



# TEACHER EDUCATOR MODULE 1: Challenging Students While Addressing Different Needs: An Introduction

## EDUCATE Project



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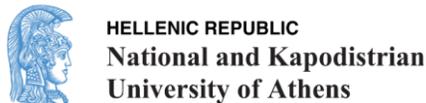
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# CASE OF PRACTICE 1

## Focusing on Mathematically Challenging Tasks

### Expected outcomes

By the end of this Case of Practice, teachers will be able to:

- Explain what mathematically challenging tasks are (LO1<sup>2</sup>);
- Identify aspects that render a mathematical task challenging (LO1);
- Describe how the opportunities to engage students in mathematically challenging work can be modified during task presentation and implementation (LO2);
- Discuss how task unfolding can offer different learning opportunities for students (LO2);

### Short Description of the Activities and How They Can Be Enacted

#### Opening Activity

 **Indicative Duration:** 40 minutes

The opening activity consists of two parts. In the first part, there is a brainstorming activity which aims to help the teacher participants generate ideas about factors that could affect students' mathematical thinking and reasoning. In the second part, teachers will be exposed to '*The Mathematics Task Framework*' (MTF, Stein *et al.*, 2000) as a tool that can help them better classify these factors and through that make more deliberate and informed decisions about the opportunities they craft for their students' thinking and reasoning.

#### 1) Part 1: Brainstorming Activity

 **Indicative Duration:** 20 minutes

In this activity, participants will be asked to cite examples of their teaching experiences or other factors that help or hinder creating a productive space for engaging students in mathematical thinking and reasoning. The videoclip viewed and/or the transcript read during the introductory activity of this module can serve as a starting point to stimulate the discussion. Give participants 4-5 minutes to respond to the questions either individually or in pairs. Then, ask participants to share their responses to the questions with the whole group. During this sharing, you might consider creating clusters of factors (e.g., student related factors, teacher related factors, classroom factors, external factors, etc.) and listing them on a flipchart as participants share their ideas and examples

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<sup>2</sup> The abbreviation LO corresponds to the Learning Objectives included in the Teacher Module 1 on p.13.



or provide a table with such headings and ask teachers to fill it. Teachers could possibly refer to factors that support high level of mathematical thinking and reasoning, including but not limited to:

- Designing, selecting, and using appropriate/challenging tasks
- Clarifying the task without explaining it
- Demonstrating a solution method and holding back from telling students how to solve the task
- Listening to the students more (and avoid telling)
- Observing students before intervening
- Providing students with the means to monitor their progress and reflect upon the process
- Allowing sufficient time (not too much, not too little) for productive struggle on the task
- Building on students' prior knowledge
- Drawing frequent conceptual connections
- Modelling high level performance
- Pressing students for justification or explanations
- Teacher's good content knowledge and knowledge for teaching
- Teacher's good understanding of the task and its educative potential

Teachers could possibly refer to factors that inhibit this kind of thinking, including but not limited to:

- Designing or selecting and using less challenging/inappropriate tasks
- Routinizing the challenging parts of a task
- Shifting emphasis from meaning, concepts or understanding to correctness or completeness
- Determining and sharing a solution to a challenging task too early
- Allocating time inappropriately (either too little or too much) for solving a task
- Pressure/attempting to cover an overcrowded curriculum
- Classroom management problems
- Teachers' tendency to step in and do the thinking for the students
- Teachers' urgency to teach the "correct concepts" to the students and make attempts to ensure that nothing goes wrong
- Students' reluctance to work on challenging tasks
- Students' lack of accountability for high level products or processes
- Teachers' difficulty in handling students' multiple solutions,

## 2) Part 2: Mathematical Task Framework



**Indicative Duration:** 20 minutes

Teachers are provided with a visual representation of the MTF which outlines three main phases of task unfolding: first, as tasks appear in curriculum materials; next, as they are set up by the teacher during the lesson (task presentation); and finally, as they are actually implemented by the teacher and students; every phase is important for student learning, but it is actually the task enactment/implementation that is the more decisive for what students learn. This representation is accompanied by a short easy-to-read text explaining what the MTF is and how it can be used. Allow teachers 5-6 minutes to observe the figure and read the text. Then initiate a discussion by asking



teachers to describe and explain the figure and what they understood from reading the text. Link the factors that emerged from the brainstorming activity by asking them to verbally classify these factors within the different phases. You can ask teachers to consider the importance of the MTF for their work in terms of helping them better classify these factors. Several examples can be discussed; for instance, a high-level task as it appears in the student textbook might turn into low demanding task because the teacher told the students exactly how to answer the question during the task presentation phase or because the students felt uncomfortable with such tasks and decided to give up. As a teacher educator, you should try to keep the focus of the activity/discussion on how different teacher actions can result in maintaining or reducing the cognitive challenge of a mathematical task. The question following the MTF intends to help teachers engage in self-assessment of their teaching and identify on which teaching phase of the three depicted in the MTF they might need more support. This can help teachers decide which module they would like/need to do next (e.g., Module 2 is related more on lesson design, Modules 4 and 3 pertain to the task presentation phase and the task implementation phase, respectively, whereas Module 5 has to do with fostering a culture for engaging all students in mathematically challenging tasks). During this discussion, you should not press teachers to discuss the challenges they face; given this is the first Case of Practice, you should respect any hesitations they might have in sharing challenges they face during their instruction; you could, however, emphasize that several teachers encounter different challenges in selecting and enacting challenging tasks, since this is really hard work.

### Activity 1 - Focusing on Task Selection



**Indicative Duration:** 30 minutes

In this activity, participants are provided with four tasks (Tasks 1 and 4 are low level tasks and Tasks 2 and 3 are high level tasks; notice that Tasks 1 and 4 mostly ask students to recall certain names of given properties and to recall and implement given algorithms in figuring out the integrals). The tasks of this activity can be replaced by other tasks of your choice if you consider them more appropriate for the participants. Allow 5-8 minutes for participants to read these tasks and then try to classify them according to how mathematically challenging they are (low vs. high), considering the corresponding target student audience. Then, invite participants to share their responses; while sharing ask for additional comments or explanations from each spokesperson, as well as, the rest of the group. Participants may want to jot down key ideas. Possible answers for this activity:

- Teachers may consider Task 1 as a high level task because many students tend to confuse the names of those mathematical properties.
- Teachers may consider Tasks 2 and 3 as high or low level due to the use of representations/images.
- Teachers may relate the tasks to the time needed to be accomplished (e.g., Task 4 can be considered as a high level task if students have about a minute to solve it).
- Teachers may consider Tasks 2 and 4 as high level because they come from higher secondary grades compared to Tasks 1 and 3.



- Task 4 might be considered as complex because it involves more advanced mathematical content (i.e., integrals). The inclusion of more advanced mathematical content does not render a task more complex.

In each case, try to focus teachers’ attention on the more substantive rather than the superficial characteristics of the tasks. These include but are not limited to the list that is presented below. As teachers identify these characteristics, they can keep track of them. You can also list the shared features on flipchart paper or a slide of a power point presentation. Substantive features of challenging tasks:

- Require complex thinking
- Require students to provide explanations and justifications that focus on the underlying mathematical ideas
- Require students to explore and understand mathematical concepts and ideas
- Allow for multiple solutions
- Have a focus on developing deeper understanding
- Are not algorithmic or do not involve reproducing known facts, rules or definitions
- Can be solved using procedures with connections to concepts or representations
- Can be linked to a daily context that is related to students’ lives.

Notice that teachers might point out that without knowing students’ prior knowledge or students in general, such classification is difficult. Acknowledge this reality and build a link to Module 2-Case 3 which focuses exactly on this issue. However, for the time being, ask teachers to focus on an “average” student of a particular class, temporarily suspending issues and concerns related to the students’ readiness levels.

In the second guiding question of this activity, teachers are asked to consider some challenges that might prevent them from using such tasks in their teaching and consider ways in which they can handle such challenges. The table presented below can provide some ideas for facilitating the discussion around these issues. If you do not have time to work on it, you can skip this question and move to the next activity.

Challenges	Possible ways to tackle the challenges
<ul style="list-style-type: none"> <li>• Student textbooks or curriculum materials do not include any challenging tasks.</li> </ul>	<ul style="list-style-type: none"> <li>• You can find many challenging tasks online or you can modify tasks included in the student textbooks so that to make them more challenging. See Appendix 2 for some ideas proposed by Deborah Ball on remodeling tasks in mathematicd textbooks.</li> </ul>
<ul style="list-style-type: none"> <li>• Some task features may be misleading, e.g., an image or a representation might make us think that this task is challenging while it does not require high level thinking from the students.</li> </ul>	<ul style="list-style-type: none"> <li>• Reflect on the level or kind of mathematical thinking or reasoning that is required by the students. (Do students need to simply apply a procedure without emphasis on conceptual understanding? Do students need to reproduce previously learned facts or rules? Does it require multiple solutions? etc.)</li> <li>• Identify the “source” of the challenge. Remember the challenge needs to come from the mathematics and not from the language level of the problem.</li> </ul>
<ul style="list-style-type: none"> <li>• Some tasks could be challenging but they might include sub-questions that lead students step-by-step to tackle the challenge.</li> </ul>	<ul style="list-style-type: none"> <li>• If you think that these sub-questions or prompts do the thinking for the students, you might consider not using them or use them only as necessary with some students but not with all (see discussion about Task Enablers in the remaining four modules).</li> </ul>



## Activity 2 - Focusing on Task Implementation



**Indicative Duration:** 30 minutes

Teachers are provided with a short extract from the *Principles and Standards* of the National Council of Teachers of Mathematics (2000) which emphasizes the key role that teachers have not only in selecting mathematically challenging tasks but also on maintaining the challenge during task presentation and implementation. The main idea is that students who are in classrooms where challenging tasks are set up and enacted at high level are likely to have more opportunities for developing mathematical thinking and reasoning. The intention of this activity is for teachers to consider which of their decisions and actions have an impact on these students' opportunities. Towards this end, they will read a task from Geometry which comes from a Greek Grade-10 mathematics classroom and they will determine its level of demand (See Appendix 11 for the transcripts)<sup>3</sup>. Then, they will compare its level of demands in the curriculum materials with its demands during task presentation and enactment (autonomous work and whole-class discussion), as captured in three short videoclips which refer to the launching and the enactment of the task. Depending on time, you can make selections to present at least one clip from each category (task presentations, autonomous work, and/or whole-class discussion). Before viewing the videoclips, you can let the teachers read the context of the videoclips or you can briefly present it depending on the time available<sup>4</sup>. Also, encourage the teachers to watch the videoclips having a specific focus in mind: while watching the clips teachers should bear in mind the guiding questions that follow the videoclips' context description<sup>5</sup>. After the teachers watch the videoclips, invite them to reflect on what they have watched based on the guiding questions. Next, allow some time for teachers to work either individually or in pairs to identify and jot down some teacher actions that contribute to presenting and enacting the task at a challenging level (or alternatively at a less challenging level than that intended by the curriculum task). The table found in the teacher materials can help them organize their ideas. You can also create the same table on a flipchart paper or a power point slide and list the shared ideas.

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<sup>3</sup> In the activities that include a mathematical task, teacher educators should ask participants to begin the activity by solving the task. For instance, in Case of Practice 1 - Activity 2, it is necessary to have an idea about the different possibilities of quadrilaterals to be able to understand the situation.

<sup>4</sup> In the activities that include introduction and discussion of videoclips, before watching a videoclip, teachers should be given a couple of minutes to read the task and determine its level of demand, the learning objectives it serves, and/or its mathematically challenging parts. This would help them to compare its level of demands in the curriculum materials with its demands during task enactment.

<sup>5</sup> In these activities, teachers should read the guiding questions that follow the videoclip's context description before watching the clips in order to have a specific focus in mind while watching it. An open-ended question should be posed by the teacher educator after watching a video clip instead of a close-ended one.



Possible teacher actions to *maintain the challenge* include but are not limited to:

Launching	Autonomous Work	Whole Class Discussion
<ul style="list-style-type: none"> <li>Asking students to explain in their own words what the task is asking them to do rather than the teacher doing this work</li> <li>Asking students to tell to a partner what the task is asking them to do</li> <li>Giving students time to read and understand a task.</li> </ul>	<ul style="list-style-type: none"> <li>Avoiding doing the thinking for students</li> <li>Asking students to explain what they did and what is causing them the difficulty</li> <li>Asking students questions rather than providing answers</li> <li>Learning to be complacent with student struggle—it is important for student learning</li> </ul>	<ul style="list-style-type: none"> <li>Being purposeful as to what can be shared and in what order</li> <li>Asking students to draw connections between the different ideas shared</li> </ul>

Possible teacher actions that result in *decreasing the challenge* include but are not limited to:

Launching	Autonomous Work	Whole Class Discussion
<ul style="list-style-type: none"> <li>Providing too many/guided explanations when presenting the task</li> <li>Explicitly/Implicitly ask students to work in a particular way or provide specific answers</li> <li>Putting special emphasis on finding the correct answers or reaching to a conclusion as fast as possible</li> </ul>	<ul style="list-style-type: none"> <li>Posing too guided questions</li> <li>Providing only one method or one type of material for solving the task</li> <li>Not encouraging students to find multiple solutions</li> <li>Teacher providing of explanations instead of asking for explanations from the students</li> <li>Providing negative feedback to incorrect or incomplete solutions</li> <li>Giving a “hint” that trivializes the challenge</li> </ul>	<ul style="list-style-type: none"> <li>Starting from presenting the correct solution</li> <li>Presenting only one correct solution</li> <li>Presenting solutions without asking students to explain, justify, or comment on their thinking</li> <li>Not asking students to compare multiple solutions</li> <li>Showing that incorrect or incomplete solutions are not appreciated/accepted</li> </ul>

## Connections to My Practice



**Indicative Duration:** 15 minutes

Because this is the first time that teachers will be asked to videotape (parts of) a lesson they teach and reflect upon, time needs to be dedicated to discussing some of the *logistics involved in videotaping*. In particular:

- Before videotaping in their classroom, teachers need to ensure that the videorecording has been approved by the National Institute Review Board (IRB) and the school principal and that active consent has been given by parents and children as appropriate. Children for whom consent has not been given should be seated outside the range of the camera.
- Teachers should avoid using children’s surnames during a lesson that is videotaped.
- Where teachers are responsible for videotaping:
  - ✓ If they don’t have a video camera, camera stand and/or lavalier microphone available in their school, they need to inform the video-club leader. They will also need an SD card for storing the recorded lesson before it is uploaded to the project server.
  - ✓ They need to arrange for a colleague to videotape one *typical* mathematics lesson (if applicable, they need to check that this person has been police vetted according to relevant school



procedures or national legislation). The video-recorder needs to be reminded about the confidential nature of all aspects of what they observe in the course of video recording.

- ✓ They need to ask the video recorder to set themselves in position in the classroom where they have a panoramic view of the room. When the lesson starts, the videorecorder needs to follow the teacher wherever they are in the room. In general the scene should be shot at a medium distance. However, when material on the board or in a child's copy is central to the lesson, the focus should be on that.
- ✓ After the next meeting of the video club, they need to delete all copies of the recording (from the camera itself, from the SD card etc.).
- If teachers are responsible for transferring/uploading the lesson:
  - ✓ After the lesson has been recorded, the recording should be transferred by "WeTransfer" to the e-mail address that is specified by the video-club leader from where it will be uploaded to the Project server. Uploading the video should be done *at least one week before the next meeting* of the video club.
- In general, teachers should specify a couple of short clips (typically 3-5 minutes each) they believe capture the focus of the current case and write a short note stating why they chose that particular clip.

For the activity of this particular case of practice, make sure that by the end of the session teachers are aware of the terms challenging tasks, task launching, student autonomous work, and whole-class discussion. If the latter three terms are not clear, please spend some time to briefly explain them. If teachers cannot locate a challenging task from their curriculum materials or the internet, they can take a less challenging task and modify it accordingly to make it challenging. Then, give directions as to what needs to be videotaped, how the setting of the videotaping is going to be, and what the focal points are.

### Closing Activity



**Indicative Duration:** 5 minutes

This activity, to be carried out individually, is meant to function as an evaluation to give you information about either what teachers have gained in this session or what issues and challenges they still face. Notice that, as in the introductory activity of this module, teachers might situate their instruction as a sequence of points rather than as a single point. If possible, it would also be interesting to direct the teachers to illustrate their initial thinking about their instruction in these four quadrants.

### Key Take-away Points of the Case of Practice 1



- The Mathematical Task Framework points to the importance of attending to the mathematical challenge of tasks at different phases, from their selection, to their presentation, and to their enactment with the students. Selecting mathematically challenging tasks is important, but, in fact, it is the latter phase (task enactment) that largely determines the opportunities for student thinking and reasoning.
- Several features can contribute toward rendering a task mathematically challenging. Although not enough in and of themselves to make a task mathematically challenging, these features include: (a) asking students to explain/justify their thinking; (b) looking for patterns and generalizations; (c) asking students to link different representations; (d) asking students to consider/discuss the underlying meaning of mathematical procedures rather than just focus on the execution of these procedures; (e) supporting students to contextualize key mathematical ideas (especially for lower-grade students).
- Some teacher actions that contribute to maintaining the mathematical challenge involve:
  - During launching: asking students to explain in their own words what the task is asking them to do rather than the teacher doing this work; asking students to tell to a partner what the task is asking them to do; giving students time to read and understand a task.
  - During student autonomous work: avoid doing the thinking for students; asking students to explain what they did and what is causing them the difficulty; asking students questions rather than providing answers; learn to accept student struggle—it is important for student learning.
  - During whole-class discussion: be purposeful as to what can be shared and in what order; ask students to draw connections between the different ideas shared.

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