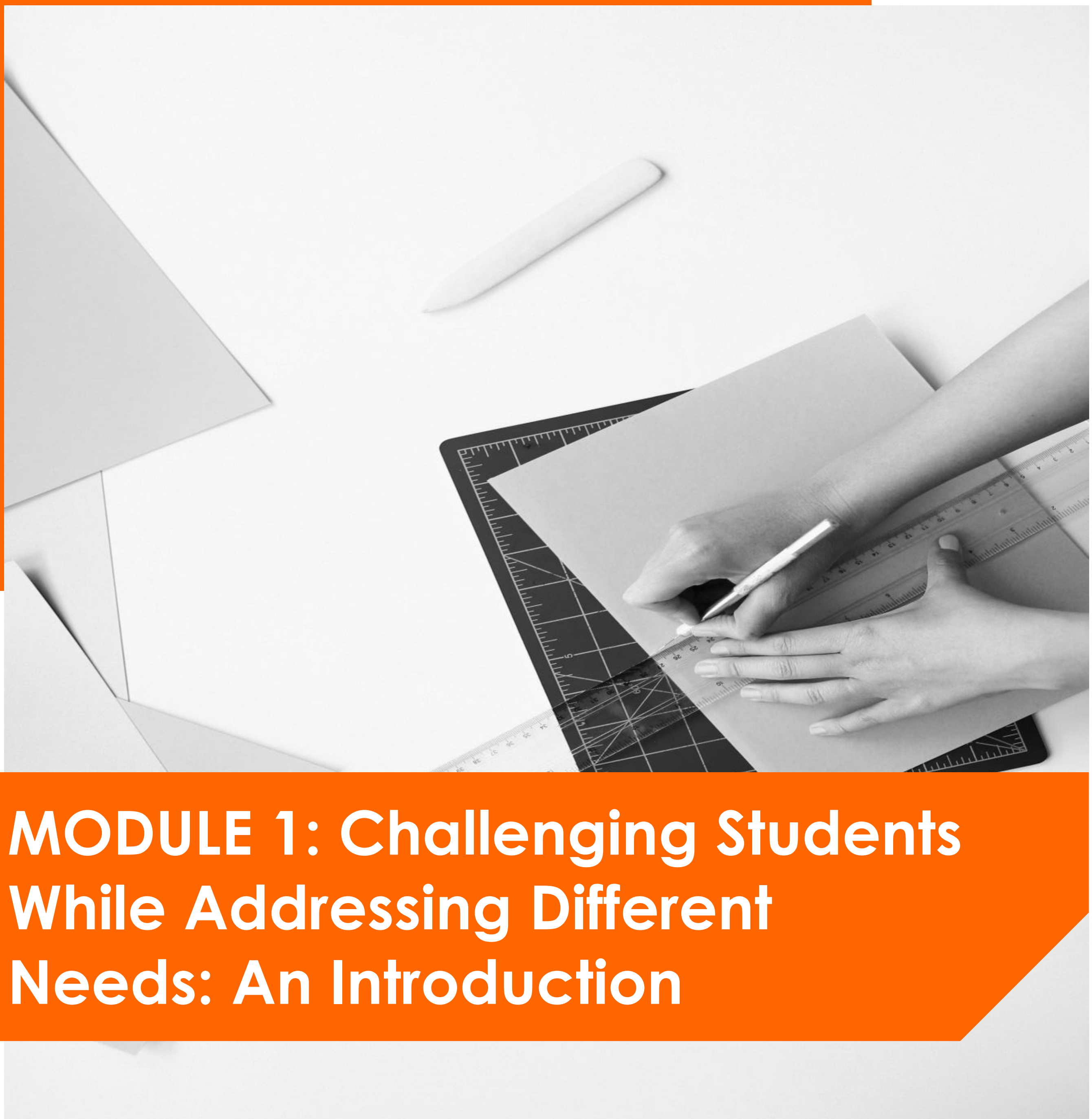

MODULE 1: Challenging Students While Addressing Different Needs: An Introduction

EDUCATE Project

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University of Cyprus
Marino Institute of Education and Trinity College Dublin
National and Kapodistrian University Athens
Instituto de Educação, Universidade de Lisboa
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SYMBOLS

Next to each activity there is one or more of the following symbols:

Individual work

Video-club setting

Read

Write or complete

Link-to-File

Watch

Reflect

Discuss

Learning Objectives

Plan

Assess

CASE OF PRACTICE 1

Focusing on Mathematically Challenging Tasks

Overview

CONTACT HOURS	2 hours
TYPE OF RESOURCES	Videoclips; Tasks; the Mathematical Task Framework
EMPHASES	Discussing how task unfolding can offer different learning opportunities for students

Activities

Opening Activity

(1) <u>Brainstorming Activity</u>

- Based on the videos you observed in the introductory activity of this module, what do you think we can do, as teachers, to create a productive space for engaging students in mathematical thinking and reasoning? What do we (often inadvertently) do that might hinder such attempts?

(2) There are several ways in which we, as teachers, might create or lessen students' opportunities to engage in mathematical thinking and reasoning. A group of U.S. researchers has proposed the *Mathematical Task Framework* (hereafter called MTF) to help us better classify these ways and through that make more deliberate and informed decisions about the opportunities we create for our students' thinking. Look at the figure below and read the brief introduction to the MTF that appears below; then consider the question that follows.

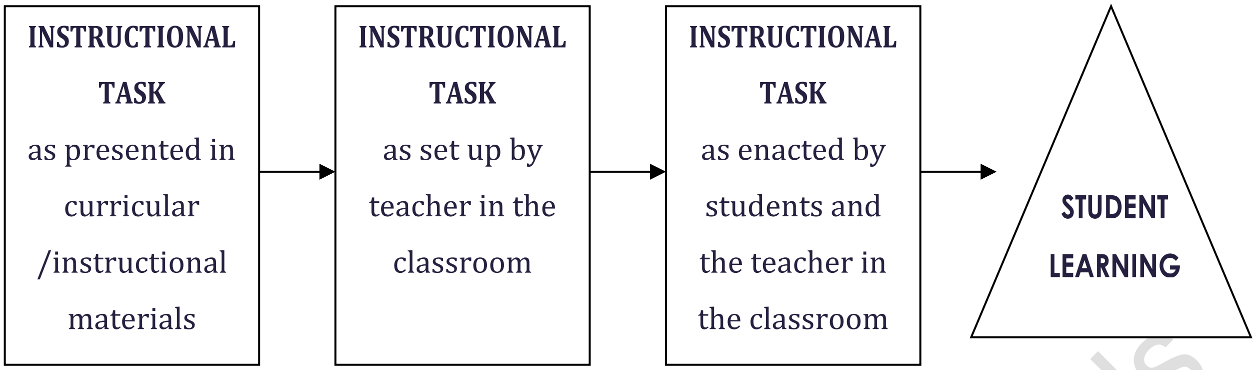

Fig.1. *The Mathematical Task Framework (adapted from Stein et al., 2000).*

About MTF: What Does it Tell Us and How Can it Be Used?

What does the MTF suggest? According to the MTF, instructional tasks pass through three stages: first, as they are presented in curriculum materials or in the handouts that the teacher prepares for her/his students (task selection); second, as they are set up by the teacher in the classroom during the launching (task presentation) of the task; and third, as they are enacted/implemented during the lesson, while the students and the teacher interact while solving these tasks (task implementation). Figure 1 captures these phases of task unfolding, emphasizing that what ultimate determines student learning is not only the *selection of mathematically challenging tasks*, but *how these tasks unfold during instruction*.

How can MTF be utilized? Over the past years, MTF has been used both as a research tool to examine instructional quality with respect to task unfolding but also as a professional development tool to sensitize teachers to the importance of attending to how the challenging aspects of a task might be altered during instruction, especially during the phases of task presentation and enactment/implementation.

Reflecting on your previous lessons, in which area(s)–(a*) task selection*, (b) *task presentation*, and (c) *task enactment*–do you feel that you face more difficulties when trying to enhance your students’ opportunities to engage in mathematically challenging work? Why do you think so?

- If you are a prospective teacher, based on the experiences you might have, in which of those areas do you anticipate that you might face more difficulties? Why do you think so?

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The activities that follow will provide you with opportunities to discuss how different decisions we make as teachers, during the phases of task selection, presentation, and enactment can create different opportunities for student learning.

Activity 1 – Focusing on Task Selection

In this activity you will be exposed to different tasks. Read them carefully and then consider the questions that follow.

Task 1 (Grade 9):

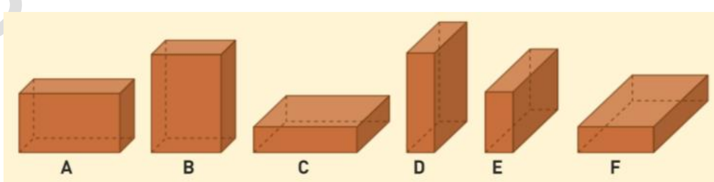
Match the following rule to its correct name:

1.	$a + b = b + a$	a.	Identity property for multiplication
2.	$(a + b) + c = a + (b + c)$	b.	Commutive property of addition
3.	$a(b + c) = ab + ac$	c.	Transitive property
4.	$a + 0 = a$	d.	Associative property of addition
5.	$a(1) = a$	e.	Identity property for addition
6.	If $a = b$, and $b = c$, then $a = c$	f.	Distributive property

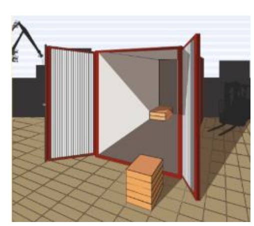
Source: Task Sorting Activity (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004, p. 71)

Task 2 (Grade 10):

Consider one container with a width of 2m, a length of 4m and a height of 2,5m to transport boxes with the shape of parallelepiped and with the following dimensions: length 70cm; width 50cm and height 30cm. Suppose that the boxes can be introduced in the container in any position, as the figure shows:



(a) If all boxes are packed as in position C, investigate the maximum number of boxes that it is possible to put into the container. Show how you got your answer.

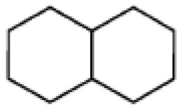


(b) If all boxes are packed in the same position inside the container, investigate which of the given positions A, B, C, D, E or F should we chose to transport the maximum number of boxes. Show how you got your answer.

Task 3 (Grade 7):

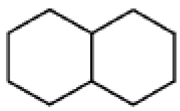
Solve each question *using the given pattern blocks*.

Find 1/2 of 1/3. Use pattern blocks. Draw your answer.



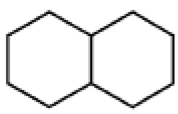
1/2 of 1/3 or 1/2 × 1/3 = _______

Find 1/3 of 1/4. Use pattern blocks. Draw your answer.



1/3 of 1/4 or 1/3 × 1/4 = _______

Find 1/4 of 1/3. Use pattern blocks. Draw your answer.



1/4 of 1/3 or 1/4 × 1/3 = _______

Source: Task Sorting Activity (Smith, Stein, Arbaugh, Brown, & Mossgrove, 2004, p. 67, adapted)

Task 4 (Grade 12):

Given that

$$\int a\,dx = ax + c, \qquad \forall a \in \mathbb{R}$$

$$\int x^r\,dx = \frac{x^{r+1}}{r+1} + c\,, \qquad \forall r \in \mathbb{R} - \{-1\}.$$

Figure out the following

$$\int 4\,dx$$
$$\int -\pi\,dx$$
$$\int x^4\,dx$$
$$\int x^{1000}\,dx$$
$$\int x^{-3}\,dx$$

Source: http://archeia.moec.gov.cy/sm/271/mathimatika_c_lyk_kk_a_tefchos.pdf (adapted)

Consider these four tasks and try to classify them according to how mathematically challenging they are (low vs. high), taking into account the corresponding target student audience.

Task	Level of Challenge (Low vs. High)
1	
2	
3	
4	

Discuss with your colleagues:

- What makes a task mathematically challenging?
- What challenges might you encounter in your practice inwhen selecting such tasks for your teaching? How might you and/or your colleagues tackle these challenges?

Activity 2 – Focusing on Task Implementation

Almost twenty years ago, the National Council of Teachers of Mathematics (NCTM) in the USA has recognized the key role that teachers have not only in selecting mathematically challenging tasks (or using such tasks from their textbooks/curriculum materials), but mostly in how they interact with these tasks with their students. In particular, NCTM (2000) noted:

> "*Worthwhile tasks alone are not sufficient for effective teaching. Teachers must also decide what aspects of a task to highlight, how to organize and orchestrate the work of the students, what questions to ask to challenge those with varied levels of expertise, and how to support students without taking over the process of thinking for them, and thus, eliminating the challenge*" (p. 19).

In this activity, we will consider how different teacher actions during task presentation and implementation might shape the opportunities afforded to students for mathematical thinking and reasoning. Toward this end, we will consider a task and discuss its enactment in the class.

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Read carefully the following task and determine its level of challenge (low vs. high).

MATHEMATICAL TASK

Geometry: The 'Quadrilaterals from Midpoints' Task

Task 1

In Figure 1, the points D, F and E are midpoints of the sides of the triangle ABC.

(a) Investigate what type of quadrilateral is the BDFE.
(b) Study how the quadrilateral BDFE changes when the triangle ABC changes. That is how the type of quadrilateral is connected to the type of the triangle.

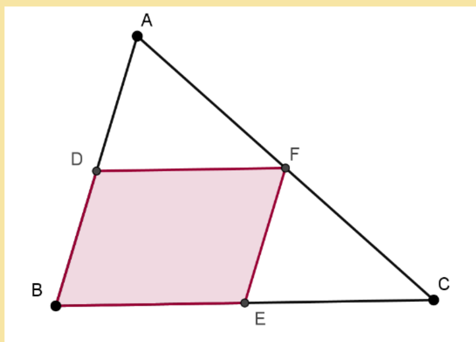

Figure 1

Watch the following video clips which refer to the launching and the enactment (student autonomous work and whole class discussion) of the task shown above.

Video clip
Context: We will look at a lesson from a Grade-10 class in Greece. In this lesson students are asked to draw on a specific theorem they studied in a previous lesson (i.e., the straight-line segment that connects the midpoints of two sides of a triangle is parallel to the third side and equal to its half), to determine the relation between the types of the triangle and its inscribed quadrilateral (see Figure 1 above). In the first part, students identified that based on the theorem, BDFE ought to be a parallelogram. In the second part, which is the focus of the video clips we will be observing, students are investigating the relationship between the type of triangle and its inscribed quadrilateral. A computer with Geogebra is available to be used by the students when they need it. We will watch three clips, one related to the

teacher's launching of the task, one related to students' autonomous work, and a third pertaining to the whole-class discussion.

Discuss with your colleagues:

- What is the level of challenge of the task, as presented in the teacher-made materials?
- Is the challenge maintained or is it modified during the unfolding of this task?
- What are the teacher actions that contribute to the maintenance or the change in the mathematical challenge each time?

Based on your discussion above, working with your colleagues:

Identify some teacher's actions that contribute to presenting and enacting the task at a mathematically challenging level.

Launching	Autonomous Work	Whole Class Discussion

Connections to (my) Practice

For our next meeting:

Select a mathematically challenging task from your textbook/curriculum materials that is included in one of the lessons you're expected to teach.

Work on this task with your students and videotape its presentation and implementation (student autonomous work and whole class discussion).

Before our next meeting, watch your videotaped lesson, and consider the level of challenge during its presentation and implementation.

Select two short excerpts (from task launching, student autonomous work, or whole-class discussion) that you would like to share with your colleagues. These excerpts should be illustrating either instances in which the mathematical challenge was maintained or instances in which it changed.

Closing Activity

Revisit the four-quadrant diagram in the introductory activity of this module, and consider where you will see your teaching being situated ***during your next lessons***:

- If you situated it in a different spot compared to that of the introductory activity, jot down two things that you learned that helped you make this (even small) shift.
- If you situated it in more or less the same spot, jot down two things you would like to learn in next meetings that you envision helping you making a more substantive shift.