer i's utility function can be written as:

$$U_i(A_i, A_j) = \theta_i \cdot V(A_i + \delta A_j) - pA_i - T \tag{5}$$

where $\theta_i \equiv 1/u'(M_i) > 0$ is the inverse of the "marginal utility of income". According to this formulation, if $u(\cdot)$ is concave then a higher level of income (M_i) is associated with a lower $u'(M_i)$ and therefore with a higher value of θ_i. We assume $M_2 \geq M_1$, implying $\theta_2 \geq \theta_1$. Since the parameter θ_i can also be interpreted as the intensity of consumer i's preferences for good A, we conclude that the wealthier consumer 2 also has a stronger preference for the good under question. In the same sense, the degree of preference heterogeneity (captured by the ratio θ_1/θ_2) can also be interpreted as the level of income inequality between consumers (see also Tirole, 1989, Ch. 3.3).

In order to get reduced form solutions, we use the specific form of benefit function $V(x) = \sqrt{x}$ for the rest of the paper.

3. Competitive and Monopolistic Equilibrium

At a competitive equilibrium, each consumer i chooses the quantity A_i so as to maximize her utility function (taking as given the price p and the quantity A_j purchased by the other consumer) and, therefore, solves the following problem:

[6] The general formula is $f(x) \approx f(x_0) + (x - x_0)f'(x_0)$. In this case, $f = u$, $x = M_i - pA_i - T$ and $x_0 = M_i$.

$$\max_{\{A_i\}} \ U_i(A_i, A_j) = \theta_i \cdot V(A_i + \delta A_j) - pA_i - T$$
$$s.t. \ A_i \geq 0$$

The solution of this problem implies the following "best-response functions" for each consumer *i*:

$$A_i(A_j) = \max\left\{\frac{\theta_i^2}{p^2} - \delta A_j, 0\right\} \tag{6}$$

In order to derive the set of individual demand functions, we impose the requirement that each consumer's choice of purchased quantity must be her best response to the quantity purchased by the other consumer (as in a Nash equilibrium):

$$A_i = A_i(A_j) \tag{7}$$

We solve the system of equations (6) and (7) to get consumers' individual demand functions $A_1(p), A_2(p)$ and the aggregate demand function *D(p)*:

$$\left(A_1(p), A_2(p), D(p)\right) = \begin{cases} \left(\dfrac{\theta_1^2 - \delta\theta_2^2}{(1-\delta)p^2}\ ,\ \dfrac{\theta_2^2 - \delta\theta_1^2}{(1-\delta)p^2}\ ,\ \dfrac{\theta_1^2 + \theta_2^2}{(1+\delta)p^2}\right) & \text{, if } \delta \leq \theta \\ \\ \left(0\ ,\ \dfrac{\theta_2^2}{p^2}\ ,\ \dfrac{\theta_2^2}{p^2}\right) & \text{, if } \theta \leq \delta \leq 1 \end{cases} \tag{8}$$

where $\theta \equiv \theta_1^2 / \theta_2^2 \in (0,1)$ is a measure of preference heterogeneity or income inequality between consumers and $D(p) = \sum_{i=1}^{n} A_i(p)$.

Note that consumer 1 purchases a zero amount of good *A* for high enough values of *δ*. In other words, if the good is similar enough to a pure public good then the free-rider problem takes the usual extreme form and only the consumer who derives the largest marginal benefit from the good purchases a positive quantity in equilibrium.

The competitive firm chooses the supplied quantity of the good so as to maximize profits and, therefore, solves the following problem:

$$\max_{\{q\}} \ \pi = (p + s - c)q$$
$$s.t. \ q \geq 0$$

The solution yields the supply function of the competitive firm:

$$q(p) = \begin{cases} 0 & \text{, if } p < c - s \\ \geq 0 & \text{, if } p = c - s \\ \infty & \text{, if } p > c - s \end{cases} \tag{9}$$

At a competitive equilibrium, aggregate demand equals aggregate supply:

$$D(p) = q(p) \tag{10}$$

The system of equations (8), (9) and (10) implies the competitive equilibrium price and allocation summarized below along with equilibrium profits, utilities and surpluses (given the subsidy *s*).

▪ **Case 1.** For $\delta \leq \theta$, the competitive equilibrium is:

$$\begin{aligned} (p^*, q^*, A_1^*, A_2^*) &= \left(c-s \ , \ \frac{\theta_1^2+\theta_2^2}{(1+\delta)(c-s)^2} \ , \ \frac{\theta_1^2-\delta\theta_2^2}{(1-\delta^2)(c-s)^2} \ , \ \frac{\theta_2^2-\delta\theta_1^2}{(1-\delta^2)(c-s)^2} \right) \\ (U_1^*, U_2^*) &= \left(\frac{(1-2\delta^2)\theta_1^2+\delta\theta_2^2}{(1-\delta^2)(c-s)} - \frac{sq^*}{2} \ , \ \frac{(1-2\delta^2)\theta_2^2+\delta\theta_1^2}{(1-\delta^2)(c-s)} - \frac{sq^*}{2} \right) \\ (CS^*, PS^*, TS^*) &= \left(\frac{(1+2\delta)(\theta_1^2+\theta_2^2)}{(1+\delta)(c-s)} - sq^* \ , \ 0 \ , \ \frac{(1+2\delta)(\theta_1^2+\theta_2^2)}{(1+\delta)(c-s)} - sq^* \right) \end{aligned} \tag{11a}$$

▪ **Case 2.** For $\theta \leq \delta \leq 1$, the competitive equilibrium is:

$$\begin{aligned} (p^*, q^*, A_1^*, A_2^*) &= \left(c-s \ , \frac{\theta_2^2}{(c-s)^2} \ , 0 \ , \ \frac{\theta_2^2}{(c-s)^2} \right) \\ (U_1^*, U_2^*) &= \left(\frac{2\theta_1\theta_2\sqrt{\delta}}{c-s} - \frac{sq^*}{2} \ , \ \frac{\theta_2^2}{c-s} - \frac{sq^*}{2} \right) \\ (CS^*, PS^*, TS^*) &= \left(\frac{\theta_2(2\theta_1\sqrt{\delta}+\theta_2)}{c-s} - sq^* \ , \ 0 \ , \ \frac{\theta_2(2\theta_1\sqrt{\delta}+\theta_2)}{c-s} - sq^* \right) \end{aligned} \tag{11b}$$

where $CS^* = \sum_{i=1}^{n} U_i^*$ is the consumer surplus, $PS^* = \pi^*$ is the producer surplus (which coincides with the firm's profit) and TS^* is the total surplus in competitive equilibrium.

On the other hand, if the market is monopolistic then the firm chooses the price of the good so as to maximize its profits given the consumers' behavior described by the aggregate demand function and, therefore, solves the following problem:

$$\max_{\{p\}} \ \pi = (p+s-c)q$$
$$s.t. \ \ q = D(p)$$

The solution of this problem yields the monopolistic price (p^M), which can then be substituted into individual demand functions to compute the equilibrium allocation summarized below along with profits, utilities and surpluses (given the subsidy s):

▪ **<u>Case 1</u>.** For $\delta \le \theta$, the monopolistic equilibrium is:

$$(p^M, q^M, A_1^M, A_2^M) = \left(2(c-s)\ ,\ \frac{\theta_1^2+\theta_2^2}{4(1+\delta)(c-s)^2}\ ,\ \frac{\theta_1^2-\delta\theta_2^2}{4(1-\delta^2)(c-s)^2}\ ,\ \frac{\theta_2^2-\delta\theta_1^2}{4(1-\delta^2)(c-s)^2} \right)$$

$$(U_1^M, U_2^M) = \left(\frac{(1-2\delta^2)\theta_1^2+\delta\theta_2^2}{2(1-\delta^2)(c-s)} - \frac{sq^M}{2}\ ,\ \frac{(1-2\delta^2)\theta_2^2+\delta\theta_1^2}{2(1-\delta^2)(c-s)} - \frac{sq^M}{2} \right) \tag{12a}$$

$$(CS^M, PS^M, TS^M) = \left(\frac{(1+2\delta)(\theta_1^2+\theta_2^2)}{2(1+\delta)(c-s)} - sq^M\ ,\ \frac{\theta_1^2+\theta_2^2}{4(1+\delta)(c-s)}\ ,\ \frac{(3+4\delta)(\theta_1^2+\theta_2^2)}{4(1+\delta)(c-s)} - sq^M \right)$$

▪ **<u>Case 2</u>.** For $\theta \le \delta \le 1$, the monopolistic equilibrium is:

$$(p^M, q^M, A_1^M, A_2^M) = \left(2(c-s)\ ,\ \frac{\theta_2^2}{4(c-s)^2}\ ,\ 0\ ,\ \frac{\theta_2^2}{4(c-s)^2} \right)$$

$$(U_1^M, U_2^M) = \left(\frac{\theta_1\theta_2\sqrt{\delta}}{c-s} - \frac{sq^M}{2}\ ,\ \frac{\theta_2^2}{2(c-s)} - \frac{sq^M}{2} \right) \tag{12b}$$

$$(CS^M, PS^M, TS^M) = \left(\frac{\theta_2(2\theta_1\sqrt{\delta}+\theta_2)}{2(c-s)} - sq^M,\ \frac{\theta_2^2}{4(c-s)}\ ,\ \frac{\theta_2(4\theta_1\sqrt{\delta}+3\theta_2)}{4(c-s)} - sq^M \right)$$

4. The First-Best Allocation and the Optimal Size of Government

The Pareto optimal allocation maximizes aggregate surplus subject to the feasibility constraints and thus solves the following problem:

$$\max_{\{q,A_1,A_2\}} TS = \sum_{i=1}^{n} \left[\theta_i V(A_i + \delta \sum_{j\ne i} A_j) \right] - cq$$

$$s.t.\ \sum_{i=1}^{n} A_i \le q$$

$$q, A_i \ge 0\ ,\ i = 1,\dots,n$$

We solve this problem to get the first-best allocation summarized below.

▪ **<u>Case 1</u>.** For $\delta \le \theta$, the Pareto efficient allocation is :

$$(q^P, A_1^P, A_2^P) = \left(\frac{(1+\delta)(\theta_1^2+\theta_2^2)}{c^2}\ ,\ \frac{(1+\delta)(\theta_1^2-\delta\theta_2^2)}{(1-\delta)c^2}\ ,\ \frac{(1+\delta)(\theta_2^2-\delta\theta_1^2)}{(1-\delta)c^2} \right) \tag{13a}$$

▪ **Case 2.** For $\theta \leq \delta \leq 1$, the Pareto efficient allocation is :

$$(q^P, A_1^P, A_2^P) = \left(\frac{(\theta_1\sqrt{\delta}+\theta_2)^2}{c^2},\ 0\ ,\ \frac{(\theta_1\sqrt{\delta}+\theta_2)^2}{c^2} \right) \qquad \text{(13b)}$$

The set of subsidies $S = (s^*, s^M)$ implementing the first-best allocation in a competitive and in a monopolistic market, respectively, is characterized by the following condition[7]:

$$q^*(s^*) = q^M(s^M) = q^P \qquad \text{(14)}$$

Equation (14) can be solved to yield the set of optimal corrective subsidies representing the optimal size of government in a competitive and in a monopolistic market:

$$(s^*, s^M) = \begin{cases} \left(\dfrac{\delta}{1+\delta}c\ ,\ \dfrac{1+2\delta}{2(1+\delta)}c \right) & \text{, if } \delta \leq \theta \\ \\ \left(\dfrac{\sqrt{\delta\theta}}{1+\sqrt{\delta\theta}}c\ ,\ \dfrac{1+2\sqrt{\delta\theta}}{2(1+\sqrt{\delta\theta})}c \right) & \text{, if } \theta \leq \delta \leq 1 \end{cases} \qquad \text{(15)}$$

Proposition 1 immediately follows from (15).

Proposition 1. **(a)** The optimal size of government is always larger in a monopolistic market than in a competitive market:

$s^M > s^*$ for all $\delta, \theta \in [0,1]$.

(b) The optimal size of government strictly increases with the publicness of the good and weakly decreases with the degree of consumers' heterogeneity (inequality) both in a competitive and in a monopolistic market:

- $\partial s^* / \partial \delta > 0$, $\partial s^M / \partial \delta > 0$ for $\delta \in [0,1]$
- $\partial s^* / \partial \theta > 0$, $\partial s^M / \partial \theta > 0$ for $\theta \in [0, \delta)$
  $\partial s^* / \partial \theta = 0$, $\partial s^M / \partial \theta = 0$ for $\theta \in (\delta, 1]$

Figures 1 and 2 graphically depict s^* and s^M as a function of δ and θ, respectively.

[7] Equivalently, s^* and s^M are the solutions to the problem of a benevolent social planner who chooses the level of subsidy so as to maximize the aggregate surplus associated with the competitive and monopolistic equilibrium found above:

$$\begin{aligned} s^* &= \arg\ \max TS^* \\ s^M &= \arg\ \max TS^M \end{aligned}$$, where TS^* and TS^M are given in (11) and (12), respectively.

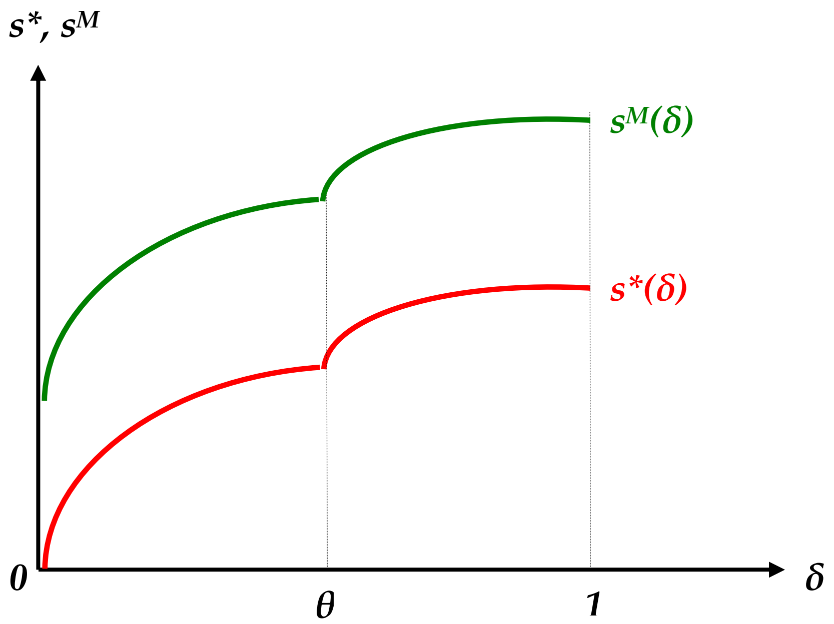

Figure 1. The optimal corrective subsidy as a function of δ

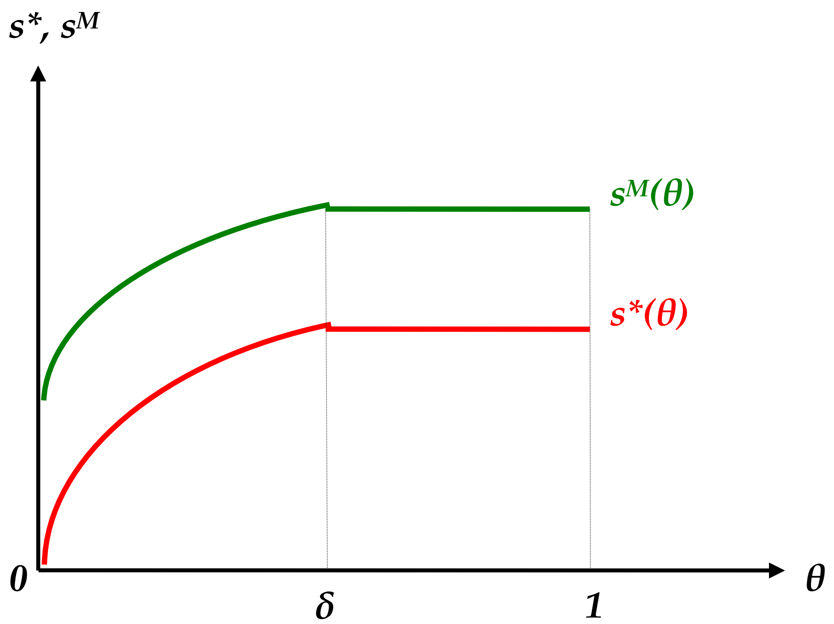

Figure 2. The optimal corrective subsidy as a function of θ

Part (a) of Proposition 1 is hardly surprising: The degree of government intervention required to implement the efficient allocation in a monopolistic market is higher than in a competitive market, since the former suffers both from the free-rider problem (as does the competitive market) and from the standard output distortion associated with a non-discriminating monopolist. As for part (b), it is also fairly intuitive that the level of corrective subsidy increases with the publicness of the good, since the free-rider problem is accentuated as the good becomes more similar to a public good. The most interesting finding is the negative relationship between consumers' heterogeneity (inequality) and the optimal size of government for $\delta \geq \theta$. This result is worth explaining in more detail. For the case of a monopolistic market, the optimal corrective subsidy (s^M) must satisfy the first-order condition[8]:

$$\frac{\partial TS^M\left(s^M(\theta_1,\theta_2),\theta_1,\theta_2\right)}{\partial s} \equiv 0$$

We differentiate both sides of the above identity with respect to θ_i and get:

$$\frac{\partial^2 TS^M}{\partial s^2}\cdot\frac{\partial s^M}{\partial \theta_i}+\frac{\partial^2 TS^M}{\partial s\partial\theta_i}\equiv 0\text{, or: }\frac{\partial s^M}{\partial \theta_i}=-\frac{\partial^2 TS^M/\partial s\partial\theta_i}{\partial^2 TS^M/\partial s^2} \tag{16}$$

where all second-order partial derivatives are evaluated at $s=s^M$.

Since the denominator of the expression in the right hand of (16) is negative due to the second-order condition for maximization, we conclude that:

$$sign\frac{\partial s^M}{\partial \theta_i}=sign\frac{\partial^2 TS^M}{\partial s\partial\theta_i}/_{s=s^M} \tag{17}$$

Consider the case $\theta \leq \delta$ first (i.e. the case where the level of inequality is relatively high). Then, consumer 1 purchases nothing in equilibrium and we know that:

$$TS^M=\frac{\theta_2(2\theta_1\sqrt{\delta}+\theta_2)}{2(c-s)}-s\frac{\theta_2^2}{4(c-s)^2}+\frac{\theta_2^2}{4(c-s)}$$

where the first term is the pre-tax consumer surplus, the second term is the cost of subsidy (i.e. the tax paid by consumers) and the third term is the firm's profit. Then, it is straightforward to see that:

$$\frac{\partial TS^M}{\partial s}=\frac{\theta_2(2\theta_1\sqrt{\delta}+\theta_2)}{2(c-s)^2}-\frac{\theta_2^2(c+s)}{4(c-s)^3}+\frac{\theta_2^2}{4(c-s)^2}\text{ and:}$$

$$\frac{\partial^2 TS^M}{\partial s\partial\theta_1}/_{s=s^M}=\frac{\theta_2\sqrt{\delta}}{2(c-s^M)^2}>0\overset{(17)}{\Rightarrow}\frac{\partial s^M}{\partial\theta_1}>0 \tag{18}$$

In this case, an increase in consumer 1's income or taste parameter θ_1 (implying a higher value of θ and therefore a lower degree of inequality) increases the marginal pre-tax consumer surplus associated with a higher value of s (due to a higher valuation of the quantity captured by consumer 1) and leaves both marginal profits and the marginal cost

[8] The analysis here follows Varian (1992, p. 491-495) and is qualitatively the same for the case of a competitive market.

of subsidy unaffected (because the equilibrium price and output do not depend on θ_1 in this case), implying that the social planner optimally chooses a higher level of the corrective subsidy s^M.

Similarly, we can find:

$$\frac{\partial^2 TS^M}{\partial s \partial \theta_2} / _{s = s^M} = \frac{1}{2(c - s^M)^2}[2\theta_1\sqrt{\delta} + 3\theta_2 - \theta_2 \frac{c + s^M}{c - s^M}] < 0 \overset{(17)}{\Rightarrow} \frac{\partial s^M}{\partial \theta_2} < 0 \tag{19}$$

In this case, an increase in consumer 2's income or taste parameter θ_2 (implying a lower value of θ and therefore a higher degree of inequality) increases the marginal pre-tax consumer surplus and the firm's marginal profit associated with a higher value of s but, at the same time, increases even more the marginal cost of subsidy, implying that the social planner optimally chooses a lower level of the corrective subsidy s^M.

From (18) and (19), we conclude that both an increase in θ_1 and a decrease in θ_2 (i.e. any increase in θ, which means a lower level of inequality between consumers) implies a higher optimal level of subsidy ($\partial s^M / \partial \theta > 0$). In sum, an increase in the level of equality calls for a bigger government to implement the efficient resource allocation in equilibrium[9].

5. Monopolistic Equilibrium with Nash Bargaining

This section characterizes and evaluates the actual level of subsidy chosen in a monopolistic market under the assumption of Nash bargaining between consumers and producers. For this purpose, we define a bargaining problem $(u, \overline{u})$ where the set $u = (CS^M(s), PS^M(s))$ represents the payoff allocations that can be settled on if there is cooperation between consumers and producers with respect to the level of subsidy $s \in [0, c]$ and the threat point $\overline{u} = (CS^M(0), PS^M(0)) \in u$ is the outcome that occurs if there is a breakdown of cooperation. In other words, if bargaining between consumers and producers collapses then the government does not intervene in the market at all and agents receive the set of payoffs corresponding to a monopolistic equilibrium without any subsidy scheme ($s=0$). The symmetric Nash bargaining solution (s^N) maximizes the

[9] For $\theta \geq \delta$, a similar analysis shows that $\partial^2 TS^M(s^M) / \partial s \partial \theta_i = 0$, implying $\partial s^M / \partial \theta_i = 0$. As a result, any change in the level of inequality leaves the optimal level of subsidy unaffected in this case ($\partial s^M / \partial \theta = 0$).

product of agents' payoff differences from the threat point and solves the following problem:

$$\max_{\{s\}} \; W = [CS^M(s) - CS^M(0)] \cdot [PS^M(s) - PS^M(0)]$$
$$s.t. \;\; 0 \le s \le c$$

where $CS^M(s)$ and $PS^M(s)$ are given in (12).

The Nash solution is summarized below.

$$s^N = \begin{cases} \dfrac{2+8\delta}{5+8\delta}c & \text{, if } \delta \le \theta \\ \\ \dfrac{2+8\sqrt{\delta\theta}}{5+8\sqrt{\delta\theta}}c & \text{, if } \theta \le \delta \le 1 \end{cases} \tag{20}$$

The results from the comparative statics analysis concerning the effect of publicness (δ) or inequality (θ) on the actual size of government (s^N) are qualitatively the same as described in Proposition 1 for the benchmark case: $\partial s^N / \partial \delta > 0$, $\partial s^N / \partial \theta \ge 0$. We proceed to evaluate the Nash solution (which is our prediction for the actual size of government in a monopolistic market) by comparing it to the optimal level of subsidy (s^M) found in (15). If $s^N > s^M$, the actual government size is inefficiently large and the good is overproduced (relative to the first-best) in equilibrium. In order to examine more closely the possibility of overproduction, we remind first that the socially optimal level of subsidy (s^M) satisfies the first-order condition:

$$\frac{\partial TS^M}{\partial s} /_{s = s^M} = 0 \text{, or: } \frac{\partial CS^M}{\partial s} /_{s = s^M} = -\frac{\partial PS^M}{\partial s} /_{s = s^M} \tag{21}$$

Then, overproduction ($s^{NB} > s^M$) will be the case if and only if:

$$\frac{\partial W}{\partial s} /_{s = s^M} = \frac{\partial CS^M}{\partial s} /_{s = s^M} \cdot \left(PS^M(s^M) - PS^M(0) \right) + \frac{\partial PS^M}{\partial s} /_{s = s^M} \cdot \left(CS^M(s^M) - CS^M(0) \right) > 0$$

$$\overset{(21)}{\Leftrightarrow} \frac{\partial PS^M}{\partial s} /_{s = s^M} \cdot \left[\left(CS^M(s^M) - CS^M(0) \right) - \left(PS^M(s^M) - PS^M(0) \right) \right] > 0$$

Since the derivative in the last expression is positive, we conclude:

$$s^{NB} > s^M \Leftrightarrow CS^M(s^M) - CS^M(0) > PS^M(s^M) - PS^M(0) \tag{22}$$

In other words, overproduction will be the case if and only if consumers' gains from the implementation of the first-best level of subsidy (relative to the case where there is no subsidy at all) exceed producers' respective gains. Note that a decrease in consumers' threat point or an increase in producers' threat point makes overproduction more likely. Therefore, the possibility of overproduction depends on our reasonable assumption that consumers receive the low enough payoff $CS^M(0)$ and producers receive the high enough payoff $PS^M(0)$ if cooperation breaks down. Intuitively, producers always benefit from a higher level of subsidy – i.e. the level of subsidy that maximizes the producer surplus is $s^P = c > s^M$. On the other hand, the level of subsidy that maximizes consumer surplus is $s^C < s^M$. Since consumers' outside option is low enough and producers' outside option is high enough, consumers are willing to accept (and producers are able to impose) a level of subsidy that is higher than s^C and, indeed, might be even higher than s^M as a Nash solution instead of ending up with their low reservation payoff.

Define the function:

$$f(\delta,\theta_1,\theta_2) \equiv \left[CS^M(s^M) - CS^M(0)\right] - \left[PS^M(s^M) - PS^M(0)\right]$$

Then, from (22) we see that $s^{NB} > s^M$ if and only if $f(\delta,\theta_1,\theta_2) > 0$. Simple calculations yield:

$$f(\delta,\theta_1,\theta_2) = \begin{cases} \dfrac{(4\delta^2-1)(\theta_1^2+\theta_2^2)}{4(1+\delta)c} & \text{, if } \delta \le \theta \\ \\ \dfrac{4\delta\theta_1^2-\theta_2^2}{4c} & \text{, if } \theta \le \delta \le 1 \end{cases} \qquad (23)$$

The above expression immediately implies the results stated in Proposition 2, which is the main finding of the paper.

Proposition 2. **(a)** For low values of θ (i.e. if the level of inequality is high enough), the equilibrium level of subsidy is inefficiently low and the good is underproduced in equilibrium for any value of publicness (δ):

• If $\theta < 1/4$, then $s^N < s^M$ (i.e. $q^M(s^N) < q^P$) for all $\delta \in [0,1]$. (see Figure 3)

(b) For intermediate or high values of θ (i.e. if the level of inequality is relatively low), the equilibrium level of subsidy is inefficiently low (the good is underproduced) when the publicness of the good is low but the equilibrium level of subsidy is inefficiently high (the good is overproduced) when the publicness of the good is high enough:

• If $1/4 \le \theta \le 1/2$, then: $s^N < s^M$ ($q^M(s^N) < q^P$) for $\delta < 1/4\theta$
$s^N > s^M$ ($q^M(s^N) > q^P$) for $1/4\theta < \delta \le 1$ (see Figure 4)

In this case, the monopolistic equilibrium is efficient for $\delta = 1/4\theta$.

• If $1/2 \le \theta \le 1$, then: $s^N < s^M$ $(q^M(s^N) < q^P)$ for $\delta < 1/2$

$s^N > s^M$ $(q^M(s^N) > q^P)$ for $1/2 < \delta \le 1$ (see Figure 5)

In this case, the monopolistic equilibrium is efficient for $\delta = 1/2$.

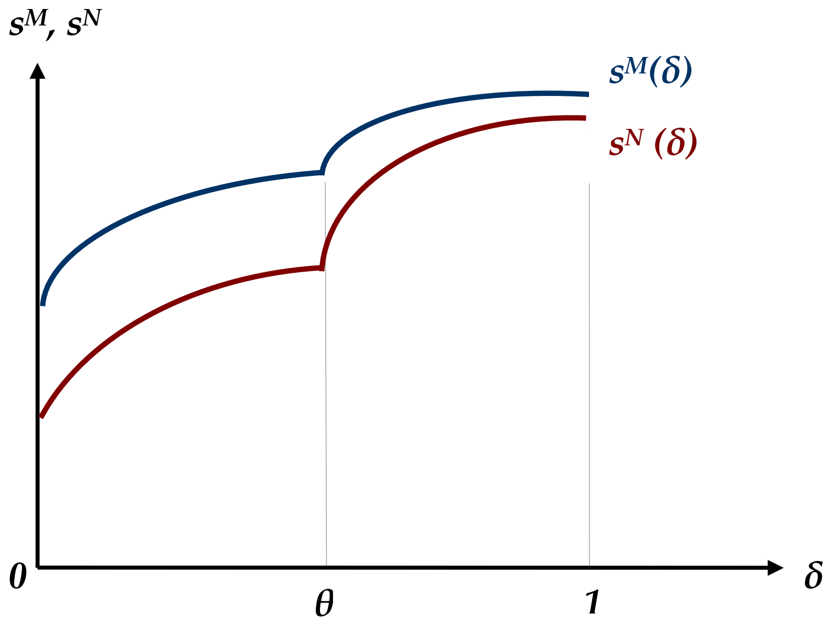

Figure 3. $\theta < 1/4$

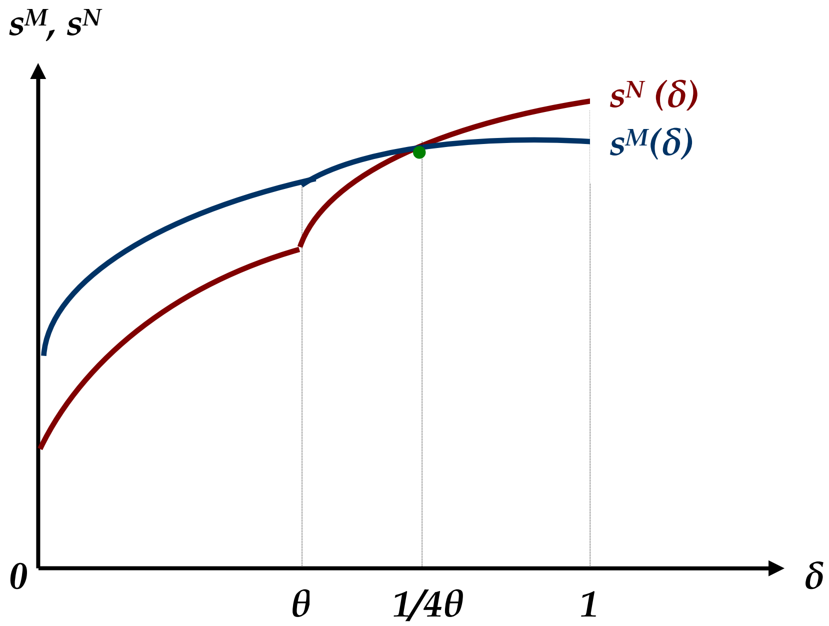

Figure 4. $1/4 \le \theta \le 1/2$

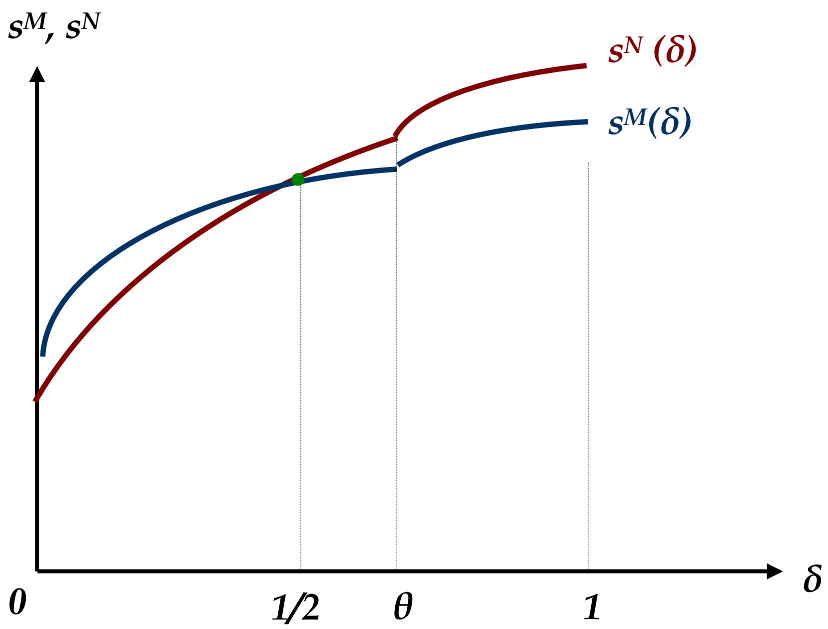

Figure 5. $1/2 < \theta \leq 1$

In sum, both the actual and the optimal government size decrease with the level of inequality and increase with the publicness of the good. If the level of inequality is relatively high, then the government size will be inefficiently low and the good is underproduced in equilibrium. But if the level of inequality is relatively low and the publicness of the good is high enough, then the government size will be inefficiently large and the good is overproduced in equilibrium. That is, quasi-public goods tend to be oversubsidized when consumers' incomes are relatively equal (i.e. when preferences are relatively homogeneous).

6. Conclusion

Most median voter models predict a positive relationship between the size of government redistribution and the level of consumers-voters' inequality. However, this theoretical prediction is not fully supported by empirical evidence. This paper has departed from the median voter approach in order to study the relationship between the level of inequality and the degree of government intervention in the market of a good which is subject to positive consumption externalities. In this context, it has been shown that the government size (represented by the level of subsidy provided to the firm) increases with the publicness of the good but may decrease with the level of consumers' inequality. In contrast to the median voter approach, we have assumed that the actual size of redistribution is the result of a compromise between consumers and producers who bargain over the level of subsidy. Then, we have shown that the actual level of subsidy (as given by the Nash solution of this bargaining problem) can be either inefficiently low or high depending on the publicness of the good and on the level of consumers' inequality. In particular, if the level of inequality is sufficiently high then the government size will be inefficiently small and the good will be underproduced in equilibrium. But if the level of inequality is relatively low and the publicness of the good is high enough, then the actual level of subsidy will be inefficiently high and the good is overproduced in equilibrium. These results may shed some light on cases of potentially oversubsidized or undersubsidized goods observed in real economic life.

A potential extension of our analysis might be to examine the optimal government size after taking into account the possibility of entry of new firms in the market. In this framework, it might also be possible to introduce informational asymmetries in order to examine the scope and welfare implications of a limit pricing policy exercised by the subsidized incumbent. These issues are left for future research.

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