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ENDOGENOUS PUBLIC POLICY AND LONG-RUN GROWTH

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Abstract

We study the determinants of voting outcomes on the provision of public consumption through marginal income taxes in the context of the simple linear growth model. We focus on how the dynamic politicoeconomic equilibrium maps the economic fundamentals to policies and long-run growth. We find that in a deterministic growth environment voters internalize, although imperfectly, the deadweight losses of taxation and vote for lower taxes when the productivity of capital is higher. Therefore, the politicoeconomic channel reinforces the positive role of productivity for growth. In a stochastic environment, we find that if business cycles are driven by productivity shocks in the endogenous growth framework, equilibrium policies imply that taxes should fall in high growth periods and rise in low-growth periods. In line with existing evidence, our model predicts procyclical public consumption and countercyclical public consumption GDP shares.

Key Words: voting, second-best taxation, endogenous growth

JEL classification: C73, D72, E61, E62, O23

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1. Introduction

The link between public policy and growth has attracted a great deal of attention not only by economists and policy makers, but is also a matter of debate in newspapers and every-day discussions. On the one hand, the opportunity to achieve a high level of long-run growth rests on the ability of the economy to implement an optimal investment strategy. This, in turn, depends on an incentive structure that allows the microeconomic units to exploit the available technological possibilities. On the other hand, fiscal policies, such as marginal income taxes, directly reduce the incentives for economic agents to follow optimal investment strategies. With marginal income taxes, markets are unable to yield the investment return that reflects the technological potential of the economy, since capital markets are distorted.

If one views fiscal policies implemented by governments as exogenous, unrelated to the economic structure, there is a direct negative causality from marginal income taxes to growth. Several papers examine the growth and welfare effects of such exogenous policy streams using calibrated or simulated models.[1] On the other hand, empirical studies, using regression analyses, are unable to confirm the causality implied by the theory that treats policies as exogenous. Many conclusions from such empirical studies are conflicting. For example, Easterly and Rebelo (1993), and Mendoza et. al. (1997), have not given compelling evidence of a strong negative link between marginal income taxes and growth, at least across large samples of countries. On the other hand, Hall and Jones (1999) conclude that differences in institutions and government policies are behind the large cross-country differences in growth rates, productivity and per capita income. Moreover, Fölster and Henrekson (2001), whose study focuses on rich countries, exploit within-country variation over time, as opposed to cross-country variation, and report a negative link between government spending and growth.

[1] See, for example, Lucas (1990), Stokey and Rebelo (1995), Mendoza et. al. (1997), Ortigueira (1998), Kneller et. al. (1999), and Rivas (2003).

By endogenizing government policies, linking them to the economic fundamentals, one can shed new light on the link between policies and growth. For example, the empirical conclusion of a weak link between marginal taxes and growth may be that taxes partly finance productive capital infrastructure and economic agents allow for higher taxes whenever they can observe such an opportunity to increase the return to private investment through more provision of infrastructure.[2] But, since in most countries the highest share of the government budget is spent on non-productive public consumption, the provision of public consumption is the key issue to explore.

In this paper, we provide a theory of public consumption goods provision in the simplest endogenous growth environment, the "linear growth" model without externalities in production. This class of linear growth models was proposed by Rebelo (1991) in order to study the link between exogenous policies, investment and growth in the long run. Without externalities in production, marginal taxation reduces the returns to accumulable capital. This reduction in the after-tax return on capital makes households act as if the available technological possibilities were shrunk in proportion to the level of the marginal tax rate. Thus, marginal taxes in the framework without externalities in production, always carry the competitive economy away from both production efficiency and Pareto efficiency.[3]

In our model, a public consumption good enters the utility function of households. Each household operates both as economic agent and voter. The public good can be financed only through marginal income taxes and under balanced fiscal budgets. The public good externality generates a political demand for positive taxes that finance the provision of the

[2] We examine the link between economic fundamentals and the provision of public infrastructure in Koulovatianos and Mirman (2004).

[3] On the contrary, in a linear growth model with externalities in production, as this of Romer (1986), the production possibility frontier is not pre-determined, but it depends on private investment choices. A fiscal policy transfering resources from consumption to investment may lead to both higher production efficiency and to a Pareto improvement.

public good. Hence, the occurence of a deadweight loss is inevitable, as marginal taxes generate disincentives for investment. But voters have the ability, depending on their economic fundamentals, to partly internalize the deadweight losses of taxation. Thus, we examine how the dynamic politicoeconomic equilibrium maps the economic fundamentals to policies and long-run growth.

We first study a deterministic growth environment. We find that in economies in which the productivity of capital is higher, voters choose lower taxes, because they (optimally) select a lower ratio of current public goods to future private consumption. This happens because, in equilibrium, marginal income taxes create a strong substitution effect, driven by the fact that it is not the stock of capital that is taxed over time, but only the capital income flow. Because of this voting behavior, the politicoeconomic channel reinforces the positive role of productivity for growth. Higher productivity gives the opportunity to voters to choose optimally lower taxes and achieve higher growth.

Our findings are in accordance with the conclusions of Hall and Jones (1999) for the link between government policies, growth and long-run differences in cross country per-capita incomes. Yet, our results do not suggest that reducing marginal income taxes exogenously leads to necessarily large improvements in the growth performance. It is, after all, institutions that creat the ability for private markets to compete with the provisions of public goods. Democracy and the pure ability of private markets to operate and compete with the public-goods provision mechanism are the ingredients of our model. Since in our model endogenous policies amplify growth differences that stem from economic fundamentals, such as preference and technology parameters, we provide an explanation for the large cross-country development differences.

We go further, by examining public policies in a stochastic environment. Interestingly, we find that if business cycles are driven by productivity shocks in the endogenous growth framework, equilibrium policies imply that taxes should fall in high growth periods and rise in low-growth periods. Exploiting growth opportunities further by decreasing taxes in periods of booms goes against the usual idea of stabilizing countercyclical policies. Moreover, the public consumption share of GDP is countercyclical in our model, also in accordance with evidence for OECD countries (only for the public consumption component of the fiscal budget as a share of GDP) reported by Hercowitz and Strawczynski (2002).

At the same time, we find that the level of public consumption is procyclical. Several empirical studies find evidence for procyclical public expenditures in general, in Latin American countries (see, for example, Gavin et. al. (1996), Gavin and Perotti (1997), Stein et. al. (1999)), and also in other economies, including many OECD countries (see Talvi and Vegh (2000) and Lane (2003)).

We stress, however, that our study is not a public finance exercise. The policy problem posed is not the choice of public finance instruments over time for an exogenously given stream of public goods. The problem we study is the voters' choice of the public goods level over time, given that the policy instrument is a marginal income tax satisfying a balanced fiscal budget. An interesting finding of Lane (2003) is that countries with more dispersed political power (a proxy for strong democratic institutions), are more likely to exhibit procyclical fiscal policies. This is in accordance with our model, which expresses a strong democratic represantation of voters' political preferences in policy making.

Related studies focusing, as we do, on time-consistent policies and introducing an externality in the form of a public consumption good, also appearing in additively-separable utility functions, are Xie (1997) and Karp and Ho Lee (2003). Xie (1997) discusses the technical issue of when optimal policies with commitment at time 0 (open-loop policies) may be time-consistent. Karp and Ho Lee (2003) examine the public finance question of setting optimal marginal taxes for a given exogenous stream of public goods, showing the way of constructing models exhibiting time-consistent open-loop policies.

In contrast to these studies, we do not discuss the technical issue of time consistency. Using our framework, we focus on studying the economic background of the interlinkages between fiscal policies and growth, when the one depends upon the other. In our model, governments are elected, or supported, by voters who are aware of the potential negative impact of taxes. These voters internalize the deadweight losses of taxation and electoral process in the most fundamental way, that is, they make perfect-foresight voting decisions.

The concept of perfect foresight voting was first proposed by Denzau and Mackay (1981) and was further articulated by Epple and Kadane (1990) in a spatial model. Simulation-based studies with perfect-foresight voting models are Krusell et. al. (1997) and Krusell and Rios-Rull (1999). Moreover, Klein et. al. (2003) suggest a generalized method for simulating models of time-consistent public-goods provision with perfect foresight.

As all the above literatures are quite specialized and rather technical, our study aims at pointing out intuitive results and raising null hypotheses. The generality of our conclusions may be an open question, calling for further theoretical or empirical investigation. Nevertheless, we believe that the observations we make in this paper capture key mechanisms through which fiscal policies depend on the fundamentals of economic environments that exhibit endogenous sustainable growth.

The plan of the paper is as follows. In section 2 we present the deterministic economy, its politico-economic equilibrium and the link between economic fundamentals and growth paths of economies with endogenous policies. In section 3 we extend our model to having stochastic productivity and we comment on the cyclical behavior of taxes.

2. The Deterministic Economy

The benchmark of our analysis is a deterministic economic environment in which a reproducible and partially storable capital good is accumulated. This capital good may be a composite of various types of physical and/or human capital. For the purpose of tractability, we focus on the dynamics of this single capital variable.

Time is discrete and the time horizon is infinite, $t = 0, 1, ...$ Households comprise a set $\mathcal{I}$ and may differ only with respect to their initial endowment of capital claims (assets). We denote assets for household $i \in \mathcal{I}$ at time 0 as $a_{i,0}$.[4] At any point in time, the aggregate capital level is given by,

$$\mathbf{k}_t = \int_{\mathcal{I}} a_{i,t}\mu_t(i)\, di \ , \quad t = 0, 1, ... \tag{1}$$

where $\mu_t(i)$ is the measure of individuals of type i at time t. We denote all aggregate variables by bold characters in order to distinguish them from variables pertaining to individual agents. We also assume that $\mathbf{k}_0 > 0$. All households are infinitely-lived. There is a single private consumable good produced through a linear aggregate production function,

$$\mathbf{y}_t = A\mathbf{k}_t \ , \tag{2}$$

with $A > 0$. The marginal return to capital is,

$$R_t = A \quad \text{and} \quad r_t = A - \delta \ , \tag{3}$$

where δ is the depreciation rate of capital, and r_t is the interest rate, which is constant over time.

A government, elected by the households each period, collects taxes in order to provide a single composite public good (parks, hospitals, theaters, art schools, defense, etc.). Nothing

[4] The set of households can be countable, finite, or a continuum. It can also be that all households have the same initial endowment (representative-agent economy), but in all cases there is a "large" number of households, making each of them having negligible impact on the aggregate economy.

is returned to the individuals in the form of a direct monetary transfer. The constitution allows only one type of taxation, the use of a common flat marginal tax on personal income, denoted by τ_t every period. Moreover, the fiscal budget should be balanced every period, so the government revenues (and expenditures) are given by,

$$\mathbf{g}_t = \tau_t (A - \delta) \mathbf{k}_t \ , \tag{4}$$

i.e., capital depreciation is tax-exempt. In order to obtain closed-form solutions, we assume log preferences. In particular, the momentary utility function for all households is,

$$u(c_t, \mathbf{g}_t) = \chi \ln(c_t) + (1 - \chi) \ln(\mathbf{g}_t) \ , \tag{5}$$

with $\chi \in (0, 1]$.[5] The assumption of additive separability and homotheticity of preferences over private and public consumption is not extreme. Amano and Wirjanto (1998) estimate a utility function of the average American household from aggregate data with constant intertemporal and intratemporal elasticity of substitution. Their regressions are based on Euler equations that stem from a permanent-income model with lump-sum taxes.[6] They find that private and public consumption are unrelated in the Edgeworth-Pareto sense (they are neither substitutes nor complements). They also find that the elasticities of intertemporal and intratemporal substitution are both about 1.56. The latter means that the assumption of natural-log preferences (unitary elasticities of intertemporal and intratemporal substitution) is not an extreme deviation from their findings.

[5] Superscripts i are dropped throughout the text unless necessary.

[6] Even though the permanent-income Euler equations of Amano and Wirjanto (1998) differ from the politico-economic equilibrium conditions of this setup, their findings should uncover similar estimates for the intra- and inter-temporal elasticities of substitution with these implied by a model distorted by marginal taxes.

2.1 Competitive equilibrium under any stream of political outcomes

We assume, for the moment, that a stream of taxes and aggregate capital, $\{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty}$, is pre-determined exogenously. Chatterjee (1994) proves that, for a general class of neoclassical growth setups, households need to know *only* the future stream of aggregate capital levels and not the future distributions of physical capital claims in order to calculate their optimal path of savings accurately.[7] The argument of Chatterjee (1994) is valid for the linear-growth model that we examine. Although it is not necessary that there be a *representative agent* in our framework (the case where all households also possess the same initial wealth), due to the fact that preferences are of the "Gorman form," enabling linear demand aggregation, our setup leads to the presence of a *representative consumer*.[8]

The problem of an individual household i, is described by,

Household Problem

$$\max_{\{(c_t,a_{t+1})\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \left[\chi \ln(c_t) + (1-\chi)\ln(\mathbf{g}_t)\right] ,$$

subject to:

$$a_{t+1} = [1 + r_t (1-\tau_t)]\, a_t - c_t \ , \tag{6}$$

(3) and (4),

given a_0, $\{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty}$.

[7] Using Chatterjee's (1994) argument, Krusell and Rios-Rull (1999) prove the same result in an economy with proportional taxation and lump-sum transfers. Interestingly, Krusell and Smith (1997) find that in neoclassical models modified in ways such that Chatterjee's (1994) theoretical argument fails, at a numerical level, households calculating distribution moments that capture future individual asset distributions, can very well approximate their optimal path by relying only on the future stream of first moments, namely on the future sequence of aggregate capital. Similar results to Chatterjee (1994) are pointed out by Caselli and Ventura (2000) in more general continuous-time frameworks.

[8] A representative consumer is a fictitious consumer whose demand functions coincide with the corresponding aggregate demand functions. Thus, Chatterjee (1994) proves the prevalence of the representative consumer for dynamic decision rules and aggregate laws of motion in a large class of optimal growth economies.

Here $\beta \in (0,1)$ and the interest rate is given by (3). In equilibrium, $\{\mathbf{k}_t\}_{t=0}^{\infty}$ should conform to every household's solution to the same problem and to market-clearing conditions. Individuals have negligible economic weight and they assume that their personal decisions do not have any impact on aggregate variables and prices. For this reason, the optimality conditions of a household i are given by,

$$\frac{c_{t+1}}{c_t} = \beta \left[1 + (A - \delta)(1 - \tau_{t+1})\right] \ , \tag{7}$$

equation (6), and the transversality condition $\lim\limits_{t\to\infty} \frac{a_{t+1}}{\prod_{s=0}^{t}[1+(A-\delta)(1-\tau_s)]} = 0$. We obtain a closed-form solution for this problem, given by the following Proposition.

Proposition 1 *The competitive equilibrium decision rules for the* ***Household Problem*** *of any individual household i, are given by,*

$$a_{t+1} = \beta \left[1 + (A - \delta)(1 - \tau_t)\right] a_t \ , \tag{8}$$

and,

$$c_t = (1 - \beta)\left[1 + (A - \delta)(1 - \tau_t)\right] a_t \ . \tag{9}$$

The aggregate-economy law of motion for capital is,

$$\mathbf{k}_{t+1} = \beta \left[1 + (A - \delta)(1 - \tau_t)\right] \mathbf{k}_t \ . \tag{10}$$

Proof. See the Appendix.

The asset holdings of all households grow at the same rate in equilibrium. Thus, as it is the case for the class of models with homothetic preferences in Chatterjee (1994), the *relative household wealth distribution* is invariant over time.

We stress the particular nature of the competitive equilibrium of this version of the linear ("AK") growth model with log-preferences. In the linear growth model with a non-unitary elasticity of intertemporal substitution for consumption, any change in future policy plans invokes a change in the current decisions of households. For example, two future policy plans at time t, $\{\tau_s\}_{s=t}^{\infty}$ and $\left(\{\tau_s\}_{s=t}^{t+n}, \widetilde{\tau}_{t+n+1}, \{\tau_s\}_{s=t+n+2}^{\infty}\right)$, with $n \geq 0$ and $\widetilde{\tau}_{t+n+1} \neq \tau_{t+n+1}$, lead to different household policies at time t, i.e. $\widetilde{a}_{t+1} \neq a_{t+1}$ and $\widetilde{c}_t \neq c_t$, if the elasticity of intertemporal substitution is not unitary. In our model, the case of unitary elasticity of intertemporal substitution, equations (8) and (9) imply that this is not the case, i.e. consumption policies at time t are exactly the same for different continuations of policy streams.

We emphasize that it is neither that households are not forward-looking, nor that one cannot study the impact of anticipated policy changes through this version of the model. Households take account of future changes in policy, but they choose to make the same decisions, as if the future policy change was not there, until they reach the period of occurrence of the change. When the economy reaches the period in which policies differ, the decision rules change, and so does the path that the economy follows. The value function of two identical economies facing, ceteris paribus, different policy plans, is therefore different since period 0. Despite the different value functions since period 0, the decision rules, up to the occurrence of the policy change, are the same in the two economies, because households do not gain extra utility from "consumption smoothing" if their elasticity of intertemporal substitution is unitary and production technology is linear.

In our model, the influence of the continuation of a tax plan, $\{\tau_s\}_{s=t+1}^{\infty}$, on decision rules is not discarded. The influence of future taxes is "fixed" into invariant parameters of the decision rules, and they are independent from the future tax plan as well as the levels of the state variables. These invariant parameters of the decision rules contain non-trivial information about the responsiveness of households to anticipated future policies. In other words, the effects of future taxes are expressed in terms of effects of the current tax rate, the parameters reflect how these effects are "enfolded" and redirected back to the current tax.

Because of its simplicity, our model is a good vehicle for studying the most important determinants of forward-looking voting behavior. As we show, our model eliminates complexities related to strategic interactions among voters for solving for politicoeconomic Markov-perfect equilibrium.

2.2 Politicoeconomic equilibrium

Since the constitution restricts governments to having a balanced budget, the only voting issue each period is the current level of the income tax rate. Majority voting over taxes takes place at the beginning of every period, i.e. *before* households and producers make their economic decisions.

Households vote simultaneously, before they act, again simultaneously, in the markets in order to make their economic decisions given the current period's electoral outcome. So, while each household decides, independently, about its best voting strategy, it takes account of the full impact of its voting action on the current electoral outcome and how this electoral outcome will affect its own current and future economic decisions, its own future voting decisions, and the current and future economic and voting decisions of other households in politicoeconomic equilibrium over the infinite horizon.

The timing of actions mimics the fact that, in most presidential and parliamentary democracies, in the beginning of each period, a government pre-announces a certain fiscal budget and it precommits to it. This fits the specification of Cohen and Michel (1988), about the time-consistent making of fiscal policies.[9] Our politico-economic equilibrium concept can be linked observationally to the real world in two ways. Either, (i) elections are held every four years (so, the duration of one period in our model is four years) and elected governments commit to their pre-announced policies for four years, or, (ii) independently from whether

[9] In particular, Cohen and Michel (1988), in the context of their continuous-time framework, call this equilibrium concept "feedback Stackelberg equilibrium with instantaneous precommitment."

there are elections or not, common voter preferences are reflected into the pre-announced fiscal budget in the beginning of each fiscal year.

The solutions given by equations (8), (9) and (10) enable us to calculate the analytical form of the value function for a household $i \in \mathcal{I}$. The value function at time $t \in \{0, 1, ...\}$ is given by,

$$V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) = \kappa_D + \frac{\ln\left(a_{i,t}\right)}{1-\beta} + \frac{\ln\left(\mathbf{k}_t\right)}{1-\beta} + \\ + \sum_{s=0}^{\infty} \beta^s \left\{ \frac{\ln\left[1 + (A-\delta)\left(1-\tau_{t+s}\right)\right]}{1-\beta} + (1-\chi)\ln\left(\tau_{t+s}\right) \right\} , \qquad (11)$$

where κ_D is a constant.[10] The optimal tax rate of a household at time t is derived from setting the partial derivative of the value function with respect to τ_t equal to zero, yielding,

$$\tau_t = \tau^D = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1 + \frac{1}{A-\delta}\right) , \quad t = 0, 1, ... \qquad (12)$$

Note that all households agree upon τ^D at all times.[11] Unanimity across households comes from the fact that the term $\frac{\ln(a_{i,t})}{1-\beta}$ in the value function of household i and the current and future policies $\{\tau_s\}_{s=t}^{\infty}$ are additively separable. This feature also leads to the conclusion that the tax rate, τ^D, given by (12), coincides with the solution of a benevolent social planner who chooses period-by-period second-best policies, given that the constitution is restricted to the use of marginal taxes and a balanced fiscal budget in each period. The social planner's objective at time $t \in \{0, 1, ...\}$ is given by $\int_{\mathcal{I}} V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) \omega(i)\, \mu_t(i)\, di$, for any set of weights $\omega(i) \geq 0$ on household i's utility. Equation (11) implies that τ^D is the second-best policy for the social planner as well.

[10]In particular,

$$\kappa_D = \chi \frac{\ln(1-\beta)}{1-\beta} + \frac{\beta \ln(\beta)}{(1-\beta)^2} + (1-\chi)\frac{\ln(A-\delta)}{1-\beta}.$$

[11]The symbol "τ^D" represents the winning tax rate in the deterministic economy.

2.3 Economic fundamentals, public policy and long-run growth

Since the equilibrium tax rate in the deterministic political economy is constant, the equilibrium growth rate is also constant.[12] In particular, from equation (8), after substituting the tax rate (12), the growth rate of all economic variables in this economy is,

$$\gamma^D = \frac{\beta(1+A-\delta)}{1+(1-\beta)(1-\chi)} - 1 \ . \tag{13}$$

The endogenous tax rate given by equation (12) has many intuitive features. If the relative weight of utility derived from the consumption of public goods is higher (lower χ), then τ^D is higher, and therefore growth is lower. More patient households (higher β) choose a lower ratio of current public goods to future private consumption (by choosing a lower τ^D), because they can compensate for relatively less public goods today, by achieving a higher growth rate. In the case of higher β, this happens because households are fundamentally more willing to save. A higher capital depreciation rate, δ, leads to a lower interest rate and less incentive to save, so voters choose a higher τ^D in order to derive more utility from a higher public-to-private goods ratio, leading to lower growth.

One of the most important economic fundamentals for explaining cross-country differences in living standards and growth rates is cross-country productivity differences. Hall and Jones (1999) stress that physical and human capital accumulation alone are inadequate to explain the biggest part of cross-country differences in per-capita income. They find that large productivity differences across countries explain most of cross-country differences in development.

In the context of our endogenous growth model, if we treat capital as a composite good, consisting of physical and human capital, differences in the productivity parameter, A, are

[12] So, since period 0 the political economy is in a steady state, there are no transitional dynamics leading to it.

crucial for growth because they also influence the level of marginal taxes. This can be seen by equation (12). When A is higher, voters internalize the growth opportunities generated by a higher capital productivity, and vote for a lower τ^D.

For example, consider a world of economies in which policies are exogenous. Suppose that the marginal tax rate in this world is constant over time, and let us denote it as τ^{exog}. A rise in the productivity parameter, A, causes growth, γ^{exog}, to increase by the gradient,

$$\frac{\partial \gamma^{exog}}{\partial A} = \beta \left(1 - \tau^{exog}\right) \ , \tag{14}$$

as it can be verified by equation (8). The gradient given by (14) determines the link between cross-country productivity differences and cross-country growth differences. Now, consider a world of political economies, where policies are endogenous in each country. A rise in the productivity parameter, A, causes growth to increase by the gradient,

$$\frac{\partial \gamma^D}{\partial A} = \beta \left(1 - \tau^D\right) - \frac{\partial \tau^D}{\partial A} \ ,$$

so, using (12) it is,

$$\frac{\partial \gamma^D}{\partial A} = \beta \left(1 - \tau^D\right) + \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)} \frac{1}{(A-\delta)^2} \ . \tag{15}$$

If we pick two countries, one from each of the two different fictitious worlds, both characterized by the same productivity level, A, *and* such that taxes are at the same level, namely $\tau^{exog} = \tau^D$, equations (14) and (15) imply that,

$$\frac{\partial \gamma^D}{\partial A} > \frac{\partial \gamma^{exog}}{\partial A} \ .$$

Thus, the politicoeconomic channel generates higher cross-country development differences that arise from structural cross-country differences in productivity.

This result is driven by the negative dependence of taxes on productivity, A. In politicoeconomic equilibrium, marginal income taxes create a strong substitution effect, namely, voters are willing to substitute current public consumption with more future private consumption. This substitution effect is driven by the fact that it is not the stock of capital that is taxed over time, but only the capital income flow. The balanced fiscal budget ties, directly, the quantity of public consumption to the *taxed* capital income flow. But the prices of future private consumption depend (negatively) also on the whole stock of capital, not only on its capital income flow. A higher productivity, A, gives better opportunities for capital accumulation and, in turn, for a sharper decline in the prices of private consumption over time.

In other words, the tax-driven mechanism of public goods provision is dominated by the competitive price mechanism of private goods provision. Due to the presence of endogenous growth in the model, and the critical dependence of growth on productivity, the higher the productivity, A, the higher the dominance of the competitive price mechanism over the public-goods provision mechanism. This is why the deadweight loss of taxation is higher when production possibilities and growth opportunities are higher, and voters perceive this. Higher productivity gives the opportunity to voters to choose optimally lower taxes and achieve higher growth.

How sensitive the results of our analysis are to different parameter values for the elasticities of intra- and inter-temporal substitution that make economic and voting decision rules perplexedly responsive to future policy paths, is an open question for quantitative investigation. We conjecture that the most important determinants of the behavior of forward-looking voters in linear-growth frameworks are captured by this version of the model and there should not be significant differences in our main qualitative results with a different elasticity of intertemporal substitution. Nevertheless, it is useful to use this particular paradigm as a stimulus for interesting quantitative questions of positive explanations of the size of government and tax distortions.

3. The Stochastic Economy

The cyclical movement of fiscal policies is an open research topic, both theoretically and empirically. The traditional Keynesian view suggests that policies should be countercyclical, namely government spending and taxes should decrease in recessions and increase during expansions. Barro (1979) suggests tax smoothing over the business cycle, so if the fiscal budget is not restricted to be balanced, the public surplus should be procyclical.

Several empirical studies find evidence for procyclical fiscal policies. Gavin et. al. (1996), Gavin and Perotti (1997), Stein et. al. (1999) find that both taxes and government expenditures in Latin American countries are procyclical. Talvi and Vegh (2000) and Lane (2003) report procyclical policies also for rich countries, and also for public consumption, the focus of this paper. Hercowitz and Strawczynski (2002) study the cyclical behavior of GDP shares of several components of government spending in OECD countries and test for possible cyclical asymmetries of the fiscal budget in periods of expansion, versus periods of contraction. Although public consumption is procyclical in the OECD (see Lane (2003)), Hercowitz and Strawczynski (2002) find that its GDP share is generally countercyclical over the cycle.

Lane (2003) claims that the fiscal cyclicality implications of endogenized public consumption in neoclasssical frameworks rest upon the specification of the utility function with respect to private versus public consumption (see p. 2664). In particular, Lane (2003) stresses that if public and private goods are separable in utility and also neither substitutes nor complements (like they are in our model), then endogenous public consumption in a neoclassical framework should be smooth over the business cycle.[13]

[13] Thus, in order to address the empirically observed procyclicality of government components, political economy models of the "voracity effect" have been suggested by Lane and Tornell (1996) and (1998) and Tornell and Lane (1998) and (1999).

In this section, we show that, in a stochastic endogenous growth environment (as opposed to an exogenous long-run growth neoclassical environment), endogenizing public consumption through perfect-foresight voting reconciliates theory with the empirical observation of public-consumption procyclicality and the countercyclicality of the public consumption GDP share. Moreover, we examine the effect of higher capital productivity volatility on average taxes.

We extend the deterministic model of the previous section to the case in which the capital productivity is random. The model is the same as in the deterministic case, with the sole difference that aggregate production is now given by,

$$\mathbf{y}_t = A_t \mathbf{k}_t \ , \tag{16}$$

where the capital productivity shock is serially uncorrelated over time, with distribution,

$$\ln(A_t) \sim N\left(\mu - \frac{\sigma^2}{2} \ , \ \sigma^2\right) \ ,$$

where $\mu > 0$ and $\sigma \geq 0$. The special case $\sigma = 0$, coincides with the model of the previous section. Parametrizing the distribution of the capital-productivity shock in this way, allows us to compare the impact of volatility on decision rules and election outcomes. In particular,

$$E(A_t) = e^{\mu} \ ,$$

while

$$Var(A_t) = e^{2\mu}\left(e^{\sigma^2} - 1\right) \ ,$$

i.e. if two countries differ only with respect to parameter σ, they have the same average capital productivity but different capital-productivity variances.

3.1 Competitive equilibrium in the stochastic economy

The interest rate in this stochastic model is,

$$r_t = A_t - \delta \ , \tag{17}$$

and the fiscal budget now is,

$$\mathbf{g}_t = \tau_t \left(A_t - \delta\right) \mathbf{k}_t \ . \tag{18}$$

The problem of an individual household i, is described by the following constrained problem,

Household Problem in the Stochastic Economy

$$\max_{\{(c_t, a_{t+1})\}_{t=0}^{\infty}} \quad E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\chi \ln\left(c_t\right) + \left(1-\chi\right) \ln\left(\mathbf{g}_t\right) \right] \right\} \ ,$$

subject to:

$$a_{t+1} = \left[1 + r_t \left(1 - \tau_t\right)\right] a_t - c_t \ , \tag{19}$$

$$\ln\left(A_t\right) \sim N\left(\mu - \frac{\sigma^2}{2} \ , \ \sigma^2\right) \ ,$$

equations (17) and (18),

given a_0, $\{(\tau_t, \mathbf{k}_t)\}_{t=0}^{\infty}$.

In competitive equilibrium, $\{\mathbf{k}_t\}_{t=0}^{\infty}$ should conform to every household's solution to the same problem and to market-clearing conditions. The necessary conditions of household i are given by,

$$\frac{1}{c_t} = \beta E_t \left[\frac{1 + \left(A_{t+1} - \delta\right)\left(1 - \tau_{t+1}\right)}{c_{t+1}} \right] \ , \tag{20}$$

equation (19), and the transversality condition $\lim_{t\to\infty} E_t \left\{ \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A_s-\delta)(1-\tau_s)]} \right\} = 0$. It turns out that we get the same solution as in the deterministic economy,

$$a_{t+1} = \beta \left[1 + \left(A_t - \delta\right)\left(1 - \tau_t\right)\right] a_t \ , \tag{21}$$

and,

$$c_t = (1-\beta)\left[1 + (A_t - \delta)(1-\tau_t)\right] a_t \ , \tag{22}$$

satisfy both necessary (and sufficient) conditions (20) and (19), and the trasversality condition.

3.2 Politicoeconomic equilibrium in the stochastic economy

As in the deterministic case, the only voting issue each period is the current level of the income tax rate. In the beginning of each period, the shock A_t is revealed, then majority voting over taxes follows, and afterwards households and producers make their economic decisions, given the electoral outcome. Each household decides, independently, about its best voting strategy, taking into account the full impact of its voting action on the current electoral outcome. Moreover, given the probability distribution of future shocks, the household also considers how the electoral outcome affects its own current and probable future economic decisions, its own probable future voting decisions, and the current and probable future economic and voting decisions of others in politicoeconomic equilibrium over the infinite horizon.

Given the analytical solutions in equations (21) and (22), the form of the value function for a household $i \in \mathcal{I}$ at time $t \in \{0, 1, ...\}$ is,

$$\begin{aligned} V\left(a_{i,t}, \mathbf{k}_t, \{\tau_s\}_{s=t}^{\infty}\right) = \kappa_S + \frac{\ln(a_{i,t})}{1-\beta} + \frac{\ln(\mathbf{k}_t)}{1-\beta} + \\ + \frac{\ln\left[1 + (A_t - \delta)(1-\tau_t)\right]}{1-\beta} + (1-\chi)\ln(\tau_t) + \\ + E_t\left\{\sum_{s=1}^{\infty} \beta^s \left\{\frac{\ln\left[1 + (A_{t+s} - \delta)(1-\tau_{t+s})\right]}{1-\beta} + (1-\chi)\ln(\tau_{t+s})\right\}\right\} \ , \end{aligned} \tag{23}$$

where κ_S is a constant.[14] Setting the partial derivative of the value function, with respect to τ_t, equal to zero yields the preferred tax rate for the single household, namely,

$$\tau_t^S = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1+\frac{1}{A_t-\delta}\right) , \quad t = 0, 1, ... \tag{24}$$

Again, there is unanimity about τ_t^S, the winning tax rate in the stochastic economy, at all times. The rationale behind unanimity across voters is the same as the one presented in the deterministic case. Moreover, following the argument made for the deterministic model, the tax policy given by (24) is also the second-best policy of a benevolent utilitarian social planner.

From equation (24) it is clear that taxes are *countercyclical*. When productivity, A_t, is high (low), households vote in favor of an decrease (increase) of the current-pulic to future-private consumption ratio, by reducing (increasing) the chosen tax rate.

In the stochastic endogenous growth model, marginal income taxes create, again, a strong substitution effect between current public and future private consumption. While current and future consumption is provided by private markets, the public-goods provision mechanism is taxes. Accumulating more capital is the key to better opportunities for having more of both private and public goods in the future. As it is revealed by the gross effective interest rate, $1+(A_t-\delta)(1-\tau_t)$, only income and not the stock of capital is taxed over time. Thus, it is the substitution effect that leads to choosing lower taxes when productivity is higher. In other words, the politicoeconomic equilibrium in the stochastic endogenous growth model implies that growth opportunities are exploited by decreasing taxes in periods of booms (and not by increasing taxes, as is suggested by a countercyclical policy).

[14] In particular,

$$\kappa_S = \chi\frac{\ln(1-\beta)}{1-\beta} + \frac{\beta\ln(\beta)}{(1-\beta)^2} + (1-\chi)\frac{E_t[\ln(A_{t+1}-\delta)]}{1-\beta} .$$

Substituting equation (24) into (18), yields the politocoeconomic equilibrium value for public consumption, namely,

$$\mathbf{g}_t = \frac{(1-\beta)(1-\chi)(1+A_t-\delta)}{1+(1-\beta)(1-\chi)}\mathbf{k}_t \ , \tag{25}$$

which implies that *government consumtion is procyclical* in equilibrium. Moreover, combining (25) with (16) yields,

$$\frac{\mathbf{g}_t}{\mathbf{y}_t} = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left(1+\frac{1-\delta}{A_t}\right) \ , \tag{26}$$

which means that *the GDP share of government consumption is countercyclical* in equilibrium. Both implications of our model are consistent with the empirical evidence discussed in the beginning of this section.

3.3 Volatility, average taxes and average growth in the political economy

Several papers, such as Ramey and Ramey (1995), Aizenman and Marion (1999), and Hnatkovska and Loayza (2003), study the link between economic volatility and long-run growth. They find that there is a negative correlation between volatility and average growth. Since there is not much theoretical background addressing these empirical findings, we examine whether volatility has an impact on average taxes and, through the channel of taxes, we also examine whether average growth is affected.[15]

Recall that the distribution of the productivity shock, A_t, is modeled so that a change in parameter σ has no effect on the average shock, $E(A_t)$, whereas volatility, $Var(A_t)$, does change. Does a different structural volatility, σ, of the productivity shock A_t, change

[15] Aizenman and Marion (1999), apart from reporting a statistically significant negative link between innovation volatility and private investment GDP shares in developing countries, they suggest some theoretical explanations for their findings. Their explanations rely upon risk aversion and non-linear budget constraints.

average taxes and average growth in politicoeconomic equilibrium? We show the dependence of average taxes on volatility in the proposition that follows.

Proposition 2 *If the volatility of capital productivity is higher, then the average tax rate is higher.*

Proof. From equation (24),

$$E(\tau_t) = \frac{(1-\beta)(1-\chi)}{1+(1-\beta)(1-\chi)}\left[1+E\left(\frac{1}{A_t-\delta}\right)\right] . \tag{27}$$

We find the dependence of $E\left(\frac{1}{A_t-\delta}\right)$ on parameter σ. Since A_t is lognormal, $(A_t-\delta)$ is also lognormal. Let parameters $\hat{\mu}$ and $\hat{\sigma}$ be defined as,

$$\ln(A_t-\delta) \sim N\left(\hat{\mu}\ ,\ \hat{\sigma}^2\right) .$$

Noticing that,

$$E(A_t-\delta) = e^{\mu}-\delta\ ,$$

$$Var(A_t-\delta) = e^{2\mu}\left(e^{\sigma^2}-1\right)\ ,$$

$$E(A_t-\delta) = e^{\hat{\mu}+\frac{1}{2}\hat{\sigma}^2}\ ,$$

and

$$Var(A_t-\delta) = e^{2\hat{\mu}+\hat{\sigma}^2}\left(e^{\hat{\sigma}^2}-1\right)\ ,$$

we express parameters $\hat{\mu}$ and $\hat{\sigma}$ as functions of μ, σ and δ. In particular,

$$e^{\hat{\sigma}^2} = \left(\frac{e^{\mu}}{e^{\mu}-\delta}\right)^2\left(e^{\sigma^2}-1\right)+1\ , \tag{28}$$

whereas,

$$e^{\hat{\mu}} = (e^{\mu}-\delta)\left[\left(\frac{e^{\mu}}{e^{\mu}-\delta}\right)^2\left(e^{\sigma^2}-1\right)+1\right]^{-\frac{1}{2}} . \tag{29}$$

Thus,

$$-\ln\left(A_t-\delta\right)\sim N\left(-\hat{\mu}\ ,\ \hat{\sigma}^2\right)\ ,$$

and,

$$E\left(\frac{1}{A_t-\delta}\right)=e^{-\hat{\mu}+\frac{1}{2}\hat{\sigma}^2}\ ,$$

so, using (28) and (29),

$$E\left(\frac{1}{A_t-\delta}\right)=\frac{1}{e^{\mu}-\delta}\left[\left(\frac{e^{\mu}}{e^{\mu}-\delta}\right)^2\left(e^{\sigma^2}-1\right)+1\right]\ .$$

This last equation combined with (27) implies that,

$$\frac{\partial E\left(\tau_t\right)}{\partial\sigma}>0\ ,$$

proving the proposition. □

Proposition 2 shows that cross-country differences in the volatility of capital productivity are linked to differences in average taxes. However, in politicoeconomic equilibrium, there is no link between volatility and average growth. Combining (21) and (24), we see that the growth rate of capital is,

$$\gamma_t^S=\frac{\beta\left(1+A_t-\delta\right)}{1+\left(1-\beta\right)\left(1-\chi\right)}-1\ . \tag{30}$$

Thus,

$$\frac{\partial E\left(\gamma_t^S\right)}{\partial\sigma}=\frac{\beta\frac{\partial E\left(A_t-\delta\right)}{\partial\sigma}}{1+\left(1-\beta\right)\left(1-\chi\right)}=0\ . \tag{31}$$

The result expressed by equation (31) comes from the fact that in equilibrium, after combining (25) with (22), the ratio $\frac{\mathbf{g}_t}{\mathbf{c}_t}$ is constant, as,

$$\frac{\mathbf{g}_t}{\mathbf{c}_t}=1-\chi\ .$$

The fluctuation over the cycle is between savings and the two types of consumption in equilibrium. For this reason, the volatility of A_t does not affect average private and public consumption, or their average share. Moreover, the fact that the volatility of A_t does affect average taxes comes from the fact that capital depreciation is tax-exempt.

The empirical studies such as Ramey and Ramey (1995), Aizenman and Marion (1999), and Hnatkovska and Loayza (2003) assert that after controling for several country-specific characteristics that may capture the cross-country variation in average productivity, $E(A_t)$, then, $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$ should be negatively correlated. Our specification of the stochastic structure of productivity, A_t, mimics this claim in the regression specification of the above empirical studies.

Our theoretical findings do not support a negative correlation between $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$, at least through the channel of taxation for the provision of public consumption. An open question is whether assuming different intratemporal and intertemporal elasticities of substitution between private and public consumption would lead to a link between volatility and growth through the politico-economic channel. Yet, another open question is whether cross country regression analysis can control for the cross-country variation in average productivity, $E(A_t)$. If not, then it is possible that the reported negative correlation between $Var\left(\gamma_t^S\right)$ and $E\left(\gamma_t^S\right)$ comes from cross-country variations in average productivity, $E(A_t)$, or, in the language of our model, from cross-country variations in parameter μ. In the last case, our model's implication about the absence of a link between volatility and growth is not necessarily erroneous.

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Appendix

Proof of proposition 1. The optimality condition (7) can be written as,

$$c_{t+1} = \beta \frac{\lambda_t}{\lambda_{t+1}} c_t \ , \quad t = 0, 1, ..., \tag{A1}$$

where λ_t is the shadow price of the final good with $\lambda_0 = 1$ (the numeraire) and:

$$\frac{\lambda_{t-1}}{\lambda_t} = 1 + (1 - \tau_t)(A - \delta) \ , \quad t = 0, 1, ..., \tag{A2}$$

with $\lambda_{-1} = 1 + (1 - \tau_0)(A - \delta)$, by default. The household budget constraint combined with (A2) becomes,

$$c_t + a_{t+1} = \frac{\lambda_{t-1}}{\lambda_t} a_t \quad \text{or} \quad \lambda_t c_t = \lambda_{t-1} a_t - \lambda_t a_{t+1} \ . \tag{A3}$$

Considering equation (A3) one period ahead, using (A1), and defining,

$$z_t \equiv \lambda_{t-1} a_t \ , \quad t = 0, 1, ...,$$

yields,

$$z_{t+2} - (1 + \beta) z_{t+1} + \beta z_t = 0 \ . \tag{A4}$$

Equation (A4) is an elementary second-order linear difference equation with two obvious solutions, $z_{t+1} = \beta z_t$ and $z_{t+1} = z_t$. The second solution is ruled out since it implies that $\lambda_t a_{t+1} = \lambda_{t-1} a_t$, so from equation (A3), it must be $c_t = 0$ in all periods, which cannot be optimal. Alternatively, $z_{t+1} = z_t$ implies that $\lambda_t a_{t+1} = \lambda_{-1} a_0$, or $\frac{\lambda_t a_{t+1}}{\lambda_{-1}} = a_0$, so from equation (A2) it must be that, $0 < a_0 = \frac{\lambda_t a_{t+1}}{\lambda_{-1}} = \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A-\delta)(1-\tau_s)]} = \lim_{t \to \infty} \frac{a_{t+1}}{\prod_{s=0}^{t} [1+(A-\delta)(1-\tau_s)]}$, i.e. a violation of the transversality condition. Therefore, only the first solution is applicable, $a_{t+1} = \beta \frac{\lambda_{t-1}}{\lambda_t} a_t$, and using equation (A2), equation (8) is proved. Equation (9) comes from combining (8) and the household budget constraint. Equation (10) comes from (8) through linear aggregation. Note that (A4) would not be linear if preferences were not "log." □